

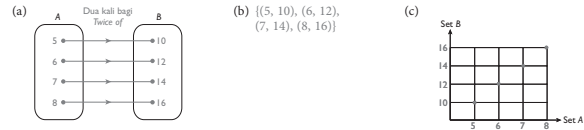
1.1 Hubungan

A. Wakilkan setiap hubungan yang berikut dalam bentuk (a) gambar rajah anak panah, (b) pasangan bertertib, dan (c) graf Cartes.

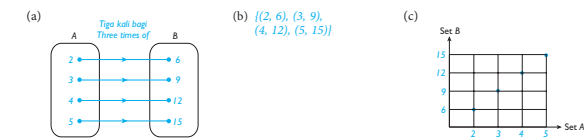
CONTOH

Diberi set $A = \{5, 6, 7, 8\}$ dan set $B = \{10, 12, 14, 16\}$. Hubungan daripada set A kepada set B ialah 'dua kali bagi'.

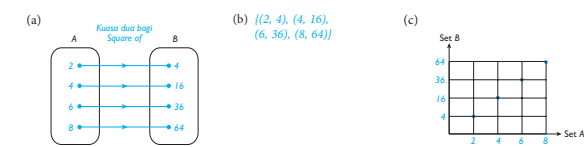
Given set $A = \{5, 6, 7, 8\}$ and set $B = \{10, 12, 14, 16\}$. The relation from set A to set B is 'twice of'.



1. Diberi set $A = \{2, 3, 4, 5\}$ dan set $B = \{6, 9, 12, 15\}$. Hubungan daripada set A kepada set B ialah 'tiga kali bagi'.

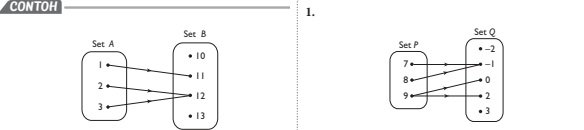


2. Diberi set $A = \{2, 4, 6, 8\}$ dan set $B = \{4, 16, 36, 64\}$. Hubungan daripada set A kepada set B ialah 'kuasa dua bagi'.



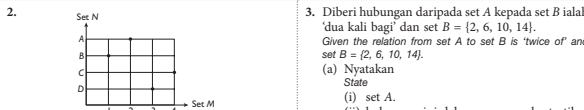
D. Selesaikan setiap yang berikut.

CONTOH



Nyatakan State
 (a) jenis hubungan ini.
 (b) julat hubungan ini.
 (c) imej bagi 2.
 (d) objek bagi 12.

- (a) Hubungan banyak kepada satu
- (b) $\{11, 12\}$
- (c) 12
- (d) 2 dan 3

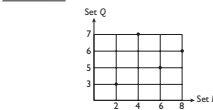


Nyatakan State
 (a) hubungan ini dalam bentuk pasangan bertertib.
 (b) jenis hubungan ini.
 (c) imej bagi 3.
 (d) domain hubungan ini.

- (a) $\{(1, B), (2, A), (3, D), (4, C)\}$
- (b) Hubungan satu kepada satu
- (c) D
- (d) $\{1, 2, 3, 4\}$

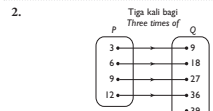
B. Nyatakan domain, kodomain, objek, imej dan julat bagi setiap hubungan yang berikut.

CONTOH



- (a) Domain / Domain = $\{2, 4, 6, 8\}$
- (b) Kodomain / Codomain = $\{3, 5, 6, 7\}$
- (c) Objek / Object = 2, 4, 6, 8
- (d) Imej / Image = 3, 5, 6, 7
- (e) Imej bagi 6 / Image of 6 = 5
- (f) Julat / Range = $\{3, 5, 6, 7\}$

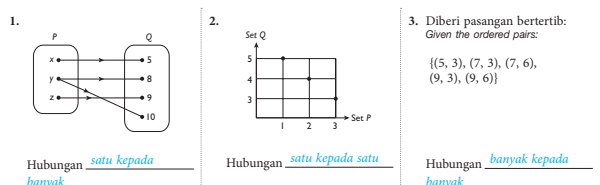
1. Diberi pasangan bertertib:
 Given the ordered pairs:
 $\{(9, 12), (10, 13), (11, 14), (12, 15)\}$
- (a) Domain / Domain = $\{9, 10, 11, 12\}$
 - (b) Kodomain / Codomain = $\{12, 13, 14, 15\}$
 - (c) Objek / Object = 9, 10, 11, 12
 - (d) Imej / Image = 12, 13, 14, 15
 - (e) Imej bagi 10 / Image of 10 = 13
 - (f) Objek bagi 15 / Object of 15 = 12
 - (g) Julat / Range = $\{12, 13, 14, 15\}$



- (a) Domain / Domain = $\{3, 6, 9, 12\}$
- (b) Kodomain / Codomain = $\{9, 18, 27, 36, 39\}$
- (c) Objek / Object = 3, 6, 9, 12
- (d) Imej / Image = 9, 18, 27, 36
- (e) Objek bagi 27 / Object of 27 = 9
- (f) Julat / Range = $\{9, 18, 27, 36\}$

3. Diberi hubungan antara set $A = \{13, 15, 26, 34\}$ dengan set $B = \{4, 6, 7, 8\}$ ditakrifkan sebagai 'hasil tambah digit'.
- Given the relation between set $A = \{13, 15, 26, 34\}$ and set $B = \{4, 6, 7, 8\}$ is defined by 'sum of digits'.
- (a) Domain / Domain = $\{13, 15, 26, 34\}$
 - (b) Kodomain / Codomain = $\{4, 6, 7, 8\}$
 - (c) Objek / Object = 13, 15, 26, 34
 - (d) Imej / Image = 4, 6, 7, 8
 - (e) Imej bagi 26 / Image of 26 = 8
 - (f) Objek bagi 7 / Object of 7 = 34
 - (g) Julat / Range = $\{4, 6, 7, 8\}$

C. Nyatakan jenis hubungan antara set P dan set Q bagi setiap yang berikut.



1.2 Fungsi

A. Tulis semua jenis hubungan dan tunjukkan setiap hubungan itu dengan gambar rajah anak panah.

Write down all types of relations and illustrate each relation with an arrow diagram.

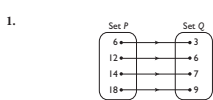
Jenis hubungan Type of relation	Gambar rajah anak panah Arrow diagram
Satu kepada satu	
Satu kepada banyak	
Banyak kepada satu	
Banyak kepada banyak	

FAKTA UTAMA

- Fungsi ialah hubungan yang setiap objeknya mempunyai hanya satu imej.
- A function is a relation where each of its objects has only one image.
- Hanya hubungan satu kepada satu dan hubungan banyak kepada satu ialah fungsi.
- Only one-to-one relations and many-to-one relations are functions.
- (a) Hubungan satu kepada satu One-to-one relation
- (b) Hubungan banyak kepada satu Many-to-one relation

B. Tulis setiap fungsi dengan menggunakan tatatanda fungsi. Seterusnya, jawab soalan yang berikut.

Write each function by using function notation. Hence, answer the following questions.



- (a) Nyatakan julat fungsi ini.
- (b) Jika 10 ialah suatu objek dalam set P, apakah imej yang sepadan dalam set Q?
- (c) Jika 10 ialah suatu imej dalam fungsi ini, nyatakan objeknya.

- Tatatanda fungsi: $f: x \rightarrow \frac{1}{2}x$
- (a) $\{3, 6, 7, 9\}$
- (b) 5
- (c) 20

2. Diberi fungsi:
 Given a function:
 $\left\{ \left(\frac{1}{3}, 3 \right), \left(\frac{1}{5}, 5 \right), \left(\frac{1}{8}, 8 \right), \left(\frac{1}{10}, 10 \right) \right\}$
- (a) Nyatakan objek fungsi ini.
 - (b) Bolehkah 0 menjadi objek dalam fungsi ini?
 - (c) Apakah imej bagi 4 jika 4 ialah suatu objek dalam fungsi ini?
- Tatatanda fungsi: $f: x \rightarrow \frac{1}{x}$
- (a) $\frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}$
 - (b) 0 tidak boleh menjadi objek dalam fungsi ini kerana imejnya tidak wujud.
 - (c) $\frac{1}{4}$

C. Bagi setiap fungsi berikut, cari imej bagi objek x yang diberikan.
For each of the following functions, find the images for the given objects x.

24

CONTOH

$f(x) = x^2 - 1$; $x = 3, -5$

$f(3) = 3^2 - 1 = 8$ ← Gantikan x dengan 3.

$f(-5) = (-5)^2 - 1 = 24$ ← Gantikan x dengan -5.

2. $f(x) = \frac{x}{3} + 6$; $x = 9, -12$

$f(9) = \frac{9}{3} + 6 = 9$

$f(-12) = \frac{-12}{3} + 6 = 2$

1. $f(x) = 2x + 5$; $x = 3, 4$

$f(3) = 2(3) + 5 = 11$

$f(4) = 2(4) + 5 = 13$

3. $f(x) = 10 - 4x$; $x = 3, -2$

$f(3) = 10 - 4(3) = -2$

$f(-2) = 10 - 4(-2) = 18$

D. Bagi setiap fungsi berikut, cari objek bagi imej yang diberikan.
For each of the following functions, find the objects for the given images.

24

CONTOH

$f(x) = 3x + 5$; Imej / Images = 4, x

$f(x) = 4$
 $3x + 5 = 4$
 $3x = -1$
 $x = -\frac{1}{3}$

$f(x) = x$
 $3x + 5 = x$
 $2x = -5$
 $x = -\frac{5}{2}$

Jika imej sesuatu fungsi ialah x, maka $f(x) = x$.

2. $f(x) = 4 - x$; Imej / Images = 6, -5

$f(x) = 6$
 $4 - x = 6$
 $x = -2$

$f(x) = -5$
 $4 - x = -5$
 $x = 9$

1. $f(x) = 3x + 2$; Imej / Images = 2, -4

$f(x) = 2$
 $3x + 2 = 2$
 $3x = 0$
 $x = 0$

$f(x) = -4$
 $3x + 2 = -4$
 $3x = -6$
 $x = -2$

3. $f(x) = x^2 - 3$; Imej / Images = 4, 2x

$f(x) = 4$
 $x^2 - 3 = 4$
 $x^2 = 7$
 $x = \pm\sqrt{7}$

$f(x) = 2x$
 $x^2 - 3 = 2x$
 $x^2 - 2x - 3 = 0$
 $(x - 3)(x + 1) = 0$
 $x = 3, -1$

E. Selesaikan masalah yang berikut.
Solve the following problems.

23/24

CONTOH

Diberi fungsi $f(x) = 2x - 4$, cari
Given the function $f(x) = 2x - 4$, find
(a) $f(3)$.

(b) nilai x jika imejnya ialah 3x.
the value of x if the image is 3x.

(a) $f(3) = 2(3) - 4 = 2$

(b) $f(x) = 3x$
 $2x - 4 = 3x$
 $-4 = 3x - 2x$
 $x = -4$

1. Diberi fungsi $f(x) = 3x + 5$, cari
Given the function $f(x) = 3x + 5$, find

(a) objek jika imejnya ialah 8,
the object if the image is 8.

(b) nilai p jika $f(p) = 11$,
the value of p if $f(p) = 11$.

(a) $f(x) = 8$
 $3x + 5 = 8$
 $3x = 3$
 $x = 1$

(b) $f(p) = 11$
 $3p + 5 = 11$
 $3p = 6$
 $p = 2$

2. Satu fungsi ditakrifkan sebagai $f(x) = 4x + m$.
A function is defined by $f(x) = 4x + m$.

(a) Cari nilai m jika $f(2) = 16$.
Find the value of m if $f(2) = 16$.

(b) Seterusnya, cari nilai x jika $f(x) = 0$.
Hence, find the value of x if $f(x) = 0$.

(a) $f(2) = 16$
 $4(2) + m = 16$
 $8 + m = 16$
 $m = 8$

(b) $f(x) = 0$
 $4x + 8 = 0$
 $4x = -8$
 $x = -2$

3. Diberi dua fungsi, $f(x) = x + 3$ dan $g(x) = 2x - 6$.
Given two functions, $f(x) = x + 3$ and $g(x) = 2x - 6$. Find the value of x if

(a) $f(x) = g(2)$.

(b) $f(x) = g(x)$.

(a) $f(x) = g(2)$
 $x + 3 = 2(2) - 6$
 $x + 3 = 2x - 6$
 $x - 2x = -6 - 3$
 $-x = -9$
 $x = 9$

(b) $f(x) = g(x)$
 $x + 3 = 2x - 6$
 $x - 2x = -6 - 3$
 $-x = -9$
 $x = 9$

F. Selesaikan masalah yang berikut.
Solve the following problems.

23/24

CONTOH

Fungsi f ditakrifkan sebagai $f: x \rightarrow mx + n$.
Jika $f(2) = 10$ dan $f(4) = 16$, cari nilai m dan nilai n.
A function f is defined as $f: x \rightarrow mx + n$. If $f(2) = 10$ and $f(4) = 16$, find the values of m and n.

$f(2) = 10$
 $2m + n = 10$ ①

$f(4) = 16$
 $4m + n = 16$ ②

② - ①: $2m = 6$
 $m = 3$

Gantikan $m = 3$ ke dalam ①.

$2(3) + n = 10$
 $6 + n = 10$
 $n = 4$

1. Fungsi f ditakrifkan sebagai $f: x \rightarrow ax + b$.
Jika $f(2) = 0$ dan $f(4) = 6$, cari nilai a dan nilai b.
A function f is defined as $f: x \rightarrow ax + b$. If $f(2) = 0$ and $f(4) = 6$, find the values of a and b.

$f(2) = 0$
 $2a + b = 0$ ①

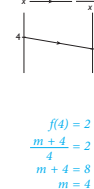
$f(4) = 6$
 $4a + b = 6$ ②

② - ①: $2a = 6$
 $a = 3$

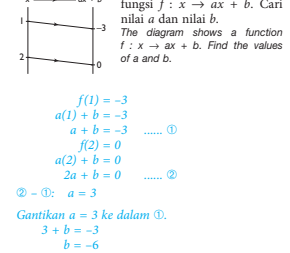
Gantikan $a = 3$ ke dalam ①.

$2(3) + b = 0$
 $6 + b = 0$
 $b = -6$

2. Rajah di sebelah menunjukkan satu fungsi:
The diagram shows a function:
 $f: x \rightarrow \frac{m+x}{x}, x \neq 0$
dengan m ialah pemalar. Cari nilai m.
where m is a constant. Find the value of m.



3. Rajah di sebelah menunjukkan fungsi $f: x \rightarrow ax + b$.
Cari nilai a dan nilai b.
The diagram shows a function $f: x \rightarrow ax + b$. Find the values of a and b.



1.3 Fungsi Gubahan

23/24/25/26/27/28

A. Cari fungsi gubahan berdasarkan fungsi f dan fungsi g yang diberikan.
Find the composite functions based on the given functions f and g.

31/32

CONTOH

$f: x \rightarrow 2x + 3$
 $g: x \rightarrow x^2 + 1$

$fg(x) = f(x^2 + 1) = 2(x^2 + 1) + 3 = 2x^2 + 2 + 3 = 2x^2 + 5$

$gf(x) = g(2x + 3) = (2x + 3)^2 + 1 = 4x^2 + 12x + 9 + 1 = 4x^2 + 12x + 10$

$fg(3) = f(3^2 + 1) = f(10) = 2(10) + 3 = 23$

Cara Lain
 $fg(3) = 2(3^2 + 1) + 3 = 2(10) + 3 = 23$

1. $f: x \rightarrow x + 4$
 $g: x \rightarrow 2x + 5$

$fg(x) = f(2x + 5) = 2x + 5 + 4 = 2x + 9$

$gf(x) = g(x + 4) = 2(x + 4) + 5 = 2x + 13$

$gf(1) = g(1 + 4) = g(5) = 2(5) + 5 = 15$

2. $f: x \rightarrow 2x - 4$
 $g: x \rightarrow 3x$

$fg(x) = f(3x) = 2(3x) - 4 = 6x - 4$

$gf(x) = g(2x - 4) = 3(2x - 4) = 6x - 12$

$gf(2) = 6(2) - 4 = 12 - 4 = 8$

3. $f: x \rightarrow 3 - x$
 $g: x \rightarrow x^2$

$f^2g(x) = f^2(x^2) = ff(x^2) = f(3 - x^2) = 3 - (3 - x^2) = x^2$

$gf^2(x) = gf(f(x)) = gf(3 - x) = g(3 - (3 - x)) = g(x) = x^2$

$gf(-2) = g(3 - (-2)) = g(5) = 25$

4. $f: x \rightarrow \frac{x+3}{x-2}, x \neq 2$
 $g: x \rightarrow \frac{x}{9}$

$fg(x) = f(\frac{x}{9}) = \frac{\frac{x}{9} + 3}{\frac{x}{9} - 2} = \frac{\frac{x+27}{9}}{\frac{x-18}{9}} = \frac{x+27}{x-18}, x \neq 18$

$gf(x) = g(\frac{x+3}{x-2}) = \frac{\frac{x+3}{x-2}}{9} = \frac{x+3}{9(x-2)}, x \neq 2$

$gf(6) = g(\frac{6+3}{6-2}) = g(\frac{9}{4}) = \frac{9}{4}$

5. $f: x \rightarrow \frac{x}{2} + 4$
 $g: x \rightarrow \frac{2}{x}, x \neq 0$

$f^2g(x) = f^2(\frac{2}{x}) = ff(\frac{2}{x}) = f(\frac{\frac{2}{x}}{2} + 4) = f(\frac{1}{x} + 4) = \frac{\frac{1}{x} + 4}{2} + 4 = \frac{1 + 12x}{2x}, x \neq 0$

$g^2f(x) = g^2(\frac{x}{2} + 4) = g(\frac{\frac{x}{2} + 4}{2}) = g(\frac{x+8}{4}) = \frac{4}{x+8}$

$fg(4) = f(\frac{2}{4}) = f(\frac{1}{2}) = \frac{1}{2} + 4 = 4\frac{1}{2}$

7

B. Cari nilai x bagi setiap fungsi gubahan yang berikut.
Find the value of x for each of the following composite functions.

32

CONTOH

Diberi: $f(x) = 2x + 5$ dan $g(x) = 3x - 2$
Given: $f(x) = 2x + 5$ and $g(x) = 3x - 2$
Cari nilai x jika $gf(x) = 1$.
Find the value of x if $gf(x) = 1$.

$gf(x) = 1$
 $g(2x + 5) = 1$
 $3(2x + 5) - 2 = 1$
 $6x + 15 - 2 = 1$
 $6x - 12 = 1$
 $6x = 13$
 $x = \frac{13}{6}$

1. Diberi: $f(x) = x + 6$ dan $g(x) = 3x + 4$
Given: $f(x) = x + 6$ and $g(x) = 3x + 4$
Jika $gf(x) = 1$, cari nilai x.
If $gf(x) = 1$, find the value of x.

$gf(x) = 1$
 $g(x + 6) = 1$
 $3(x + 6) + 4 = 1$
 $3x + 18 + 4 = 1$
 $3x = -21$
 $x = -7$

2. Diberi: $f(x) = 3 - x$ dan $g(x) = 2x + 6$
Given: $f(x) = 3 - x$ and $g(x) = 2x + 6$
Jika $fg(x) = -9$, cari nilai x.
If $fg(x) = -9$, find the value of x.

$fg(x) = -9$
 $f(2x + 6) = -9$
 $3 - (2x + 6) = -9$
 $-2x - 3 = -9$
 $2x = 6$
 $x = 3$

3. Diberi: $f(x) = 3x + 2$
Given: $f(x) = 3x + 2$
Cari nilai x jika $f^2(x) - 26 = 0$.
Find the value of x if $f^2(x) - 26 = 0$.

$f^2(x) - 26 = 0$
 $ff(3x + 2) - 26 = 0$
 $3(3x + 2) + 2 - 26 = 0$
 $9x + 6 + 2 - 26 = 0$
 $9x = 18$
 $x = 2$

4. Diberi: $f(x) = 2x + 4$ dan $g(x) = 3x + 2$
Given: $f(x) = 2x + 4$ and $g(x) = 3x + 2$
Cari nilai x jika $fg(x) = 2x$.
Find the value of x if $fg(x) = 2x$.

$fg(x) = 2x$
 $f(3x + 2) = 2x$
 $2(3x + 2) + 4 = 2x$
 $6x + 4 + 4 = 2x$
 $4x = -8$
 $x = -2$

5. Diberi: $f(x) = 3 - \frac{x}{4}$ dan $g(x) = 2x + 1$
Given: $f(x) = 3 - \frac{x}{4}$ and $g(x) = 2x + 1$
Jika $gf(x) = f(x)$, cari nilai x.
If $gf(x) = f(x)$, find the value of x.

$gf(x) = f(x)$
 $g(3 - \frac{x}{4}) = 3 - \frac{x}{4}$
 $2(3 - \frac{x}{4}) + 1 = 3 - \frac{x}{4}$
 $6 - \frac{x}{2} + 1 = 3 - \frac{x}{4}$
 $4 = \frac{x}{4}$
 $x = 16$

8

C. Diberi fungsi f dan fungsi gubahan fg , cari fungsi g .
Given the function f and composite function fg , find the function g .

CONTOH

$f(x) = 5x + 1$; $fg(x) = 5x^2 - 4$

$fg(x) = 5x^2 - 4$ ← Perhatikan bahawa g ialah fungsi dalam.
 $5g(x) + 1 = 5x^2 - 4$
 $5g(x) = 5x^2 - 5$
 $g(x) = \frac{5x^2 - 5}{5}$
 $g(x) = x^2 - 1$

1. $f(x) = x - 3$; $fg(x) = 2x$

$fg(x) = 2x$
 $g(x) - 3 = 2x$
 $g(x) = 2x + 3$

2. $f(x) = 4x + 5$; $fg(x) = 2x + \frac{7}{2}$

$fg(x) = 2x + \frac{7}{2}$
 $4g(x) + 5 = 2x + \frac{7}{2}$
 $4g(x) = 2x - \frac{3}{2}$
 $g(x) = \frac{1}{4}(2x - \frac{3}{2})$
 $g(x) = \frac{x}{2} - \frac{3}{8}$

3. $f(x) = \frac{2}{x}$, $x \neq 0$; $fg(x) = \frac{1}{2x+3}$, $x \neq -\frac{3}{2}$

$fg(x) = \frac{1}{2x+3}$
 $\frac{2}{x} = \frac{1}{2x+3}$
 $g(x) = 4x + 6$

D. Selesaikan masalah yang berikut.
Solve the following problems.

CONTOH

Diberi $f(x) = 5x - 4$ dan $fg(x) = 1 - 5x$, cari
Given $f(x) = 5x - 4$ and $fg(x) = 1 - 5x$, find

- (a) $g(x)$.
 (b) $g(-2)$.
- (a) $fg(x) = 1 - 5x$
 $5g(x) - 4 = 1 - 5x$
 $5g(x) = 5 - 5x$
 $g(x) = 1 - x$
- (b) $g(-2) = 1 - (-2)$
 $= 1 + 2$
 $= 3$

1. Diberi $f(x) = 2x - 4$ dan $fg(x) = 6(x + 1)$, cari
Given $f(x) = 2x - 4$ and $fg(x) = 6(x + 1)$, find

- (a) $g(x)$.
 (b) $g(-1)$.
- (a) $fg(x) = 6(x + 1)$
 $2g(x) - 4 = 6x + 6$
 $2g(x) = 6x + 10$
 $g(x) = 3x + 5$
- (b) $g(-1) = 3(-1) + 5$
 $= -3 + 5$
 $= 2$

2. Diberi $f(x) = 2 - 3x$ dan $fg(x) = -x - 1$, cari
Given $f(x) = 2 - 3x$ and $fg(x) = -x - 1$, find

- (a) $g(x + 2)$.
 (b) $g(3)$.
- (a) $fg(x) = -x - 1$
 $2 - 3g(x) = -x - 1$
 $g(x) = \frac{x + 3}{3}$
 $g(x + 2) = \frac{(x + 2) + 3}{3}$
 $= \frac{x + 5}{3}$
- (b) $g(3) = \frac{3 + 3}{3}$
 $= 2$

3. Diberi $f(x) = 4(x + 1)$ dan $fg(x) = 4(x^2 + 4)$, cari
Given $f(x) = 4(x + 1)$ and $fg(x) = 4(x^2 + 4)$, find $gf(x)$.

$fg(x) = 4(x^2 + 4)$
 $4g(x) + 4 = 4(x^2 + 4)$
 $g(x) = x^2 + 3$

$gf(x) = g[4(x + 1)]$
 $= [4(x + 1) + 1]^2 + 3$
 $= 16(x + 1)^2 + 3$

E. Diberi fungsi g dan fungsi gubahan fg , cari fungsi f .
Given the function g and composite function fg , find the function f .

CONTOH

$g(x) = x^2 - 4$; $fg(x) = 2x^2 - 1$

$fg(x) = 2x^2 - 1$ ← Perhatikan bahawa f ialah fungsi luar. Dalam Bahagian C, fungsi g ialah fungsi dalam.
 $f(x^2 - 4) = 2x^2 - 1$
 Katakan $y = x^2 - 4$.
 Maka, $x^2 = y + 4$
 $f(y) = 2(y + 4) - 1$
 $= 2y + 8 - 1$
 $= 2y + 7$

Maka, $f(x) = 2x + 7$ ← Tulis jawapan dalam sebutan x dengan menukar pemboleh ubah y kepada x .

1. $g(x) = 2x - 4$; $fg(x) = 2x - 9$

$fg(x) = 2x - 9$
 $f(2x - 4) = 2x - 9$
 Katakan $y = 2x - 4$
 Maka, $2x = y + 4$
 $f(y) = (y + 4) - 9$
 $= y - 5$
 Maka, $f(x) = x - 5$

2. $g(x) = \frac{x}{2}$; $fg(x) = \frac{x^2}{4} + 2$

$fg(x) = \frac{x^2}{4} + 2$
 $f(\frac{x}{2}) = \frac{x^2}{4} + 2$
 Katakan $y = \frac{x}{2}$
 Maka, $x = 2y$
 $f(y) = \frac{(2y)^2}{4} + 2$
 $= y^2 + 2$
 Maka, $f(x) = x^2 + 2$

3. $g(x) = 5x - 4$; $fg(x) = 6 - 5x$

$fg(x) = 6 - 5x$
 $f(5x - 4) = 6 - 5x$
 Katakan $y = 5x - 4$
 Maka, $5x = y + 4$
 $f(y) = 6 - (y + 4)$
 $= 6 - y - 4$
 $= 2 - y$
 Maka, $f(x) = 2 - x$

F. Selesaikan masalah yang berikut.
Solve the following problems.

CONTOH

Diberi bahawa $f(x) = ax + 3$ dan $g(x) = 4 + 2x$.
Cari nilai a jika $fg(3) = 23$.
Given $f(x) = ax + 3$ and $g(x) = 4 + 2x$. Find the value of a if $fg(3) = 23$.

$fg(3) = 23$
 $f(4 + 2(3)) = 23$
 $f(10) = 23$
 $a(10) + 3 = 23$
 $10a = 20$
 $a = 2$

1. Diberi bahawa $f(x) = mx + 2$, $g(x) = nx - 4$ dan $fg(x) = 12x - 3$. Cari nilai m dan nilai n .
Given $f(x) = mx + 2$, $g(x) = nx - 4$ and $fg(x) = 12x - 3$. Find the values of m and n .

$fg(x) = 12x - 3$
 $f(nx - 4) = 12x - 3$
 $m(nx - 4) + 2 = 12x - 3$
 $mnx - 4m + 2 = 12x - 3$
 Maka,
 $-4m + 2 = -3$ dan $mn = 12$
 $-4m = -5$ $\frac{5}{4}n = 12$
 $m = 1\frac{1}{4}$ $n = 9\frac{3}{5}$

2. Diberi dua fungsi, $f(x) = x + 1$ dan $g(x) = ax + 3$.
Cari

- (a) $f(5)$.
 (b) nilai a jika $gf(5) = 21$.
 the value of a if $gf(5) = 21$.
- (a) $f(5) = 5 + 1$
 $= 6$
- (b) $gf(5) = 21$
 $g(6) = 21$
 $a(6) + 3 = 21$
 $6a = 18$
 $a = 3$

3. Diberi bahawa fungsi $f(x) = \frac{4}{x}$, $x \neq 0$, dan fungsi gubahan $fg(x) = 2x$. Cari

- Given the function $f(x) = \frac{4}{x}$, $x \neq 0$, and composite function $fg(x) = 2x$. Find
- (a) $g(x)$. (b) $gf(2)$.
- (a) $fg(x) = 2x$
 $\frac{4}{x} = 2x$
 $g(x) = \frac{2}{x}$, $x \neq 0$
- (b) $gf(2) = g(\frac{4}{2})$
 $= g(2)$
 $= \frac{2}{2}$
 $= 1$

4. Diberi bahawa $f(x) = ax + b$, $a > 0$ dan $f^2(x) = 9x + 20$. Cari nilai a dan nilai b .

Given $f(x) = ax + b$, $a > 0$ and $f^2(x) = 9x + 20$. Find the values of a and b .

$f^2(x) = 9x + 20$
 $f(ax + b) = 9x + 20$
 $a(ax + b) + b = 9x + 20$
 $a^2x + ab + b = 9x + 20$
 Maka, $a^2 = 9$
 $a = 3$ ($a > 0$)
 $ab + b = 20$
 $3b + b = 20$
 $4b = 20$
 $b = 5$

5. Diberi bahawa $f(x) = x + 4$, $g(x) = 4 - 2x$ dan $fg(x) = a - bx$. Cari nilai a dan nilai b .

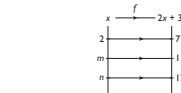
Given $f(x) = x + 4$, $g(x) = 4 - 2x$ and $fg(x) = a - bx$. Find the values of a and b .

$fg(x) = a - bx$
 $f(4 - 2x) = a - bx$
 $4 - 2x + 4 = a - bx$
 $8 - 2x = a - bx$
 Maka, $a = 8$ dan $b = 2$.

1.4 Fungsi Songsang

A. Cari nilai m dan nilai n dengan menggunakan pemetaan songsang.
Find the values of m and n by using inverse mapping.

CONTOH

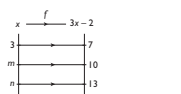


$f^{-1}(11) = m$ $f^{-1}(17) = n$
 $ff^{-1}(11) = f(m)$ $ff^{-1}(17) = f(n)$
 $11 = 2m + 3$ $17 = 2n + 3$
 $8 = 2m$ $14 = 2n$
 $m = 4$ $n = 7$

FAKTA UTAMA

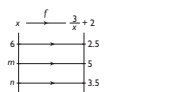
- Jika fungsi f memetakan x kepada y , maka fungsi yang memetakan y kembali kepada x ialah fungsi songsang bagi f dan ditulis sebagai f^{-1} .
If a function f maps x to y , then the function that maps y back to x is the inverse function of f and is written as f^{-1} .
- $f(x) = y$
 $f^{-1}(y) = x$
 $x = f^{-1}(y)$ atau/or $f^{-1}(y) = x$
- $f^{-1}(x) \neq \frac{1}{x}$

1.



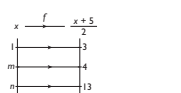
$f^{-1}(10) = m$ $f^{-1}(13) = n$
 $ff^{-1}(10) = f(m)$ $ff^{-1}(13) = f(n)$
 $10 = f(m)$ $13 = f(n)$
 $10 = 3m - 2$ $13 = 3n - 2$
 $12 = 3m$ $15 = 3n$
 $m = 4$ $n = 5$

2.



$f^{-1}(5) = m$ $f^{-1}(3.5) = n$
 $ff^{-1}(5) = f(m)$ $ff^{-1}(3.5) = f(n)$
 $5 = f(m)$ $3.5 = f(n)$
 $5 = \frac{3}{m} + 2$ $3.5 = \frac{3}{n} + 2$
 $\frac{3}{m} = 3$ $1.5 = \frac{3}{n}$
 $m = 1$ $n = 2$

3.



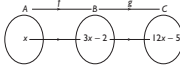
$f^{-1}(4) = m$ $f^{-1}(13) = n$
 $ff^{-1}(4) = f(m)$ $ff^{-1}(13) = f(n)$
 $4 = \frac{m+5}{2}$ $13 = \frac{n+5}{2}$
 $8 = m + 5$ $26 = n + 5$
 $m = 3$ $n = 21$

PRAKTIS FORMATIF Kertas 2

ANALISIS SOALAN SPM				
Subtopik	2012	2014	2015	2016
1.1	-	-	-	-
1.2	-	S.3(a)(i)	-	-
1.3	-	S.3(a)	-	-
1.4	-	S.3(a)(i)	-	-

Jawab semua soalan.
Answer all the questions.

1. Dalam rajah di bawah, fungsi f memetakan set A kepada set B dan fungsi g memetakan set B kepada set C.
In the diagram, the function f maps set A to set B and the function g maps set B to set C.



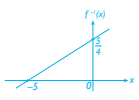
Cari
Find
(a) dalam sebutan x , fungsi in terms of x , the function
(i) yang memetakan set B kepada set A. which maps set B to set A.
(ii) $g(x)$.
(b) nilai x dengan keadaan $fg(x) = 5x + 14$. the value of x such that $fg(x) = 5x + 14$.

(a) (i) Fungsi yang memetakan set B kepada set A ialah $f^{-1}(x)$.
Berdasarkan rajah yang diberi, $f(x) = 3x - 2$.
Katakan $y = f^{-1}(x)$
Maka, $f(y) = x$
 $3y - 2 = x$
 $3y = x + 2$
 $y = \frac{x+2}{3}$
Maka, $f^{-1}(x) = \frac{x+2}{3}$
(ii) Berdasarkan rajah yang diberi,
 $gf(x) = 12x - 5$
Maka, $g(3x - 2) = 12x - 5$
Katakan $u = 3x - 2$
Maka, $x = \frac{u+2}{3}$ dan
 $g(u) = 12\left(\frac{u+2}{3}\right) - 5$
 $= 4u + 8 - 5$
 $= 4u + 3$
Maka, $g(x) = 4x + 3$
(b) $fg(x) = 5x + 14$
 $f(4x + 3) = 5x + 14$
 $3(4x + 3) - 2 = 5x + 14$
 $12x + 9 - 2 = 5x + 14$
 $7x = 7$
 $x = 1$

2. Diberi dua fungsi, $f: x \rightarrow 4x - 5$ dan $g: x \rightarrow \frac{x}{4} - 2$. Cari
Given two functions, $f: x \rightarrow 4x - 5$ and $g: x \rightarrow \frac{x}{4} - 2$. Find

(a) $f^{-1}(x)$, lakarkan graf $f^{-1}(x)$ dan kemudiannya nyatakan domain x .
 $f^{-1}(x)$, sketch the graph of $f^{-1}(x)$ and then state the domain of x .
(b) $f^{-1}g(x)$.
(c) $h(x)$ dengan keadaan $hg(x) = x - 8$.
 $h(x)$ such that $hg(x) = x - 8$.

(a) Diberi $f(x) = 4x - 5$.
Katakan $y = f^{-1}(x)$
 $f(y) = x$
 $4y - 5 = x$
 $4y = x + 5$
 $y = \frac{x+5}{4}$
Maka, $f^{-1}(x) = \frac{x+5}{4}$
Lakarkan graf $f^{-1}(x)$:

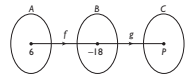


Domain x ialah semua nilai nyata.
(b) $f^{-1}g(x) = f^{-1}\left(\frac{x}{4} - 2\right)$
 $= \frac{\left(\frac{x}{4} - 2\right) + 5}{4}$
 $= \frac{x + 12}{16}$
(c) $hg(x) = x - 8$
 $h\left(\frac{x}{4} - 2\right) = x - 8$
Katakan $u = \frac{x}{4} - 2$,
maka $x = 4u + 8$.
 $h(u) = (4u + 8) - 8$
 $= 4u$
Maka, $h(x) = 4x$.

FOKUS KBAT

Kemahiran Kognitif: Menganalisis
Konteks: Fungsi Gubahan, Fungsi Songsang

Dalam rajah di bawah, fungsi f memetakan set A kepada set B dan fungsi g memetakan set B kepada set C, dengan keadaan p ialah pemalar.
In the diagram, the function f maps set A to set B and the function g maps set B to set C, where p is a constant.



Diberi $f(x) = kx^2 - 5x$, dengan keadaan k ialah pemalar.
Given that $f(x) = kx^2 - 5x$, where k is a constant.
(a) Tentukan sama ada hubungan yang memetakan set B kepada set A ialah suatu fungsi atau bukan. Nyatakan sebab anda.
Determine whether the relation which maps set B to set A is a function. State your reason.
(b) Cari nilai k .
Find the value of k .
(c) Seterusnya, cari fungsi yang memetakan set A kepada set C, dalam sebutan x , dengan keadaan fungsi yang memetakan set C kepada set B ialah $\frac{18x}{x-1}$.
Hence, find the function which maps set A to set C, in terms of x , where the function which maps set C to set B is $\frac{18x}{x-1}$.

Info KBAT
Fungsi yang memetakan set A kepada set C ialah g^1 manakala fungsi yang memetakan set C kepada set B ialah g^2 .
The function which maps set A to set C is g^1 while the function which maps set C to set B is g^2 .

(a) Bukan fungsi.
Kerana hubungan yang memetakan set B kepada set A ialah hubungan satu kepada banyak.
(b) $f(x) = kx^2 - 5x$
 $f(6) = -18$
 $k(6^2) - 5(6) = -18$
 $36k - 30 = -18$
 $36k = 12$
 $k = \frac{1}{3}$
(c) $g^1(x) = \frac{18x}{x-1}$
Katakan $g(x) = y$
Maka, $g^1(y) = x$
 $\frac{18y}{y-1} = x$
 $18y = xy - x$
 $xy - 18y = x$
 $y(x - 18) = x$
 $y = \frac{x}{x-18}$
 $g(x) = \frac{x}{x-18}$ dan $f(x) = \frac{1}{3}x^2 - 5x$
 $gf(x) = g\left(\frac{1}{3}x^2 - 5x\right)$
 $= \frac{\frac{1}{3}x^2 - 5x}{\frac{1}{3}x^2 - 5x - 18}$
 $= \frac{x^2 - 15x}{x^2 - 15x - 54}, x \neq -3, 18$

BAB 2 PERSAMAAN KUADRATIK QUADRATIC EQUATIONS

2.1 Persamaan Kuadratik dan Punca-puncunya

A. Tulis nilai-nilai a , b dan c bagi setiap persamaan kuadratik yang berikut.
Write down the values of a , b and c for each of the following quadratic equations.

- $3x^2 + 4x = 0$
 $a = 3, b = 4, c = 0$
- $x^2 - x + 3 = 0$
 $a = 1, b = -1, c = 3$
- $-2x^2 + 9 = 0$
 $a = -2, b = 0, c = 9$

B. Tentukan sama ada setiap persamaan yang berikut ialah persamaan kuadratik atau bukan. Nyatakan sebabnya dengan jelas.
Determine whether each of the following equations is a quadratic equation. State the reason clearly.

- $4(x-1) = 2x + 3$
 $4(x-1) = 2x + 3$
 $4x - 4 = 2x + 3$
 $2x - 7 = 0$
Bukan persamaan kuadratik.
Kerana kuasa tertinggi x ialah 1.
- $(x-4)(2x+3) = 0$
 $(x-4)(2x+3) = 0$
 $2x^2 + 3x - 8x - 12 = 0$
 $2x^2 - 5x - 12 = 0$
Ya. Kerana kuasa tertinggi x ialah 2.
- $\frac{2}{x^2} + x - 3 = 0$
Bukan persamaan kuadratik.
Kerana kuasa tertinggi x ialah 1.

C. Tulis semula setiap persamaan kuadratik dalam bentuk $ax^2 + bx + c = 0$.
Rewrite each quadratic equation in the general form, $ax^2 + bx + c = 0$.

- CONTOH**
- $x(x-3) = 2x^2 + 5$
 $x^2 - 3x = 2x^2 + 5$
 $x^2 + 3x + 5 = 0$
 - $(x+2)^2 = 4x$
 $x^2 + 4x + 4 = 4x$
 $x^2 + 4 = 0$
 - $5x + 6 - x^2 = 3x(x+2)$
 $5x + 6 - x^2 = 3x^2 + 6x$
 $4x^2 + x - 6 = 0$

D. Selesaikan setiap yang berikut.
Solve each of the following.

- Cari nilai m jika $x = 2$ ialah punca persamaan $x^2 + mx - 8 = 0$.
Find the value of m if $x = 2$ is a root of the equation $x^2 + mx - 8 = 0$.
Gantikan $x = 2$ ke dalam $x^2 + mx - 8 = 0$.
 $2^2 + m(2) - 8 = 0$
 $4 + 2m - 8 = 0$
 $2m = 4$
 $m = 2$
- Cari nilai m dan nilai n jika 1 dan -4 ialah punca persamaan $mx^2 + nx - 4 = 0$.
Find the values of m and n if 1 and -4 are the roots of the equation $mx^2 + nx - 4 = 0$.
Gantikan $x = 1$.
 $m(1)^2 + n(1) - 4 = 0$
 $m + n = 4$ ①
Gantikan $x = -4$.
 $m(-4)^2 + n(-4) - 4 = 0$
 $16m - 4n = 4$
 $4m - n = 1$ ②
① + ②: $5m = 5$ dan $1 + n = 4$
 $m = 1$ dan $n = 3$

2.2 Penyelesaian Persamaan Kuadratik

A. Selesaikan persamaan kuadratik yang berikut secara penfaktoran.
Solve the following quadratic equations by factorization.

- CONTOH**
- $x(x-11) = -30$
 $x(x-11) = -30$
 $x^2 - 11x + 30 = 0$
 $(x-6)(x-5) = 0$
 $x-6 = 0$ atau $x-5 = 0$
 $x = 6$ atau $x = 5$
 - $x^2 - 2x - 3 = 0$
 $x^2 - 2x - 3 = 0$
 $(x-3)(x+1) = 0$
 $x-3 = 0$ atau $x+1 = 0$
 $x = 3$ atau $x = -1$
 - $2x^2 + 5x - 12 = 0$
 $2x^2 + 5x - 12 = 0$
 $(2x-3)(x+4) = 0$
 $2x-3 = 0$ atau $x+4 = 0$
 $x = \frac{3}{2}$ atau $x = -4$
 - $2x(x+2) = -2 - x$
 $2x^2 + 4x + x + 2 = 0$
 $2x^2 + 5x + 2 = 0$
 $(2x+1)(x+2) = 0$
 $2x+1 = 0$ atau $x+2 = 0$
 $x = -\frac{1}{2}$ atau $x = -2$
 - $x(2x+9) = 18$
 $2x^2 + 9x - 18 = 0$
 $(2x-3)(x+6) = 0$
 $2x-3 = 0$ atau $x+6 = 0$
 $x = \frac{3}{2}$ atau $x = -6$

B. Selesaikan persamaan kuadratik yang berikut secara penyempurnaan kuasa dua.
Solve the following quadratic equations by completing the square.

- CONTOH**
- $x^2 + 6x - 12 = 0$
 $x^2 + 6x - 12 = 0$
 $x^2 + 6x = 12$
 $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 12 + \left(\frac{6}{2}\right)^2$
 $(x+3)^2 = 12 + 3^2$
 $(x+3)^2 = 21$
 $x+3 = \pm\sqrt{21}$
 $x = -3 \pm\sqrt{21}$
 $= -3 + \sqrt{21}$ atau $-3 - \sqrt{21}$
 $= 1.583$ atau -7.583 (4 A.B.)
 - $x^2 - 8x + 10 = 0$
 $x^2 - 8x + 10 = 0$
 $x^2 - 8x = -10$
 $x^2 - 8x + \left(\frac{-8}{2}\right)^2 = -10 + \left(\frac{-8}{2}\right)^2$
 $(x-4)^2 = -10 + 16$
 $(x-4)^2 = 6$
 $x-4 = \pm\sqrt{6}$
 $x = 4 \pm\sqrt{6}$
 $= 4 + \sqrt{6}$ atau $4 - \sqrt{6}$
 $= 6.450$ atau 1.551
 - $-x^2 - 4x + 2 = 0$
 $-x^2 - 4x + 2 = 0$
 $x^2 + 4x = 2$
 $x^2 + 4x + \left(\frac{4}{2}\right)^2 = 2 + \left(\frac{4}{2}\right)^2$
 $(x+2)^2 = 2 + 4$
 $(x+2)^2 = 6$
 $x+2 = \pm\sqrt{6}$
 $x = -2 \pm\sqrt{6}$
 $= -2 + \sqrt{6}$ atau $-2 - \sqrt{6}$
 $= 0.4495$ atau -4.4495
 - $-2x^2 + 12x - 8 = 0$
 $-2x^2 + 12x - 8 = 0$
 $x^2 - 6x + 4 = 0$
 $x^2 - 6x = -4$
 $x^2 - 6x + \left(\frac{-6}{2}\right)^2 = -4 + \left(\frac{-6}{2}\right)^2$
 $(x-3)^2 = -4 + 9$
 $(x-3)^2 = 5$
 $x-3 = \pm\sqrt{5}$
 $x = 3 \pm\sqrt{5}$
 $= 3 + \sqrt{5}$ atau $3 - \sqrt{5}$
 $= 5.236$ atau 0.7639

C. Selesaikan persamaan kuadrat dengan menggunakan rumus: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Solve the quadratic equations by using the formula: 21(d)

CONTOH

$$-2x^2 - x + 5 = 0$$

$$a = -2, b = -1, c = 5$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-2)(5)}}{2(-2)}$$

$$= \frac{1 \pm \sqrt{41}}{-4}$$

$$= -1.851 \text{ atau } 1.351$$

Gunakan kalkulator untuk mendapat nilai ini.

2. $2x^2 - 5x - 4 = 0$

$$a = 2, b = -5, c = -4$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{57}}{4}$$

$$= 3.1375 \text{ atau } -0.6375$$

4. $-3x^2 - 6x + 10 = 0$

$$a = -3, b = -6, c = 10$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-3)(10)}}{2(-3)}$$

$$= \frac{6 \pm \sqrt{136}}{-6}$$

$$= -3.0817 \text{ atau } 1.0817$$

1. $x^2 + 4x - 3 = 0$

$$a = 1, b = 4, c = -3$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{28}}{2(1)}$$

$$= 0.6458 \text{ atau } -4.6458$$

3. $3x^2 + 18x + 5 = 0$

$$a = 3, b = 18, c = 5$$

$$x = \frac{-18 \pm \sqrt{18^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{-18 \pm \sqrt{264}}{6}$$

$$= -0.2920 \text{ atau } -5.7080$$

5. $6x^2 - 7x + 2 = 0$

$$a = 6, b = -7, c = 2$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(2)}}{2(6)}$$

$$= \frac{7 \pm \sqrt{1}}{12}$$

$$= \frac{2}{3} \text{ atau } \frac{1}{2}$$

D. Bentukkan persamaan kuadrat dengan setiap pasangan punca x_1 dan x_2 yang diberikan.
Form a quadratic equation with each given pair of roots, x_1 and x_2 . 22

CONTOH

1. $x_1 = -3, x_2 = 4$

$$[x - (-3)][x - 4] = 0$$

$$(x + 3)(x - 4) = 0$$

$$x^2 - 4x + 3x - 12 = 0$$

$$x^2 - x - 12 = 0$$

1. $x_1 = 3, x_2 = 5$

$$(x - 3)(x - 5) = 0$$

$$x^2 - 8x + 15 = 0$$

2. $x_1 = 4, x_2 = -6$

$$(x - 4)(x - (-6)) = 0$$

$$(x - 4)(x + 6) = 0$$

$$x^2 + 2x - 24 = 0$$

3. $x_1 = -5, x_2 = -7$

$$[x - (-5)][x - (-7)] = 0$$

$$(x + 5)(x + 7) = 0$$

$$x^2 + 12x + 35 = 0$$

E. Diberi α dan β ialah punca persamaan kuadrat. Bentukkan persamaan kuadrat baharu dengan hasil tambah dan hasil darab punca-puncunya seperti berikut.
Given α and β are the roots of quadratic equations. Form the new quadratic equations with the given sum and product of the roots as follows. 22

CONTOH

$\alpha + \beta = 1, \alpha\beta = \frac{2}{9}$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (1)x + \frac{2}{9} = 0$$

$$9x^2 - 9x + 2 = 0$$

$\alpha + \beta = \text{HTP}$
 $\alpha\beta = \text{HDP}$

1. $\alpha + \beta = 2, \alpha\beta = -15$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - 2x + (-15) = 0$$

$$x^2 - 2x - 15 = 0$$

2. $\alpha + \beta = -7, \alpha\beta = 12$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-7)x + 12 = 0$$

$$x^2 + 7x + 12 = 0$$

3. $\alpha + \beta = \frac{5}{2}, \alpha\beta = \frac{3}{2}$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$$

$$2x^2 - 5x + 3 = 0$$

F. Cari hasil tambah dan hasil darab punca-punca bagi setiap persamaan kuadrat yang berikut.
Find the sum and the product of the roots of each of the following quadratic equations. 22

CONTOH

1. $x^2 - 6x + 8 = 0$
 $2x^2 + 5x - 6 = 0$

$$2x^2 + 5x - 6 = 0$$

$$x^2 + \frac{5}{2}x - 3 = 0$$

$$x^2 - \left(\frac{5}{2}\right)x + (-3) = 0$$

HTP = $-\frac{5}{2}$
HDP = -3

1. $x^2 - 6x + 8 = 0$

$$x^2 - 6x + 8 = 0$$

HTP = 6
HDP = 8

2. $x^2 + 7x + 6 = 0$

$$x^2 + 7x + 6 = 0$$

$$x^2 - (-7)x + 6 = 0$$

HTP = -7
HDP = 6

3. $x^2 - 5x - 6 = 0$

$$x^2 - 5x - 6 = 0$$

$$x^2 - 5x + (-6) = 0$$

HTP = 5
HDP = -6

4. $-x^2 + 9x + 20 = 0$

$$-x^2 + 9x + 20 = 0$$

$$x^2 - 9x - 20 = 0$$

$$x^2 - 9x + (-20) = 0$$

HTP = 9
HDP = -20

5. $3x^2 + 12x - 21 = 0$

$$3x^2 + 12x - 21 = 0$$

$$x^2 + 4x - 7 = 0$$

$$x^2 - (-4)x + (-7) = 0$$

HTP = -4
HDP = -7

G. Selesaikan masalah yang berikut.
Solve the following problems. 21:22

1. Gunakan hasil tambah dan hasil darab punca-punca untuk menentukan sama ada 2 dan $\frac{5}{2}$ adalah punca-punca persamaan kuadrat $2x^2 + 4x + 5 = 0$ atau bukan.
Use the sum and the product of roots to determine whether 2 and $\frac{5}{2}$ are the roots of the quadratic equation $2x^2 + 4x + 5 = 0$.

$$2x^2 + 4x + 5 = 0$$

$$x^2 + 2x + \frac{5}{2} = 0$$

$$x^2 - (-2)x + \frac{5}{2} = 0$$

HTP = -2
HDP = $\frac{5}{2}$

Jika punca-punca persamaan ialah 2 dan $\frac{5}{2}$, maka
HTP = $2 + \frac{5}{2} = \frac{9}{2} \neq -2$
HDP = $2 \times \frac{5}{2} = 5 \neq \frac{5}{2}$
Oleh kerana HTP dan HDP tidak sama, maka 2 dan $\frac{5}{2}$ bukan punca persamaan kuadrat.

2. Jika punca-punca persamaan kuadrat $x^2 + 3x + 2 = 0$ ialah r_1 dan r_2 , bentukkan persamaan kuadrat baharu dengan punca-punca $\frac{1}{r_1}$ dan $\frac{1}{r_2}$.
If the roots of the quadratic equation $x^2 + 3x + 2 = 0$ are r_1 and r_2 , form a new quadratic equation with $\frac{1}{r_1}$ and $\frac{1}{r_2}$ as the roots.

$$x^2 + 3x + 2 = 0$$

$$x^2 - (-3)x + 2 = 0$$

$$r_1 + r_2 = -3 \text{ dan } r_1 r_2 = 2$$

Untuk persamaan kuadrat baharu:

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2} = \frac{-3}{2}$$

$$\frac{1}{r_1} \times \frac{1}{r_2} = \frac{1}{r_1 r_2} = \frac{1}{2}$$

Persamaan kuadrat baharu ialah

$$x^2 - \left(-\frac{3}{2}\right)x + \frac{1}{2} = 0$$

$$x^2 + \frac{3}{2}x + \frac{1}{2} = 0$$

$$2x^2 + 3x + 1 = 0$$

3. Diberi r_1 dan r_2 ialah punca-punca persamaan kuadrat $2x^2 + 8x + 6 = 0$. Cari nilai $r_1^2 + r_2^2$ tanpa menyelesaikan persamaan itu.
Given r_1 and r_2 are the roots of the quadratic equation $2x^2 + 8x + 6 = 0$. Find the value of $r_1^2 + r_2^2$ without solving the equation. KBA1

$$2x^2 + 8x + 6 = 0$$

$$x^2 + 4x + 3 = 0$$

$$x^2 - (-4)x + 3 = 0$$

$r_1 + r_2 = -4$
 $r_1 r_2 = 3$

$$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1 r_2$$

$$(-4)^2 = r_1^2 + r_2^2 + 2(3)$$

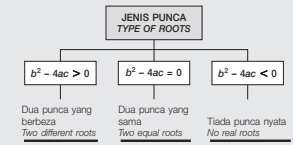
$$r_1^2 + r_2^2 = 16 - 6 = 10$$

2.3 Jenis Punca Persamaan Kuadrat (23A1)(18)(18)(23)(18)

FAKTA UTAMA

Jenis punca bagi suatu persamaan kuadrat $ax^2 + bx + c = 0$ boleh ditentukan dengan nilai $b^2 - 4ac$.
The type of roots of a quadratic equation $ax^2 + bx + c = 0$ can be determined by the value of $b^2 - 4ac$.

Peta pokok berikut menunjukkan jenis punca:
The following tree map shows the types of roots:



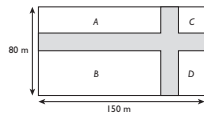
A. Cari nilai $b^2 - 4ac$. Seterusnya, tentukan jenis punca bagi setiap persamaan kuadrat yang berikut.
Find the value of $b^2 - 4ac$. Hence, determine the type of roots of each of the following quadratic equations. 3(1)

Persamaan Equation	Nilai $b^2 - 4ac$ Value of $b^2 - 4ac$	Jenis punca Type of roots
1. $x^2 + 5x + 6 = 0$	$b^2 - 4ac = 5^2 - 4(1)(6) = 25 - 24 = 1 > 0$	Dua punca yang berbeza
2. $2x^2 - 7x + 8 = 0$	$b^2 - 4ac = (-7)^2 - 4(2)(8) = 49 - 64 = -15 < 0$	Tiada punca nyata atau tiada punca
3. $x^2 + 6x + 9 = 0$	$b^2 - 4ac = 6^2 - 4(1)(9) = 36 - 36 = 0$	Dua punca yang sama atau satu punca sahaja
4. $-12x + 4x^2 = -9$	$-12x + 4x^2 = -9$ $4x^2 - 12x + 9 = 0$ $b^2 - 4ac = (-12)^2 - 4(4)(9) = 144 - 144 = 0$	Dua punca yang sama atau satu punca sahaja
5. $7x + 3 = 5x^2$	$7x + 3 = 5x^2$ $5x^2 - 7x - 3 = 0$ $b^2 - 4ac = (-7)^2 - 4(5)(-3) = 49 + 60 = 109 > 0$	Dua punca yang berbeza

FOKUS KBAT

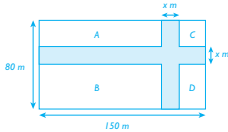
Kemahiran Kognitif: Menganalisis
Konteks: Penyelesaian Persamaan Kuadratik

Rajah di bawah menunjukkan suatu kawasan kediaman yang berbentuk segi empat tepat. Kawasan A, B, C dan D ialah tapak perumahan manakala bahagian berlorek mewakili jalan dengan lebar yang sama.
The diagram shows a residential area which has a rectangular shape. The areas, A, B, C and D, are housing sites while the shaded region represents roads with the same width.



Jika jumlah luas tapak perumahan ialah 10 656 m², berapakah lebar jalan itu?
If the total area of the housing sites is 10 656 m², what is the width of the roads?

Katakan lebar jalan = x m



Luas segi empat tepat - Luas kawasan berlorek = Jumlah luas tapak perumahan

$$150(80) - (80x + 150x - x^2) = 10\,656$$

$$12\,000 - 80x - 150x + x^2 = 10\,656$$

$$x^2 - 230x + 1\,344 = 0$$

$$(x - 6)(x - 224) = 0$$

$$x = 6 \text{ atau } x = 224$$

Berdasarkan rajah, $x < 80$. Maka $x = 6$.

Lebar jalan itu ialah 6 m.

Info KBAT

Jumlah luas tapak perumahan = Luas segi empat tepat
- Luas kawasan berlorek
Total area of the housing sites = Area of a rectangle
- Area of the shaded region

BAB 3

**FUNGSI KUADRATIK
QUADRATIC FUNCTIONS**

3.1 Fungsi Kuadratik dan Grafnya

FAKTA UTAMA

- Fungsi kuadratik dalam pemboleh ubah x ialah fungsi dengan kuasa tertinggi x ialah 2.
- A quadratic function in the variable x is a function with the highest power of x is 2.
- Bentuk am: $f(x) = ax^2 + bx + c$, dengan keadaan a, b dan c ialah pemalar dan $a \neq 0$.
- General form: $f(x) = ax^2 + bx + c$, where a, b dan c are constants and $a \neq 0$.
- Bentuk dan kedudukan graf fungsi kuadratik.
- Shapes and positions of graphs of quadratic functions:

Nilai $b^2 - 4ac$ Value of $b^2 - 4ac$	Kedudukan graf $f(x)$ Position of graph of $f(x)$
$b^2 - 4ac > 0$	<ul style="list-style-type: none"> Graf $f(x)$ menyalang paksi-x pada dua titik yang berlainan. Graph of $f(x)$ intersects the x-axis at two different points.
$b^2 - 4ac = 0$	<ul style="list-style-type: none"> Graf $f(x)$ menyentuh paksi-x pada satu titik sahaja. Graph of $f(x)$ touches the x-axis at only one point. Paksi-x ialah tangen kepada graf $f(x)$. The x-axis is a tangent to the graph of $f(x)$.
$b^2 - 4ac < 0$	<ul style="list-style-type: none"> Graf $f(x)$ tidak menyalang atau menyentuh paksi-x. Graph of $f(x)$ does not intersect or touch the x-axis. Graf $f(x)$ berada di atas atau di bawah paksi-x. Graph of $f(x)$ is above or below the x-axis.

A. Tentukan sama ada setiap fungsi berikut ialah fungsi kuadratik atau bukan.
Determine whether each of the following functions is a quadratic function.

CONTOH

- (a) $f(x) = 3x^2 + 2x - 4$ (b) $f(x) = x^{-2} + 4x + 6$ (c) $f(x) = 3x + 2$
Ya ← Kuasa tertinggi x ialah 2. Bukan ← Kuasa tertinggi x ialah 1. Bukan ← Kuasa tertinggi x ialah 1.

1. $f(x) = 2x^2 + 3x - 1$ 2. $f(x) = -2x^2 + 4x + 5$ 3. $f(x) = x^3 - 2x + 4$
Ya Ya Bukan

B. Nyatakan nilai-nilai a, b dan c dalam setiap fungsi kuadratik yang berikut.
State the values of a, b and c in each of the following quadratic functions.

1. $f(x) = x^2 + x + 1$ 2. $f(x) = 3x^2 - 4x + 5$ 3. $f(x) = x(2x - 3)$
 $a = 1, b = 1, c = 1$ $a = 3, b = -4, c = 5$ $a = 2, b = -3, c = 0$

C. Berdasarkan graf fungsi kuadratik yang diberi, lengkapkan jadual berikut.
Based on the given graphs of quadratic functions, complete the following table.

Graf bagi $f(x)$ Graph of $f(x)$	Nilai $b^2 - 4ac$ Value of $b^2 - 4ac$	Jenis punca bagi $f(x) = 0$ Type of roots of $f(x) = 0$
	$b^2 - 4ac < 0$	$f(x) = 0$ tiada punca nyata.
1.	$b^2 - 4ac > 0$	$f(x) = 0$ mempunyai dua punca nyata yang berbeza.
2.	$b^2 - 4ac > 0$	$f(x) = 0$ mempunyai dua punca nyata yang berbeza.
3.	$b^2 - 4ac = 0$	$f(x) = 0$ mempunyai dua punca nyata yang sama.
4.	$b^2 - 4ac < 0$	$f(x) = 0$ tiada punca nyata.

D. Bagi setiap fungsi kuadratik $f(x)$ yang diberikan, tentukan jenis punca bagi $f(x) = 0$. Seterusnya, lakarkan graf $f(x)$ terhadap paksi- x .
For each given quadratic function $f(x)$, determine the type of the roots of $f(x) = 0$. Hence, sketch the graph of $f(x)$ with respect to the x -axis.

Fungsi kuadratik Quadratic function	Jenis punca bagi $f(x) = 0$ Type of roots of $f(x) = 0$	Graf bagi $f(x)$ Graph of $f(x)$
CONTOH $f(x) = x^2 - 6x + 9$	$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$ Dua punca nyata yang sama	
1. $f(x) = x^2 + 2x - 3$	$b^2 - 4ac = 2^2 - 4(1)(-3) = 4 + 12 = 16 > 0$ Dua punca nyata yang berbeza.	
2. $f(x) = x^2 - 8x + 16$	$b^2 - 4ac = (-8)^2 - 4(1)(16) = 64 - 64 = 0$ Dua punca nyata yang sama.	
3. $f(x) = -3x^2 + 5x - 6$	$b^2 - 4ac = 5^2 - 4(-3)(-6) = 25 - 72 = -47 < 0$ Tiada punca nyata.	
4. $f(x) = -x^2 - 4x + 6$	$b^2 - 4ac = (-4)^2 - 4(-1)(6) = 16 + 24 = 40 > 0$ Dua punca nyata yang berbeza.	

E. Cari julat nilai p jika graf fungsi $f(x)$ menyalang paksi- x pada dua titik yang berlainan.
Find the range of the values of p if the graph of function $f(x)$ intersects the x -axis at two different points.

CONTOH

$f(x) = x^2 - 4x + p$
 $(-4)^2 - 4(1)(p) > 0 \leftarrow b^2 - 4ac > 0$
 $16 - 4p > 0$
 $-4p > -16$
 $p < 4$

1. $f(x) = x^2 - 3x - p$
 $(-3)^2 - 4(1)(-p) > 0$
 $9 + 4p > 0$
 $4p > -9$
 $p < -\frac{9}{4}$

2. $f(x) = px^2 + 6x + 4$
 $6^2 - 4(p)(4) > 0$
 $36 - 16p > 0$
 $-16p > -36$
 $p < 2\frac{1}{4}$

3. $f(x) = x^2 - 6x + p - 1$
 $(-6)^2 - 4(1)(p - 1) > 0$
 $36 - 4p + 4 > 0$
 $-4p > -40$
 $p < 10$

F. Cari julat nilai p jika graf fungsi $f(x)$ tidak menyalang paksi- x .
Find the range of the values of p if the graph of function $f(x)$ does not intersect the x -axis.

CONTOH

$f(x) = px^2 + 6x + 5$
 $6^2 - 4(p)(5) < 0 \leftarrow b^2 - 4ac < 0$
 $36 - 20p < 0$
 $-20p < -36 \Rightarrow p > 1\frac{4}{5}$

1. $f(x) = 10x^2 + 20x + p$
 $20^2 - 4(10)(p) < 0$
 $400 - 40p < 0$
 $-40p < -400$
 $p > 10$

2. $f(x) = px^2 + 12x + 12$
 $12^2 - 4(p)(12) < 0$
 $144 - 48p < 0$
 $-48p < -144$
 $p > 3$

3. $f(x) = 3x^2 - 6x + p$
 $(-6)^2 - 4(3)(p) < 0$
 $36 - 12p < 0$
 $-12p < -36$
 $p > 3$

G. Cari nilai-nilai m jika graf fungsi $f(x)$ menyentuh paksi- x pada satu titik sahaja.
Find the values of m if the graph of function $f(x)$ touches the x -axis at only one point.

CONTOH

$f(x) = x^2 + mx + 4$
 $m^2 - 4(1)(4) = 0 \leftarrow b^2 - 4ac = 0$
 $m^2 - 16 = 0$
 $m^2 = 16$
 $m = \pm\sqrt{16}$
 $m = -4 \text{ atau } 4$

1. $f(x) = 3x^2 + mx + 3$
 $m^2 - 4(3)(3) = 0$
 $m^2 - 36 = 0$
 $m^2 = 36$
 $m = \pm\sqrt{36}$
 $m = -6 \text{ atau } 6$

2. $f(x) = x^2 + mx + 3 - m$
 $m^2 - 4(1)(3 - m) = 0$
 $m^2 + 4m - 12 = 0$
 $(m + 6)(m - 2) = 0$
 $m = -6 \text{ atau } 2$

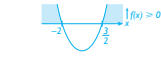
3. $f(x) = 2x^2 + (m + 4)x + 8$
 $(m + 4)^2 - 4(2)(8) = 0$
 $m^2 + 8m + 16 - 64 = 0$
 $m^2 + 8m - 48 = 0$
 $(m + 12)(m - 4) = 0$
 $m = -12 \text{ atau } 4$

2. $2x^2 + x \geq 6$

$$\begin{aligned} 2x^2 + x &\geq 6 \\ 2x^2 + x - 6 &\geq 0 \\ (2x - 3)(x + 2) &\geq 0 \end{aligned}$$

Katakan $(2x - 3)(x + 2) = 0$.

$$x = \frac{3}{2} \text{ atau } -2$$



Untuk $2x^2 + x \geq 6$, $x \leq -2$ atau $x \geq \frac{3}{2}$.

B. Selesaikan masalah yang berikut.
Solve the following problems.

1. (a) Cari julat nilai x jika $4x \leq x^2$.
Find the range of values of x if $4x \leq x^2$.
- (b) Cari julat nilai p jika $x^2 - (p + 1)x - p^2 + 1 = 0$ mempunyai dua punca nyata.
Find the range of values of p if $x^2 - (p + 1)x - p^2 + 1 = 0$ has two real roots.
- (a) $4x \leq x^2$
 $x^2 - 4x \geq 0$
 $x(x - 4) \geq 0$
- Katakan $x(x - 4) = 0$.
- $$x = 0 \text{ atau } 4$$
-
- Untuk $4x \leq x^2$, $x \leq 0$ atau $x \geq 4$.
- (b) $x^2 - (p + 1)x - p^2 + 1 = 0$
 $b^2 - 4ac > 0$
 $[-(p + 1)]^2 - 4(1)(-p^2 + 1) > 0$
 $p^2 + 2p + 1 + 4p^2 - 4 > 0$
 $5p^2 + 2p - 3 > 0$
 $(5p - 3)(p + 1) > 0$
- Katakan $(5p - 3)(p + 1) = 0$.
- $$p = \frac{3}{5} \text{ atau } -1$$
-
- Maka, $p < -1$ atau $p > \frac{3}{5}$.

3. $x + 4 < (x + 4)(2x - 1)$

$$\begin{aligned} x + 4 &< 2x^2 - x + 8x - 4 \\ 0 &< 2x^2 + 6x - 8 \\ 2x^2 + 6x - 8 &> 0 \\ x^2 + 3x - 4 &> 0 \\ (x + 4)(x - 1) &> 0 \end{aligned}$$

Katakan $(x + 4)(x - 1) = 0$.

$$x = -4 \text{ atau } 1$$

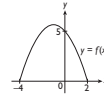
Untuk $x + 4 < (x + 4)(2x - 1)$,
 $x < -4$ atau $x > 1$

2. (a) Diberi $f(x) = 4x^2 - 16$, cari julat nilai x dengan keadaan $f(x)$ sentiasa positif.
Given $f(x) = 4x^2 - 16$, find the range of values of x such that $f(x)$ is always positive.
- (b) Cari julat nilai x yang memuaskan ketaksamaan $(x - 3)^2 \leq x - 3$.
Find the range of values of x which satisfies the inequality $(x - 3)^2 \leq x - 3$.
- (a) Untuk $f(x)$ sentiasa positif, $f(x) > 0$.
Maka, $4x^2 - 16 > 0$
 $x^2 - 4 > 0$
 $(x + 2)(x - 2) > 0$
-
- Untuk $4x^2 - 16 > 0$,
 $x < -2$ atau $x > 2$
- (b) $(x - 3)^2 \leq x - 3$
 $x^2 - 6x + 9 \leq x - 3$
 $x^2 - 7x + 12 \leq 0$
 $(x - 3)(x - 4) \leq 0$
-
- Untuk $(x - 3)^2 \leq x - 3$,
 $3 \leq x \leq 4$

PRAKTIK FORMATIF Kertas 1

Jawab semua soalan.
Answer all the questions.

1. Rajah di bawah menunjukkan graf fungsi kuadratik $y = f(x)$.
The diagram shows the graph of a quadratic function $y = f(x)$.



Nyatakan
State

- (a) punca-punca bagi persamaan $f(x) = 0$,
the roots of the equation $f(x) = 0$.
- (b) persamaan paksi simetri bagi lengkung itu,
the equation of the axis of symmetry of the curve.

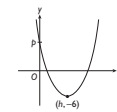
3. Graf fungsi kuadratik $f(x) = px^2 - 4x + 2$, dengan keadaan p ialah pemalar, tidak bersilang dengan paksi-x. Cari julat nilai p .
The graph of a quadratic function $f(x) = px^2 - 4x + 2$, where p is a constant, does not intersect the x-axis. Find the range of the values of p .

Subtopik	ANALISIS SOALAN SPM			
	2013	2014	2015	2016
S.1	S.5	-	S.3b	-
S.2	S.6	S.4b(i)	S.3b	-
S.3	-	-	-	-
S.4	-	S.4b	S.4	S.1b

$$\begin{aligned} b^2 - 4ac < 0 \\ (-4)^2 - 4(p)(2) < 0 \\ 16 - 8p < 0 \\ 8p > 16 \\ p > 2 \end{aligned}$$

$$\begin{aligned} b^2 - 4ac < 0 \\ (-4)^2 - 4(p)(2) < 0 \\ 16 - 8p < 0 \\ 8p > 16 \\ p > 2 \end{aligned}$$

4. Rajah di bawah menunjukkan graf fungsi kuadratik $f(x) = (x - 3)^2 - 2k$, dengan keadaan k ialah pemalar. The diagram shows the graph of a quadratic function $f(x) = (x - 3)^2 - 2k$, where k is a constant.



Diberi $(h, -6)$ ialah titik minimum graf itu.
Given $(h, -6)$ is the minimum point of the graph.

- (a) Nyatakan nilai h dan nilai k .
State the values of h and k .
- (b) Cari nilai p .
Find the value of p .

- (a) Graf fungsi $f(x) = (x - 3)^2 - 2k$ mempunyai titik minimum $(3, -2k)$.
Maka, $(3, -2k) = (h, -6)$

Jadi, $h = 3$ dan $-2k = -6$
 $k = 3$

(b) $f(x) = (x - 3)^2 - 6$
Apabila $x = 0$, $f(x) = (0 - 3)^2 - 6 = 3$
Maka, $p = 3$

PRAKTIK FORMATIF Kertas 2

Subtopik	ANALISIS SOALAN SPM			
	2013	2014	2015	2016
S.1	-	-	-	-
S.2	-	-	-	S.2a(i)
S.3	-	-	-	S.2b
S.4	-	-	-	-

Jawab semua soalan.
Answer all the questions.

1. Lengkung fungsi kuadratik $f(x) = 2(x - h)^2 - 2k$ menyalang paksi-x pada titik-titik $(2, 0)$ dan $(4, 0)$.
The curve of a quadratic function $f(x) = 2(x - h)^2 - 2k$ intersects the x-axis at points $(2, 0)$ and $(4, 0)$. The straight line $y = -2$ is the tangent to the vertex of the curve.

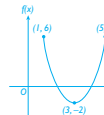


- (a) Cari nilai h dan nilai k .
Find the values of h and k .
- (b) Seterusnya, lakar graf $f(x)$ untuk $1 \leq x \leq 5$.
Hence, sketch the graph of $f(x)$ for $1 \leq x \leq 5$.
- (c) Jika graf itu dipantulkan pada paksi-x, tulis persamaan bagi lengkung itu.
If the graph is reflected about the x-axis, write the equation of the curve.

(a) Lengkung fungsi itu menyalang paksi-x pada $(2, 0)$ dan $(4, 0)$.
Maka, titik minimum lengkung terletak pada $x = \frac{2 + 4}{2} = 3$.

Juga diberi $y = -2$ ialah garis tangen kepada bucu lengkung. Ini bermakna titik $(3, -2)$ ialah bucu lengkung itu.
Daripada fungsi $f(x) = 2(x - h)^2 - 2k$, koordinat bucu lengkung ialah $(h, -2k)$.

Maka, $h = 3$ dan $k = 1$.
(b) $f(x) = 2(x - 3)^2 - 2$
 $f(1) = 2(1 - 3)^2 - 2 = 6$
 $f(5) = 2(5 - 3)^2 - 2 = 6$
Lengkung itu melalui titik $(1, 6)$ dan $(5, 6)$.



(c) Persamaan baharu bagi lengkung ialah $f(x) = -2(x - 3)^2 + 2$
 $f(x) = -2(x - 3)^2 + 2$

5. Graf fungsi kuadratik $f(x) = px^2 - 4x + q$, dengan keadaan p dan q ialah pemalar, mempunyai satu titik maksimum.
The graph of a quadratic function $f(x) = px^2 - 4x + q$, where p and q are constants, has a maximum point.
- (a) Diberi p ialah suatu integer, nyatakan nilai p yang maksimum.
Given p is an integer, state the maximum value of p .
- (b) Dengan menggunakan jawapan dari (a), cari nilai q apabila graf itu menyentuh paksi-x pada suatu titik sahaja.
Using the answer from (a), find the value of q when the graph touches the x-axis at only one point.
- (a) Fungsi itu mempunyai titik maksimum, maka $p < 0$.
Jika p ialah integer, maka nilai maksimum p ialah -1 .
- (b) $f(x) = -x^2 - 4x + q$
 $b^2 - 4ac = 0$ jika graf fungsi itu menyentuh paksi-x pada satu titik sahaja.
 $(-4)^2 - 4(1)q = 0$
 $16 + 4q = 0$
 $4q = -16$
 $q = -4$

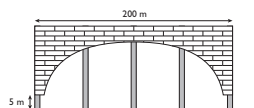
6. Rajah di bawah menunjukkan graf fungsi kuadratik $f(x) = (x - 2)^2 - 16$.
The diagram shows the graph of the quadratic function $f(x) = (x - 2)^2 - 16$.
-
- Nyatakan
State
- (a) koordinat titik minimum bagi lengkung itu,
the coordinates of the minimum point of the curve.
- (b) persamaan paksi simetri bagi lengkung itu,
the equation of the axis of symmetry of the curve.
- (c) julat nilai x apabila $f(x)$ ialah negatif,
the range of values of x when $f(x)$ is negative.
- (a) $(-2, -16)$
- (b) $x = 2$ atau $x - 2 = 0$
- (c) $-2 < x < 6$

7. Cari julat nilai x untuk $(x - 2)^2 \leq 14 - x$.
Find the range of values of x for $(x - 2)^2 \leq 14 - x$.
- $$\begin{aligned} (x - 2)^2 &\leq 14 - x \\ x^2 - 4x + 4 &\leq 14 - x \\ x^2 - 3x - 10 &\leq 0 \\ (x + 2)(x - 5) &\leq 0 \\ -2 &\leq x < 5 \end{aligned}$$
-
8. Diberi $f(x) = -2x^2 - x + 13$, cari julat nilai x untuk $f(x) \leq -2$.
Given that $f(x) = -2x^2 - x + 13$, find the range of values of x for $f(x) \leq -2$.
- $$\begin{aligned} f(x) &\leq -2 \\ -2x^2 - x + 13 &\leq -2 \\ 2x^2 + x - 15 &\geq 0 \\ (2x - 5)(x + 3) &\geq 0 \\ x &\leq -3 \text{ atau } x \geq \frac{5}{2} \end{aligned}$$
-
9. Cari julat nilai x untuk $3x^2 + 11x \leq 4$.
Find the range of values of x for $3x^2 + 11x \leq 4$.
- $$\begin{aligned} 3x^2 + 11x &\leq 4 \\ 3x^2 + 11x - 4 &\leq 0 \\ (3x - 1)(x + 4) &\leq 0 \\ -4 &\leq x \leq \frac{1}{3} \end{aligned}$$
-
10. Diberi fungsi kuadratik $f(x) = x^2 + wx + 2w - 3$, dengan keadaan w ialah pemalar, adalah sentiasa positif apabila $p < w < q$. Cari nilai p dan nilai q .
Given a quadratic function $f(x) = x^2 + wx + 2w - 3$, where w is a constant, is always positive when $p < w < q$. Find the values of p and q .
- $$\begin{aligned} f(x) &= x^2 + wx + 2w - 3 \\ a &= 1, b = w, c = 2w - 3 \\ b^2 - 4ac &< 0 \\ w^2 - 4(1)(2w - 3) &< 0 \\ w^2 - 8w + 12 &< 0 \\ (w - 2)(w - 6) &< 0 \\ 2 &< w < 6 \end{aligned}$$
- Maka, $p = 2$ dan $q = 6$.

FOKUS KBAT

Kemahiran Kognitif: Mengaplikasi dan Mencipta
Konteks: Fungsi Kuadratik dan Grafnya, Nilai Maksimum

Rajah di bawah menunjukkan pandangan sisi sebuah jambatan. Bahagian melengkung di bawah jambatan itu disokong oleh lima batang tiang keluli. Tinggi, dalam m, setiap tiang keluli itu diwakili oleh fungsi $h(x) = ax^2 + \frac{1}{2}x + k$, dengan keadaan x ialah jarak sesuatu tiang dari tiang pertama, dan a dan k ialah pemalar.
The diagram shows the side view of a bridge. The curved part at the bottom of the bridge is supported by five steel poles. The height, in m, of each steel pole is represented by a function $h(x) = ax^2 + \frac{1}{2}x + k$, where x is the distance of a pole from the first pole, and a and k are constants.



Diberi panjang jambatan itu ialah 200 m dan panjang tiang pertama ialah 5 m. Hitung panjang bagi tiang yang paling tinggi.
Given the length of the bridge is 200 m and the length of the first pole is 5 m. Find the length of the highest pole.

$$h(x) = ax^2 + \frac{1}{2}x + k$$

Pada titik $(0, 5)$: $5 = a(0)^2 + \frac{1}{2}(0) + k$
 $k = 5$

Maka, $h(x) = ax^2 + \frac{1}{2}x + 5$

$$= a\left(x^2 + \frac{1}{2a}x\right) + 5$$

$$= a\left[x^2 + \frac{1}{2a}x + \left(\frac{1}{4a}\right)^2 - \left(\frac{1}{4a}\right)^2\right] + 5$$

$$= a\left[\left(x + \frac{1}{4a}\right)^2 - \frac{1}{16a^2}\right] + 5$$

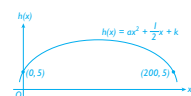
$$= a\left[x + \frac{1}{4a}\right]^2 - \frac{1}{16a} + 5$$

Pada titik maksimum, $x = \frac{0 + 200}{2} = 100$

Maka, $\frac{1}{4a} = -100$
 $a = -\frac{1}{400}$

Panjang tiang paling tinggi = $-\frac{1}{16}\left(-\frac{1}{400}\right)$
 $= \frac{25}{4} + 5$
 $= 30 \text{ m}$

Info KBAT
Anggarkan hujung atas tiang pertama sebagai titik $(0, 5)$ yang terletak pada fungsi $h(x)$.
Assume that the upper end of the first pole is at point $(0, 5)$ that lies on the function $h(x)$.



BAB 4

PERSAMAAN SERENTAK SIMULTANEOUS EQUATIONS

4.1 Penyelesaian Persamaan Serentak dalam Dua Anu: Satu Persamaan Linear dan Satu Persamaan Tak Linear

A. Selesaikan persamaan serentak yang berikut.

Solve the following simultaneous equations.

CONTOH

$x - 3y + 7 = 0$ ① ← Persamaan linear
 $x^2 + y^2 - 4xy - 13 = 0$ ② ← Persamaan tak linear
Dari ①: $x = 3y - 7$ ③ ← Ungkapkan x dalam sebutan y .
Gantikan ③ ke dalam ②:
 $(3y - 7)^2 + y^2 - 4(3y - 7)y - 13 = 0$
 $(3y - 7)^2 + y^2 - 12y^2 + 28y - 13 = 0$
 $9y^2 - 42y + 49 + y^2 - 12y^2 + 28y - 13 = 0$
 $-2y^2 - 14y + 36 = 0$
Permudahkan persamaan dan faktorkan. $\rightarrow y^2 + 7y - 18 = 0$
 $\rightarrow (y - 2)(y + 9) = 0$
 $y = 2 \text{ atau } -9$

Gantikan nilai-nilai y ke dalam ③.
Apabila $y = 2$, $x = 3(2) - 7 = -1$
Apabila $y = -9$, $x = 3(-9) - 7 = -34$
Penyelesaian ialah $x = -1, y = 2$ dan $x = -34, y = -9$.

1. $2x + y = 6$ ①
 $x^2 - 3y^2 + 4x + 3 = 0$ ②
Dari ①: $y = 6 - 2x$ ③
Gantikan ③ ke dalam ②:
 $x^2 - 3(6 - 2x)^2 + 4x + 3 = 0$
 $x^2 - 108 + 72x - 12x^2 + 4x + 3 = 0$
 $11x^2 - 76x + 105 = 0$
 $(11x - 21)(x - 5) = 0$
 $x = \frac{21}{11}$ atau 5

Gantikan nilai-nilai x ke dalam ③.
Apabila $x = \frac{21}{11}$, $y = 6 - 2\left(\frac{21}{11}\right) = \frac{24}{11}$
Apabila $x = 5$, $y = 6 - 2(5) = -4$
Penyelesaian ialah $x = 1, \frac{10}{11}, y = 2, \frac{2}{11}$ dan $x = 5, y = -4$.

2. $2x - 5y + 16 = 0$ ①
 $3xy - 4x - 24y = 10$ ②
Dari ①: $x = \frac{5y - 16}{2}$ ③
Gantikan ③ ke dalam ②:
 $3\left(\frac{5y - 16}{2}\right)y - 4\left(\frac{5y - 16}{2}\right) - 24y = 10$
 $\frac{15}{2}y^2 - 24y - 10y + 32 - 24y = 10$
 $15y^2 - 116y + 44 = 0$
 $(3y - 22)(5y - 2) = 0$
 $y = \frac{22}{3}$ atau $\frac{2}{5}$

Gantikan nilai-nilai y ke dalam ③.
Apabila $y = \frac{22}{3}$, $x = \frac{5\left(\frac{22}{3}\right) - 16}{2} = \frac{31}{3}$
Apabila $y = \frac{2}{5}$, $x = \frac{5\left(\frac{2}{5}\right) - 16}{2} = -7$
Penyelesaian ialah $x = 10, \frac{1}{3}, y = 7, \frac{1}{3}$ dan $x = -7, y = \frac{2}{5}$.

3. $2x - \frac{y}{3} = 1$ ①
 $4x^2 + y^2 - 10xy = 7$ ②
Dari ①: $y = 6x - 3$ ③
Gantikan ③ ke dalam ②:
 $4x^2 + (6x - 3)^2 - 10x(6x - 3) = 7$
 $4x^2 + 36x^2 - 36x + 9 - 60x^2 + 30x = 7$
 $-20x^2 - 6x + 2 = 0$
 $10x^2 + 3x - 1 = 0$
 $(2x + 1)(5x - 1) = 0$
 $x = -\frac{1}{2}$ atau $\frac{1}{5}$

Gantikan nilai-nilai x ke dalam ③.
Apabila $x = -\frac{1}{2}$, $y = 6\left(-\frac{1}{2}\right) - 3 = -6$
Apabila $x = \frac{1}{5}$, $y = 6\left(\frac{1}{5}\right) - 3 = -\frac{9}{5}$
Penyelesaian ialah $x = -\frac{1}{2}, y = -6$ dan $x = \frac{1}{5}, y = -\frac{4}{5}$.

B. Selesaikan persamaan serentak yang berikut. Berikan jawapan betul kepada tiga tempat perpuluhan.
Solve each of the following simultaneous equations. Give the answers correct to three decimal places.

CONTOH

$\frac{x}{y} + \frac{y}{x} = 2$ ① ← Persamaan tak linear
 $x - \frac{y}{2} = 3$ ② ← Persamaan linear

Dari ①: $\frac{x^2 + 5y}{16} = 2$
 $x^2 + 5y = 2xy$ ③
Dari ②: $y = 2x - 6$ ④
Gantikan ④ ke dalam ③:
 $x^2 + 5(2x - 6) = 2x(2x - 6)$
 $x^2 + 10x - 30 = 4x^2 - 12x$
 $3x^2 - 22x + 30 = 0$
 $x = \frac{-(-22) \pm \sqrt{(-22)^2 - 4(3)(30)}}{2(3)} = \frac{22 \pm \sqrt{124}}{6}$
 $= 5.523 \text{ atau } 1.811$

Gantikan nilai-nilai x ke dalam ④.
Apabila $x = 5.523$, $y = 2(5.523) - 6 = 5.046$
Apabila $x = 1.811$, $y = 2(1.811) - 6 = -2.378$
Penyelesaian ialah $x = 5.523, y = 5.046$ dan $x = 1.811, y = -2.378$.

2. $\frac{3}{y} + \frac{1}{x} = 4$ ①
 $y - 2x = 3$ ②

Dari ①: $\frac{3x + y}{xy} = 4$
 $3x + y = 4xy$ ③
Dari ②: $y = 2x + 3$ ④

Gantikan ④ ke dalam ③:
 $3x + (2x + 3) = 4x(2x + 3)$
 $5x + 3 = 8x^2 + 12x$
 $8x^2 + 7x - 3 = 0$
 $x = \frac{-7 \pm \sqrt{7^2 - 4(8)(-3)}}{2(8)}$
 $= \frac{-7 \pm \sqrt{145}}{16}$
 $= 0.315 \text{ atau } -1.190$

Gantikan nilai-nilai x ke dalam ④.
Apabila $x = 0.315$, $y = 2(0.315) + 3 = 3.630$
Apabila $x = -1.190$, $y = 2(-1.190) + 3 = 0.620$
Penyelesaian ialah $x = 0.315, y = 3.630$ dan $x = -1.190, y = 0.620$.

1. $x + 3y + 1 = 0$ ①
 $x^2 - 10y^2 + xy + 4 = 0$ ②
Dari ①: $x = -3y - 1$ ③
Gantikan ③ ke dalam ②:
 $(-3y - 1)^2 - 10y^2 + (-3y - 1)y + 4 = 0$
 $9y^2 + 6y + 1 - 10y^2 - 3y^2 - y + 4 = 0$
 $-4y^2 + 5y + 5 = 0$

$y = \frac{-5 \pm \sqrt{5^2 - 4(-4)(5)}}{2(-4)}$
 $= \frac{-5 \pm \sqrt{105}}{-8}$
 $= -0.656 \text{ atau } 1.906$
Gantikan nilai-nilai y ke dalam ③.
Apabila $y = -0.656$, $x = -3(-0.656) - 1 = 0.968$
Apabila $y = 1.906$, $x = -3(1.906) - 1 = -6.718$
Penyelesaian ialah $x = 0.968, y = -0.656$ dan $x = -6.718, y = 1.906$.

3. $\frac{3x}{2} + \frac{4y}{3} = 5x^2 - 4y = x + 1$

$\frac{3x}{2} + \frac{4y}{3} = x + 1$ ①
 $5x^2 - 4y = x + 1$ ②
Dari ①: $y = \frac{6 - 3x}{8}$ ③
Gantikan ③ ke dalam ②:
 $5x^2 - 4\left(\frac{6 - 3x}{8}\right) = x + 1$
 $5x^2 - 3 + \frac{3}{2}x = x + 1$
 $10x^2 + x - 8 = 0$
 $x = \frac{-1 \pm \sqrt{1^2 - 4(10)(-8)}}{2(10)} = \frac{-1 \pm \sqrt{321}}{20}$
 $= 0.846 \text{ atau } -0.946$

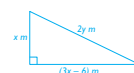
Gantikan nilai-nilai x ke dalam ③.
Apabila $x = 0.846$, $y = \frac{6 - 3(0.846)}{8} = 0.433$
Apabila $x = -0.946$, $y = \frac{6 - 3(-0.946)}{8} = 1.105$
Penyelesaian ialah $x = 0.846, y = 0.433$ dan $x = -0.946, y = 1.105$.

C. Selesaikan masalah yang berikut.

Solve the following problems.

1. David mempunyai sebidang tanah yang berbentuk segi tiga bersudut tegak. Sempanan paling panjang tanah itu ialah 2y m. Dua sempanan lagi masing-masing ialah x dan (3x - 6) m. Dia menggunakan dawai pagar sepanjang 60 m untuk memagari tanah itu. Cari panjang, dalam m, setiap sempanan tanah itu.
David had a piece of right-angled triangle shaped land. The longest side of the land is 2y m. The other two sides are x m and (3x - 6) m respectively. He fenced off the land with 60 m of fencing wire. Find the length, in m, of each side of the land.

Perimeter tanah = 60 m
 $2y + x + (3x - 6) = 60$
 $2y + 4x = 66$
 $y + 2x = 33$ ①

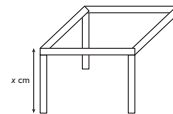


Berdasarkan teorem Pythagoras,
 $x^2 + (3x - 6)^2 = (2y)^2$ ②
Gantikan ① ke dalam ②:
 $x^2 + (3x - 6)^2 = 4(33 - 2x)^2$
 $x^2 + 9x^2 - 36x + 36 = 4(356 - 528x + 16x^2)$
 $6x^2 - 492x + 4(320) = 0$
 $x^2 - 82x + 720 = 0$
 $(x - 10)(x - 72) = 0$
 $x = 10 \text{ atau } 72$

Tetapi $x < 60$.
Maka, $x = 10$
 $2y + 2(33 - 2(10)) = 66$
 $3x - 6 = 3(10) - 6 = 24$

Panjang sempanan tanah itu ialah 10 m, 24 m dan 26 m.

2. Rajah di sebelah menunjukkan sebuah rangka meja yang dibina daripada lapan batang rod keluli. Tinggi meja itu ialah x cm dan luas permukaan atas meja ialah 4 000 cm². Panjang permukaan atas meja melebihi tingginya sebanyak 20 cm. Diberi jumlah panjang rod keluli yang digunakan ialah 500 cm. Cari ukuran, dalam cm, panjang dan lebar permukaan atas meja itu.
The diagram shows the frame of a table which is constructed from eight steel rods. The height of the table is x cm and the area of the table top is 4 000 cm². The length of the table top exceeds its height by 20 cm. Given the total length of steel rods used is 500 cm. Find the measurements, in cm, of the length and width of the table top.



Katakan:
Lebar permukaan atas meja = y cm
Panjang permukaan atas meja = (x + 20) cm
Jumlah panjang rod keluli = 500 cm
 $2y + 2(x + 20) + 4x = 500$
 $2y + 2x + 40 + 4x = 500$
 $2y + 6x = 460$
 $y + 3x = 230$
 $y = 230 - 3x$ ①
Luas permukaan atas meja = 4 000 cm²
 $(x + 20)y = 4 000$ ②

Gantikan ① ke dalam ②:
 $(x + 20)(230 - 3x) = 4 000$
 $230x - 3x^2 + 4 600 - 60x = 4 000$
 $-3x^2 + 170x + 600 = 0$
 $3x^2 - 170x - 600 = 0$
 $(3x + 10)(x - 60) = 0$
 $x = -\frac{10}{3}$ atau 60
Tetapi $x > 0$.
Maka, $x = 60$
 $x + 20 = 60 + 20 = 80$
 $y = 230 - 3(60) = 50$

Maka, panjang permukaan atas meja ialah 80 cm dan lebarnya ialah 50 cm.

PRAKTIK FORMATIF Kertas 2

ANALISIS SOALAN SPM				
Subtopik	2013	2014	2015	2016
	41	S.1	S.1	S.1
				S.3

Jawab semua soalan.
Answer all the questions.

1. Selesaikan persamaan serentak $x - 2y - 1 = 0$ dan $2x^2 + 3y^2 - 6xy = -4$. Berikan jawapan betul kepada tiga tempat perpuluhan.
Solve the simultaneous equations $x - 2y - 1 = 0$ and $2x^2 + 3y^2 - 6xy = -4$. Give the answers correct to three decimal places.

$$\begin{aligned} x - 2y - 1 &= 0 & [5] \\ x &= 1 + 2y & \text{..... ①} \\ 2x^2 + 3y^2 - 6xy &= -4 & \text{..... ②} \end{aligned}$$

Gantikan ① ke dalam ②.

$$\begin{aligned} 2(1 + 2y)^2 + 3y^2 - 6(1 + 2y)y &= -4 \\ 2(1 + 4y + 4y^2) + 3y^2 - 6(y + 2y^2) &= -4 \\ 2 + 8y + 8y^2 + 3y^2 - 6y - 12y^2 + 4 &= 0 \\ -y^2 + 2y + 6 &= 0 \\ y^2 - 2y - 6 &= 0 \end{aligned}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{28}}{2}$$

$$= 3.646 \text{ atau } -1.646$$

Apabila $y = 3.646$,
 $x = 1 + 2(3.646) = 8.292$
Apabila $y = -1.646$,
 $x = 1 + 2(-1.646) = -2.292$
Penyelesaian ialah $x = 8.292, y = 3.646$ dan $x = -2.292, y = -1.646$.

2. Selesaikan persamaan serentak yang berikut:
Solve the following simultaneous equations:

$$\begin{aligned} 3x + y &= 4 & [5] \\ x^2 + 3y^2 + 5xy - 9 &= 0 & \text{..... ②} \end{aligned}$$

Gantikan ① ke dalam ②.

$$\begin{aligned} x^2 + 3(4 - 3x)^2 + 5x(4 - 3x) - 9 &= 0 \\ x^2 + 3(16 - 24x + 9x^2) + 20x - 15x^2 - 9 &= 0 \\ 13x^2 - 52x + 39 &= 0 \\ x^2 - 4x + 3 &= 0 \\ (x - 3)(x - 1) &= 0 \\ x &= 3 \text{ atau } 1 \end{aligned}$$

Apabila $x = 3, y = 4 - 3(3) = -5$
Apabila $x = 1, y = 4 - 3(1) = 1$
Penyelesaian ialah $x = 3, y = -5$ dan $x = 1, y = 1$.

3. Selesaikan persamaan serentak yang berikut:
Solve the following simultaneous equations:

$$\begin{aligned} 3x + y &= 9 & [5] \\ 2x^2 - xy - y &= -1 & \text{..... ②} \end{aligned}$$

Gantikan ① ke dalam ②.

$$\begin{aligned} 2x^2 - x(9 - 3x) - (9 - 3x) &= -1 \\ 2x^2 - 9x + 3x^2 - 9 + 3x &= -1 \\ 5x^2 - 6x - 8 &= 0 \\ (5x + 4)(x - 2) &= 0 \\ x &= -\frac{4}{5} \text{ atau } 2 \end{aligned}$$

Apabila $x = -\frac{4}{5}, y = 9 - 3(-\frac{4}{5}) = 11\frac{2}{5}$
Apabila $x = 2, y = 9 - 3(2) = 3$
Penyelesaian ialah $x = -\frac{4}{5}, y = 11\frac{2}{5}$ dan $x = 2, y = 3$.

4. Selesaikan persamaan serentak yang berikut:
Solve the following simultaneous equations:

$$\begin{aligned} 2x - y - 4 &= 0 & [5] \\ x^2 - 2y^2 - 3y + 1 &= 0 & \text{..... ②} \end{aligned}$$

Beri jawapan betul kepada tiga tempat perpuluhan.
Give the answers correct to three decimal places.

Gantikan ① ke dalam ②.

$$\begin{aligned} x^2 - 2(2x - 4)^2 - 3(2x - 4) + 1 &= 0 \\ x^2 - 2(4x^2 - 16x + 16) - (6x - 12) + 1 &= 0 \\ x^2 - 8x^2 + 32x - 32 - 6x + 12 + 1 &= 0 \\ -7x^2 + 26x - 19 &= 0 \\ 7x^2 - 26x + 19 &= 0 \\ (7x - 19)(x - 1) &= 0 \\ x &= -2.7143 \text{ atau } 1 \end{aligned}$$

Apabila $x = 2.7143$,
 $y = 2(2.7143) - 4 = 1.4286$
Apabila $x = 1$,
 $y = 2(1) - 4 = -2$
Penyelesaian ialah $x = 2.714, y = 1.429$ dan $x = 1, y = -2$.

5. Selesaikan persamaan serentak yang berikut:
Solve the following simultaneous equations:

$$\begin{aligned} 3x - y + 5 &= 0 & [5] \\ 4x^2 + y^2 - 2xy &= 12 & \text{..... ②} \end{aligned}$$

Gantikan ① ke dalam ②.

$$\begin{aligned} 4x^2 + (3x + 5)^2 - 2x(3x + 5) &= 12 \\ 4x^2 + 9x^2 + 30x + 25 - 6x^2 - 10x - 12 &= 0 \\ 7x^2 + 20x + 13 &= 0 \\ (7x + 13)(x + 1) &= 0 \\ x &= -\frac{13}{7} \text{ atau } -1 \end{aligned}$$

Apabila $x = -\frac{13}{7}, y = 3(-\frac{13}{7}) + 5 = -\frac{4}{7}$
Apabila $x = -1, y = 3(-1) + 5 = 2$
Penyelesaian ialah $x = -\frac{13}{7}, y = -\frac{4}{7}$ dan $x = -1, y = 2$.

6. Selesaikan persamaan serentak $y - 2x = 8$ dan $y - xy = 7x$. Berikan jawapan betul kepada dua tempat perpuluhan.
Solve the simultaneous equations $y - 2x = 8$ and $y - xy = 7x$. Give the answers correct to two decimal places.

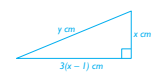
$$\begin{aligned} y - 2x &= 8 & [5] \\ y - xy &= 7x & \text{..... ②} \end{aligned}$$

Gantikan ① ke dalam ②.

$$\begin{aligned} (2x + 8) - x(2x + 8) &= 7x \\ 2x + 8 - 2x^2 - 8x &= 7x \\ 2x^2 + 13x - 8 &= 0 \\ x &= \frac{-13 \pm \sqrt{13^2 - 4(2)(-8)}}{2(2)} \\ &= \frac{-13 \pm \sqrt{233}}{4} \\ &= 0.5661 \text{ atau } -7.0661 \end{aligned}$$

Apabila $x = 0.5661$,
 $y = 2(0.5661) + 8 = 9.1322$
Apabila $x = -7.0661$,
 $y = 2(-7.0661) + 8 = -6.1322$
Penyelesaian ialah $x = 0.57, y = 9.13$ dan $x = -7.07, y = -6.13$.

7. Seutas dawai yang panjangnya 30 cm dibengkokkan untuk menjadi sebuah segi tiga bersudut tegak. Sisi terpanjang segi tiga itu ialah y cm dan dua sisi yang lain masing-masing ialah x cm dan $3(x - 1)$ cm. Cari panjang, dalam cm, setiap sisi segi tiga itu.
A piece of wire of length 30 cm is bent to form a right-angled triangle. The longest side of the triangle is y cm and the other two sides are x cm and $3(x - 1)$ cm respectively. Find the length, in cm, of each side of the triangle.



Perimeter segi tiga = 30 cm
Maka, $x + 3(x - 1) + y = 30$
 $x + 3x - 3 + y = 30$
 $y = 33 - 4x$ ①

Berdasarkan teorem Pythagoras,
 $x^2 + [3(x - 1)]^2 = y^2$
 $x^2 + 9(x^2 - 2x + 1) = y^2$
 $10x^2 - 18x + 9 = y^2$ ②

Gantikan ① ke dalam ②.

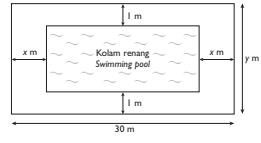
$$\begin{aligned} 10x^2 - 18x + 9 &= (33 - 4x)^2 \\ 10x^2 - 18x + 9 &= 1089 - 264x + 16x^2 \\ 6x^2 - 246x + 1080 &= 0 \\ x^2 - 41x + 180 &= 0 \\ (x - 5)(x - 36) &= 0 \\ x &= 5 \text{ atau } 36 \end{aligned}$$

Tetapi $x < 30$.
Maka, $x = 5$
 $3(x - 1) = 3(5 - 1) = 12$
 $y = 33 - 4(5) = 13$
Maka, panjang sisi segi tiga itu ialah 5 cm, 12 cm dan 13 cm.

FOKUS KBAT

Kemahiran Kognitif: Mengaplikasi dan Menganalisis
Konteks: Penyelesaian Persamaan Serentak, Ketaaksamaan

Rajah di bawah menunjukkan sebuah kolam renang. Lantai di sekeliling kolam renang itu ditutupi dengan jubin. The diagram shows a swimming pool. The floor that surrounds the swimming pool is covered with tiles.



Panjang lantai ialah 30 m dan panjang kolam renang itu adalah melebihi 20 m. Jika perimeter dan luas kolam renang masing-masing ialah 84 m dan 416 m², cari nilai x dan nilai y . Seterusnya, tentukan luas kawasan lantai yang ditutupi dengan jubin.
The length of the floor is 30 m and the length of the swimming pool is more than 20 m. If the perimeter and area of the swimming pool are 84 m and 416 m² respectively, find the values of x and y . Hence, determine the area of the floor that is covered with tiles.

Panjang kolam renang = $(30 - 2x)$ m
Lebar kolam renang = $(y - 2)$ m

Perimeter kolam renang = 84 m
 $2(30 - 2x) + 2(y - 2) = 84$
 $(30 - 2x) + (y - 2) = 42$
 $-2x + y = 14$ ①
 $y = 2x + 14$ ②

Luas kolam renang = 416 m²
 $(30 - 2x)(y - 2) = 416$
 $(15 - x)(y - 2) = 208$ ②

Gantikan ② ke dalam ①.
 $(15 - x)[(2x + 14) - 2] = 208$
 $(15 - x)(2x + 12) = 208$
 $(15 - x)(x + 6) = 104$
 $15x + 90 - x^2 - 6x = 104$
 $-x^2 + 9x - 14 = 0$
 $x^2 - 9x + 14 = 0$
 $(x - 2)(x - 7) = 0$
 $x = 2$ atau 7

Info KBAT
Bentuk dua persamaan yang masing-masing melibatkan perimeter dan luas kolam renang.
Form two equations which involve the perimeter and area of the swimming pool respectively.

Diberi panjang kolam renang > 20 m.
Maka, $30 - 2x > 20$
 $-2x < -10$
 $2x < 10$
 $x < 5$
Jadi, $x = 2$.
Apabila $x = 2, y = 2(2) + 14 = 18$
Luas kawasan yang ditutupi dengan jubin
= $30 \times 18 -$ Luas kolam renang
= $(30 \times 18) - 416$
= $540 - 416$
= 124 m²

BAB 5 INDEKS DAN LOGARITMA
INDICES AND LOGARITHMS

5.1 Indeks dan Hukum Indeks
A. Permutasikan setiap yang berikut.
Simplify each of the following.

CONTOH

(a) $x^2 \times x^3 + x^2 = x^{2+3-2} = x^3$
(b) $(3g^2h^3)^4 = 3^4 \times g^{2 \times 4} \times h^{3 \times 4} = 81g^8h^{12}$
(c) $\frac{(4p^3q)^2 \times pq^5}{2p^4q} = \frac{16p^6q^2 \times pq^5}{2p^4q} = \frac{8p^6 \times 1 \times 4 \times q^{2+5-1}}{1} = 8p^6q^6$

FAKTA UTAMA
Hukum Indeks:
Laws of indices:
• $a^m \times a^n = a^{m+n}$
• $a^m \div a^n = a^{m-n}$
• $(a^m)^n = a^{mn}$
• $(ab)^n = a^n b^n$
• $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

1. $k^6 \times k^9 = k^{6+9} = k^{15}$	2. $y^7 \div y^5 = y^{7-5} = y^2$	3. $m^3 n^2 \times m^2 n = m^{3+2} n^{2+1} = m^5 n^3$
4. $(2a^3 b^2)^5 = 2^5 \times a^{3 \times 5} \times b^{2 \times 5} = 32a^{15} b^{10}$	5. $(h^2)^3 \div h^2 \times h^4 = h^{2 \times 3} \div h^2 \times h^4 = h^{6-2+4} = h^8$	6. $\frac{r^8 s^3 \times r^2 s}{r^2 s} = r^{8+2-2} s^{3+1-1} = r^8 s^3$
7. $\frac{ab^7 \times (6a)^3}{24a^4 b^6} = \frac{ab^7 \times 216a^3}{24a^4 b^6} = \frac{216}{24} a^{1+3-4} b^{7-6} = 9a^0 b = 9a^0 b$	8. $\frac{20p^2 q \times p^3 q}{(2p^3)^4 \times pq^2} = \frac{20p^2 q \times p^3 q}{16p^{12} \times pq^2} = \frac{20}{16} p^{2+3-12} q^{1+1-2} = \frac{5}{4} p^{-7} q^0 = \frac{5p^7}{4q^7}$	9. $\frac{(x^2 y \times xy^2)^2}{x^{11} y^5} = \frac{(x^4 + y^3)^2}{x^{11} y^5} = \frac{x^8 y^6}{x^{11} y^5} = x^{8-11} y^{6-5} = x^{-3} y = \frac{y}{x^3}$

B. Tanpa menggunakan kalkulator, cari nilai bagi setiap yang berikut.
Without using a calculator, find the value of each of the following. 1.1

CONTOH

$$9^{\frac{3}{2}} \times 3^{-6} = (3^{\frac{3}{2}})^2 \times 3^{-6}$$

$$= 3^3 \times 3^{-6}$$

$$= 3^{3+(-6)}$$

$$= 3^{-3}$$

$$= \frac{1}{3^3}$$

$$= \frac{1}{27}$$

1. $5^5 \times 125^2 + 25^4 = 5^5 \times (5^3)^2 + (5^2)^4$

$$= 5^5 \times 5^6 + 5^8$$

$$= 5^{5+6} + 5^8$$

$$= 5^8 + 5^8$$

$$= 2 \times 5^8$$

$$= 125$$

2. $216^{\frac{1}{3}} \times 4^{-1} \times 9^{\frac{1}{2}} = (6^3)^{\frac{1}{3}} \times \frac{1}{4} \times (3^2)^{\frac{1}{2}}$

$$= 6 \times \frac{1}{4} \times 3$$

$$= \frac{3}{2} \times 27$$

$$= 40\frac{1}{2}$$

3. $\sqrt[3]{8 \times 16^{\frac{1}{2}}} = \frac{(2^3)^{\frac{1}{3}} \times (2^4)^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}}}$

$$= \frac{2^1 \times 2^2}{2^1}$$

$$= \frac{2^3 \times 2^2}{2^1}$$

$$= \frac{2^{3+2}}{2^1}$$

$$= \frac{2^5}{2^1}$$

$$= 2^{5-1}$$

$$= 2^4$$

$$= 16$$

C. Permudahkan dan cari nilai bagi setiap yang berikut.
Simplify and find the value of each of the following. 1.1

CONTOH

$$\frac{20^n \times 25^{1+0.5n}}{2^{2n-1}} = \frac{(2^2 \times 5)^n \times (5^2)^{1+0.5n}}{2^{2n-1}}$$

$$= \frac{2^{2n} \times 5^{n+1} \times 5^{n+0.5n}}{2^{2n-1}}$$

$$= \frac{2^{2n} \times 5^{2n+1}}{2^{2n-1}}$$

$$= 2^{2n-(2n-1)} \times 5^{2n+1}$$

$$= 2^1 \times 5^{2n+1}$$

$$= 10 \times 5^{2n}$$

1. $\frac{217^{n+1} \times 3^{1-n}}{7^n} = \frac{(3 \times 7)^{n+1} \times 3^{1-n}}{7^n}$

$$= \frac{3^{n+1} \times 7^{n+1} \times 3^{1-n}}{7^n}$$

$$= \frac{3^{n+1+1-n} \times 7^{n+1}}{7^n}$$

$$= \frac{3^2 \times 7^{n+1}}{7^n}$$

$$= 9 \times 7^{n+1-n}$$

$$= 9 \times 7^1$$

$$= 63$$

2. $\frac{45^{n+1} \times 3^{-2n}}{5^{n-1}} = \frac{(3^2 \times 5)^{n+1} \times 3^{-2n}}{5^{n-1}}$

$$= \frac{3^{2n+2} \times 5^{n+1} \times 3^{-2n}}{5^{n-1}}$$

$$= \frac{3^{2n+2-2n} \times 5^{n+1-(n-1)}}{1}$$

$$= 3^2 \times 5^2$$

$$= 225$$

3. $\frac{56^x}{7^{x+1} \times 4^{\frac{1}{2}x-1}} = \frac{(2^3 \times 7)^x}{7^{x+1} \times 2^{2x-2}}$

$$= \frac{2^{3x} \times 7^x}{7^{x+1} \times 2^{2x-2}}$$

$$= \frac{2^{3x} \times 7^x}{7^{x+1} \times 2^{2x-2}}$$

$$= \frac{2^{3x-(2x-2)} \times 7^{x-(x+1)}}{1}$$

$$= \frac{2^{x+2} \times 7^{-1}}{1}$$

$$= \frac{2^{x+2}}{7}$$

49

D. Diberi $2^n = x$ dan $3^n = y$, ungkapkan setiap yang berikut dalam sebutan x dan/atau y .
Given $2^n = x$ and $3^n = y$, express each of the following in terms of x and/or y . 1.1

CONTOH

1. $9^n + 32^n = (3^2)^n + (2^5)^n$

$$= 3^{2n} + 2^{5n}$$

$$= (2^2 \times 3)^n + (2^5)^n$$

$$= 2^{2n} \times 3^n + 2^{5n}$$

$$= x^2 y + y^2$$

2. $(12)^{n+1} = 12^n \times 12^1$

$$= (2^2 \times 3)^n \times 12$$

$$= 2^{2n} \times 3^n \times 12$$

$$= 12x^2 y$$

3. $6^n - 81^{2n} = (2 \times 3)^n - (3^4)^{2n}$

$$= 2^n \times 3^n - 3^{8n}$$

$$= xy - y^2$$

4. $54^n \times 32^n = (2 \times 3^3)^n \times (2^5)^n$

$$= 2^n \times 3^{3n} \times 2^{5n}$$

$$= 2^{n+5n} \times 3^{3n}$$

$$= 2^{6n} \times 3^{3n}$$

$$= (2^2 \times 3)^n \times 3^n$$

$$= 12x^n \times y^n$$

$$= 12x^n y^n$$

5. $24^{n+1} \times 243^n = 24^n \times 24^1 \times (3^5)^n$

$$= (2^3 \times 3)^n \times 24 \times 3^{5n}$$

$$= 2^{3n} \times 3^n \times 24 \times 3^{5n}$$

$$= 2^3 \times 3^{n+5n} \times 24$$

$$= 2^3 \times 3^{6n} \times 24$$

$$= 2^3 \times 3^{6n} \times 2^3 \times 3^3$$

$$= 2^{3+3} \times 3^{6n+3}$$

$$= 2^6 \times 3^{6n+3}$$

$$= 2^6 \times 3^3 \times 3^{6n}$$

$$= 2^6 \times 3^3 \times (2 \times 3)^{2n}$$

$$= 2^6 \times 3^3 \times 2^{2n} \times 3^{2n}$$

$$= 2^{6+2n} \times 3^{3+2n}$$

$$= 2^{2n+6} \times 3^{2n+3}$$

6. $36^{n+1} + 2(12^n) = \frac{36^n \times 36^1}{2 \times 12^n} + 2 \times 12^n$

$$= \frac{18 \times 36^n}{12^n} + 2 \times 12^n$$

$$= 18 \left(\frac{36}{12}\right)^n + 2 \times 12^n$$

$$= 18(3^n)^n + 2 \times 12^n$$

$$= 18(3^{3n}) + 2 \times 12^n$$

$$= 18 \times 3^{3n} + 2 \times 12^n$$

E. Permudahkan setiap ungkapan yang berikut.
Simplify each of the following expressions. 1.1

CONTOH

$$3^n - 3^{n+1} + 3^{n+2} = 3^n - 3^n(3) + 3^n(3^2)$$

$$= 3^n - 3^n(3) + 3^n(9)$$

$$= 3^n(1 - 3 + 9)$$

$$= 7(3^n)$$

1. $2^{n+5} - 2^{n+2} = 2^n(2^5) - 2^n(2^2)$

$$= 2^n(32) - 2^n(4)$$

$$= 2^n(32 - 4)$$

$$= 2^n(28)$$

$$= 2^n(2^2 \times 7)$$

$$= 7(2^{n+2})$$

2. $3^7 + 4(3^{7+2}) - 3^{7+3} = 3^7 + 4(3^7)(3^2) - 3^7(3^3)$

$$= 3^7 + 3^7(36) - 3^7(27)$$

$$= 3^7(1 + 36 - 27)$$

$$= 10(3^7)$$

3. $7(6^n) + 36^{\frac{n}{2}} + 6^{n+1} = 7(6^n) + (6^2)^{\frac{n}{2}} + 6^n(6)$

$$= 7(6^n) + 6^n + 6^n(6)$$

$$= 6^n(7 + 1 + 6)$$

$$= 14(6^n)$$

4. $5^{x+1} + 5^{x-1} = 5^x(5) + \frac{5^x}{5}$

$$= 5^x(5 + \frac{1}{5})$$

$$= 5^x(\frac{26}{5})$$

$$= \frac{26}{5}(5^{x-1})$$

5. $4^{n+1} - 2^{2n+1} + 8^{\frac{2}{3}n} = 4^n(4) - 2^{2n}(2) + (2^3)^{\frac{2}{3}n}$

$$= 4^n(4) - 4^n(2) + 2^{2n}$$

$$= 4^n(4 - 2) + 4^n(2) + 4^n$$

$$= 4^n(4 - 2 + 1)$$

$$= 3(4^n)$$

50

5.2 Logaritma dan Hukum Logaritma 1.1

A. Ungkapkan setiap yang berikut dalam bentuk logaritma.
Express each of the following in the logarithmic form. 1.1

CONTOH

$$625 = 5^4 \leftarrow y = a^x$$

$$\log_5 625 = 4 \leftarrow \log_a y = x$$

1. $343 = 7^3$

$$\log_7 343 = 3$$

2. $32 = 2^5$

$$\log_2 32 = 5$$

3. $\frac{1}{9} = 3^{-2}$

$$\log_3 \frac{1}{9} = -2$$

4. $M = x^3$

$$\log_x M = 3$$

5. $8^n = k$

$$\log_8 k = n$$

B. Ungkapkan setiap yang berikut dalam bentuk indeks.
Express each of the following in the index form. 1.1

CONTOH

$$\log_5 64 = 6 \leftarrow \log_a y = x$$

$$64 = 2^6 \leftarrow y = a^x$$

1. $\log_5 25 = 2$

$$25 = 5^2$$

2. $\log_3 243 = 5$

$$243 = 3^5$$

3. $\log_5 \frac{1}{125} = -3$

$$\frac{1}{125} = 5^{-3}$$

4. $\log_a 5 = n$

$$5 = a^n$$

5. $x = \log_7 y$

$$y = 7^x$$

C. Cari nilai bagi setiap yang berikut.
Find the value of each of the following. 1.1

CONTOH

(a) $\log_{10} 2.7 = 0.4314$

Tekanan/Press: $\log \frac{2}{10} \frac{7}{10}$

Beri jawapan akhir dalam 4 angka bererti.

1. $\log_{10} 13.8 = 1.140$

2. $\log_{10} \frac{5}{8} = -0.2041$

3. $\log_3 81 = \log_3 3^4 = 4$

4. $\log_5 3 = 4 \log_5 3 \leftarrow \log_a a = 1$

$$= 4(1)$$

$$= 4$$

5. $\log_3 2^7 = 7 \log_3 2 = 7$

6. $\log_5 0.2 = \log_5 \frac{1}{5} = -1$

51

D. Cari nilai x bagi setiap yang berikut.
Find the value of x in each of the following. 1.1

CONTOH

(a) $\log_4 x = -3$

$$x = 4^{-3}$$

$$= \frac{1}{4^3}$$

$$= \frac{1}{64}$$

(b) $\log_6 36 = 2$

$$36 = 6^2$$

Indeksnya sama. Samakan asasnya.

3. $\log_5 125 = 3$

$$125 = 5^3$$

$$x = 5$$

4. $\log_7 7 = -\frac{1}{2}$

$$7 = 7^{-\frac{1}{2}}$$

$$x = 7^{-2}$$

$$= \frac{1}{49}$$

E. Permudahkan setiap yang berikut.
Simplify each of the following. 1.1

CONTOH

(a) $\log_2 2 + 3 \log_5 6 - \log_5 12$

$$= \log_2 2 + \log_5 6^3 - \log_5 12$$

$$= \log_2 \left(\frac{2 \times 6^3}{12}\right)$$

$$= \log_2 36$$

(b) $\log_3 x^2 - \log_9 \frac{1}{y} + \log_3 8$

$$= 2 \log_3 x - \log_9 y^{-1} + \log_3 2^3$$

$$= 2(1) - (-1) \log_3 y + 3 \log_3 2$$

$$= 2 + 1 + 3$$

$$= 6$$

FAKTA UTAMA
Hukum logaritma:
Laws of logarithms:
• $\log_a xy = \log_a x + \log_a y$
• $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
• $\log_a x^m = m \log_a x$

1. $2 \log_5 12 + \log_5 10 - \log_5 15$

$$= \log_5 12^2 + \log_5 10 - \log_5 15$$

$$= \log_5 \left(\frac{12^2 \times 10}{15}\right)$$

$$= \log_5 96$$

2. $\frac{1}{3} \log_8 8 - 2 \log_8 6 + \frac{3}{2} \log_8 9$

$$= \log_8 (8^{\frac{1}{3}}) - \log_8 6^2 + \log_8 (9^{\frac{3}{2}})$$

$$= \log_8 2 - \log_8 36 + \log_8 27$$

$$= \log_8 \left(\frac{2 \times 27}{36}\right)$$

$$= \log_8 \left(\frac{3}{2}\right)$$

3. $6 \log_m xy - 3 \log_m x - 2 \log_m y$

$$= \log_m (xy)^6 - \log_m x^3 - \log_m y^2$$

$$= \log_m \frac{x^6 y^6}{x^3 y^2}$$

$$= \log_m x^3 y^4$$

4. $\log_6 \sqrt{b} + 2 \log_6 c^3 - \log_6 32$

$$= \log_6 b^{\frac{1}{2}} + 3 \log_6 c - \log_6 2^5$$

$$= \frac{1}{2} \log_6 b + 6 \log_6 c - 5 \log_6 2$$

$$= \frac{1}{2} + 6 - 5$$

$$= \frac{3}{2}$$

52

F. Selesaikan setiap yang berikut.

Solve each of the following.

23

CONTOH

Diberi $\log_a 2 = x$ dan $\log_a 5 = y$, ungkapkan setiap yang berikut dalam sebutan x dan y .
Given $\log_a 2 = x$ and $\log_a 5 = y$, express each of the following in terms of x and y .

(a) $\log_a 40$
 $= \log_a (2^3 \times 5)$
 $= \log_a 2^3 + \log_a 5$
 $= 3 \log_a 2 + \log_a 5$
 $= 3x + y$

(b) $\log_a 0.08a$
 $= \log_a \frac{2}{25} a$
 $= \log_a 2 + \log_a a - \log_a 5^2$
 $= \log_a 2 + 1 - 2 \log_a 5$
 $= x + 1 - 2y$

(c) $\log_a \frac{\sqrt{125}}{16a^2}$
 $= \log_a \sqrt{125} - \log_a 16 - \log_a a^2$
 $= \log_a 5^{\frac{3}{2}} - \log_a 2^4 - \log_a a$
 $= \frac{3}{2} \log_a 5 - \log_a 2^4 - \log_a a$
 $= \frac{3}{2} \log_a 5 - 4 \log_a 2 - \log_a a$
 $= \frac{3}{2} y - 4x - 7$

1. Diberi $\log_3 3 = x$ dan $\log_3 7 = y$, ungkapkan setiap yang berikut dalam sebutan x dan y .
Given $\log_3 3 = x$ and $\log_3 7 = y$, express each of the following in terms of x and y .

(a) $\log_3 63$
 $= \log_3 (3^2 \times 7)$
 $= \log_3 3^2 + \log_3 7$
 $= 2 \log_3 3 + \log_3 7$
 $= 2x + y$

(b) $\log_3 \frac{49a}{81}$
 $= \log_3 49 + \log_3 a - \log_3 81$
 $= \log_3 7^2 + \log_3 a - \log_3 3^4$
 $= 2 \log_3 7 + 1 - 4 \log_3 3$
 $= 2y + 1 - 4x$

(c) $\log_3 \frac{\sqrt{343a^2}}{3}$
 $= \log_3 \sqrt{343} + \log_3 \sqrt{a^2} - \log_3 a$
 $= \log_3 7^{\frac{3}{2}} + \log_3 a^{\frac{1}{2}} - \log_3 a$
 $= \frac{3}{2} \log_3 7 + \frac{1}{2} \log_3 a - \log_3 a$
 $= \frac{3}{2} y + \frac{1}{2} x - x$

2. Diberi $\log_3 2 = m$ dan $\log_3 5 = n$, ungkapkan setiap yang berikut dalam sebutan m dan n .
Given $\log_3 2 = m$ and $\log_3 5 = n$, express each of the following in terms of m and n .

(a) $\log_3 90$
 $= \log_3 (2 \times 5 \times 3^2)$
 $= \log_3 2 + \log_3 5 + \log_3 3^2$
 $= \log_3 2 + \log_3 5 + 2(1)$
 $= m + n + 2$

(b) $\log_3 0.3$
 $= \log_3 \frac{3}{10}$
 $= \log_3 3 - \log_3 2 \times \log_3 5$
 $= \log_3 3 - \log_3 2 - \log_3 5$
 $= 1 - m - n$

(c) $\log_3 3.75$
 $= \log_3 \frac{15}{4}$
 $= \log_3 3 + \log_3 5 - \log_3 2^2$
 $= 1 + \log_3 5 - 2 \log_3 2$
 $= 1 + n - 2m$

3. Diberi $\log_4 4 = p$ dan $\log_4 5 = q$, ungkapkan setiap yang berikut dalam sebutan p dan q .
Given $\log_4 4 = p$ and $\log_4 5 = q$, express each of the following in terms of p and q .

(a) $\log_4 100$
 $= \log_4 (4 \times 5^2)$
 $= \log_4 4 + \log_4 5^2$
 $= \log_4 4 + 2 \log_4 5$
 $= p + 2q$

(b) $\log_4 0.8x^3$
 $= \log_4 \frac{4}{5} x^3$
 $= \log_4 4 + \log_4 x^3 - \log_4 5$
 $= \log_4 4 + 3 \log_4 x - \log_4 5$
 $= p + 3 - q$

(c) $\log_4 \frac{12.5}{\sqrt{x}}$
 $= \log_4 \frac{25}{2\sqrt{x}}$
 $= \log_4 25 - \log_4 2 - \log_4 x^{\frac{1}{2}}$
 $= \log_4 5^2 - \log_4 2^{\frac{1}{2}} - \frac{1}{2} \log_4 x$
 $= 2 \log_4 5 - \frac{1}{2} \log_4 2 - \frac{1}{2} \log_4 x$
 $= 2q - \frac{1}{2} p - \frac{1}{2}$

53

5.3 Penukaran Asas Logaritma

A. Cari nilai setiap logaritma dengan menukarkan asasnya kepada asas 10.

Find the value of each logarithm by changing the base to base 10.

31

CONTOH

(a) $\log_7 8.2 = \frac{\log_{10} 8.2}{\log_{10} 7} = 1.081$

(b) $\log_5 \frac{3}{5} = \frac{\log_{10} (\frac{3}{5})}{\log_{10} 2} = -0.7370$

FAKTA UTAMA

• $\log_a b = \frac{\log_c b}{\log_c a}$
 • $\log_a b = \frac{1}{\log_b a}$

1. $\log_5 13 = \frac{\log_{10} 13}{\log_{10} 5} = 1.594$

2. $\log_8 4.53 = \frac{\log_{10} 4.53}{\log_{10} 8} = 0.7265$

3. $\log_9 \frac{7}{9} = \frac{\log_{10} (\frac{7}{9})}{\log_{10} 6} = -0.1403$

B. Bagi setiap yang berikut, ungkapkan P dalam sebutan Q.

For each of the following, express P in terms of Q.

31

CONTOH

$\log_3 P + \log_9 Q = 2$
 $\log_3 P + \frac{\log_3 Q}{\log_3 9} = \log_3 3^2$
 $\log_3 P + \frac{\log_3 Q}{2} = \log_3 9$
 $\log_3 P + \log_3 Q^{\frac{1}{2}} = \log_3 9$
 $\log_3 P = \log_3 9 - \log_3 \sqrt{Q}$
 $\log_3 P = \log_3 \frac{9}{\sqrt{Q}}$
 $P = \frac{9}{\sqrt{Q}}$

1. $\log_2 P + \log_4 Q = 1$
 $\log_2 P + \frac{\log_2 Q}{\log_2 4} = \log_2 2$
 $\log_2 P + \frac{\log_2 Q}{2} = \log_2 2$
 $\log_2 P + \frac{1}{2} \log_2 Q = \log_2 2$
 $\log_2 P + \log_2 Q^{\frac{1}{2}} = \log_2 2$
 $\log_2 P = \log_2 2 - \log_2 \sqrt{Q}$
 $\log_2 P = \log_2 \frac{2}{\sqrt{Q}}$
 $P = \frac{2}{\sqrt{Q}}$

2. $\log_5 P - 6 \log_{25} Q = 3$
 $\log_5 P - \frac{6 \log_5 Q}{\log_5 25} = \log_5 5^3$
 $\log_5 P - \frac{6 \log_5 Q}{2} = \log_5 125$
 $\log_5 P - 3 \log_5 Q = \log_5 125$
 $\log_5 P - \log_5 Q^3 = \log_5 125$
 $\log_5 P = \log_5 125 + \log_5 Q^3$
 $\log_5 P = \log_5 125Q^3$
 $P = 125Q^3$

3. $4 \log_{49} P - 2 \log_7 Q - 1 = 0$
 $\frac{4 \log_7 P}{\log_7 49} - 2 \log_7 Q - \log_7 7 = 0$
 $\frac{4 \log_7 P}{2} - \log_7 Q^2 - \log_7 7 = 0$
 $2 \log_7 P - \log_7 Q^2 - \log_7 7 = 0$
 $\log_7 P^2 - \log_7 Q^2 - \log_7 7 = 0$
 $\log_7 P^2 = \log_7 7 + \log_7 Q^2$
 $\log_7 P^2 = \log_7 7Q^2$
 $P^2 = 7Q^2$
 $P = \sqrt{7}Q$

55

G. Bagi setiap persamaan yang berikut, ungkapkan y dalam sebutan x.

For each of the following equations, express y in terms of x.

23

CONTOH

$\log_5 xy + 1 = 2 \log_5 x - \log_5 y$
 $\log_5 xy + \log_5 5 = \log_5 x^2 - \log_5 y$
 $\log_5 5xy = \log_5 \frac{x^2}{y}$
 $5xy = \frac{x^2}{y}$
 $y^2 = \frac{x^2}{5} \rightarrow y = \sqrt{\frac{x}{5}}$

1. $3 \log_2 x + 2 \log_2 y = 1$
 $\log_2 x^3 + \log_2 y^2 = 1$
 $\log_2 x^3 y^2 = 2^1$
 $x^3 y^2 = 2$
 $y^2 = \frac{2}{x^3}$
 $y = \sqrt{\frac{2}{x^3}}$

2. $\log_3 xy - 2 = 3 \log_3 x - \log_3 y$
 $\log_3 xy - \log_3 3^2 = \log_3 x^3 - \log_3 y$
 $\log_3 xy - \log_3 9 = \log_3 x^3 - \log_3 y$
 $\log_3 \frac{xy}{9} = \log_3 \frac{x^3}{y}$
 $\frac{xy}{9} = \frac{x^3}{y}$
 $y^2 = 9x^2$
 $y = 3x$

3. $\log_4 (x + y) - 5 \log_4 x = 3$
 $\log_4 (x + y) - \log_4 x^5 = 3$
 $\log_4 \frac{x + y}{x^5} = 3$
 $\frac{x + y}{x^5} = 4^3$
 $x + y = 64x^5$
 $y = 64x^5 - x$

H. Selesaikan setiap yang berikut dengan menggunakan hukum logaritma.

Solve each of the following by using the laws of logarithms.

23

CONTOH

Diberi: $\log_3 x^2 y = 1$ dan/and $\log_3 x^3 y^3 = 4$
 Cari nilai bagi $\log_3 x$ dan $\log_3 y$.
 Find the values of $\log_3 x$ and $\log_3 y$.

$\log_3 x^2 y = 1$
 $\log_3 x^2 + \log_3 y = 1$
 $2 \log_3 x + \log_3 y = 1$ ①

($\times 3$) $\begin{cases} 2 \log_3 x + \log_3 y = 1 & \text{..... ①} \\ 6 \log_3 x + 3 \log_3 y = 3 & \text{..... ②} \end{cases}$

$\log_3 x^2 y^3 = 4$
 $\log_3 x^3 + \log_3 y^3 = 4$
 $4 \log_3 x + 3 \log_3 y = 4$ ③

② - ③: $2 \log_3 x = -1$
 $\log_3 x = -\frac{1}{2}$

Gantikan $\log_3 x = -\frac{1}{2}$ ke dalam ①.
 $2(-\frac{1}{2}) + \log_3 y = 1$
 $-1 + \log_3 y = 1$
 $\log_3 y = 2$

Diberi: $\log_2 xy = 3$ dan/and $\log_2 \frac{x^3}{y^2} = -5$
 Cari nilai bagi $\log_2 x$ dan $\log_2 y$.
 Find the values of $\log_2 x$ and $\log_2 y$.

$\log_2 xy = 3$
 $\log_2 x + \log_2 y = 3$ ①

($\times 2$) $\begin{cases} \log_2 x + \log_2 y = 3 & \text{..... ①} \\ 2 \log_2 x + 2 \log_2 y = 6 & \text{..... ②} \end{cases}$

$\log_2 \frac{x^3}{y^2} = -5$
 $\log_2 x^3 - \log_2 y^2 = -5$
 $3 \log_2 x - 2 \log_2 y = -5$ ③

② + ③: $5 \log_2 x = 1$
 $\log_2 x = \frac{1}{5}$

Gantikan $\log_2 x = \frac{1}{5}$ ke dalam ①.
 $\frac{1}{5} + \log_2 y = 3$
 $\log_2 y = 3 - \frac{1}{5}$
 $\log_2 y = \frac{14}{5}$

54

C. Selesaikan masalah yang berikut.

Solve the following problems.

31

CONTOH

(a) Diberi $\log_4 m = p$, ungkapkan $\log_2 16m^5$ dalam sebutan p .
 Given $\log_4 m = p$, express $\log_2 16m^5$ in terms of p .

$\log_2 16m^5 = \frac{\log_4 16m^5}{\log_4 2} = \frac{\log_4 16 + \log_4 m^5}{\log_4 2}$
 $= \frac{\log_4 2^4 + 5 \log_4 m}{\log_4 2}$
 $= \frac{\log_4 2^4 + 5 \log_4 m}{2}$
 $= \frac{2 \log_4 4 + 5p}{2}$
 $= \frac{2 + 5p}{2}$
 $= 1 + 2.5p$
 $= 4 + 10p$

(b) Diberi $\log_3 4 = x$ dan $\log_3 5 = y$, ungkapkan $\log_3 0.6$ dalam sebutan x dan y .
 Given $\log_3 4 = x$ and $\log_3 5 = y$, express $\log_3 0.6$ in terms of x and y .

$\log_3 0.6 = \frac{\log_3 6}{\log_3 10}$
 $= \frac{\log_3 3 + \log_3 2}{\log_3 2 + \log_3 5}$
 $= \frac{1 + \log_3 2}{\log_3 2 + y}$
 $= \frac{1 + \frac{x}{2}}{\frac{x}{2} + y}$
 $= \frac{2 + x}{x + 2y}$

1. Diberi $\log_3 h = k$, ungkapkan $\log_9 27h^4$ dalam sebutan k .
 Given $\log_3 h = k$, express $\log_9 27h^4$ in terms of k .

$\log_9 27h^4 = \frac{\log_3 27h^4}{\log_3 9}$
 $= \frac{\log_3 27 + \log_3 h^4}{\log_3 3^2}$
 $= \frac{\log_3 3^3 + 4 \log_3 h}{2 \log_3 3}$
 $= \frac{3 \log_3 3 + 4k}{2}$
 $= \frac{3 + 4k}{2}$

2. Diberi $\log_2 2 = x$ dan $\log_2 3 = y$, ungkapkan $\log_{24} 49$ dalam sebutan x dan y .
 Given $\log_2 2 = x$ and $\log_2 3 = y$, express $\log_{24} 49$ in terms of x and y .

$\log_{24} 49 = \frac{\log_2 49}{\log_2 24}$
 $= \frac{\log_2 7^2}{\log_2 (2^3 \times 3)}$
 $= \frac{2 \log_2 7}{\log_2 2^3 + \log_2 3}$
 $= \frac{2 \log_2 7}{3 + y}$
 $= \frac{2}{3x + y}$

3. Diberi $\log_{25} m = n$, ungkapkan $\log_m \frac{m^2}{125}$ dalam sebutan n .
 Given $\log_{25} m = n$, express $\log_m \frac{m^2}{125}$ in terms of n .

$\log_m \frac{m^2}{125} = \log_m m^2 - \log_m 125$
 $= 2 \log_m m - \frac{\log_{25} 125}{\log_{25} m}$
 $= 2 - \frac{\log_{25} 25^{\frac{3}{2}}}{\log_{25} m}$
 $= 2 - \frac{\frac{3}{2} \log_{25} 25}{n}$
 $= 2 - \frac{3}{2n}$
 $= \frac{4n - 3}{2n}$

4. Diberi $\log_2 4 = a$ dan $\log_2 4 = b$, ungkapkan $\log_2 1.8$ dalam sebutan a dan b .
 Given $\log_2 4 = a$ and $\log_2 4 = b$, express $\log_2 1.8$ in terms of a and b .

$\log_2 1.8 = \log_2 \frac{3^2}{5} = \log_2 3^2 - \log_2 5$
 $= 2 \log_2 3 - \log_2 5$
 $= \frac{2}{2} - \frac{1}{\log_2 2}$
 $= \frac{2}{2} - \frac{1}{\log_2 4^{\frac{1}{2}}}$
 $= \frac{2}{2} - \frac{1}{\frac{1}{2} \log_2 4}$
 $= \frac{4}{b} - \frac{2}{a}$

56

5.4 Penyelesaian Persamaan yang Melibatkan Indeks dan Logaritma

A. Selesaikan persamaan yang berikut.
Solve the following equations.

CONTOH

(a) $9^{x+1} = 27$
 $(3^2)^{x+1} = 3^3$
 $3^{2x+2} = 3^3$
 $2x + 2 = 3$ — Jika $a^x = a^y$,
 $2x = 1$ Maka $x = y$.
 $x = \frac{1}{2}$

(b) $625(5^x) = \frac{1}{125^4}$
 $5^4(5^x) = (5^{-3})^4$
 $5^{4+x} = 5^{-12}$
 $4 + x = -12$
 $4x = -4$
 $x = -1$

(c) $2^x + 2^{x+1} = 24$
 $2^x + (2^x)(2) = 24$
 $2^x(1+2) = 24$
 $2^x(3) = 24$
 $2^x = 8$
 $2^x = 2^3$
 $x = 3$

1. $343^{x-1} = 49$
 $(7^3)^{x-1} = 7^2$
 $7^{3x-3} = 7^2$
 $3x - 3 = 2$
 $3x = 5$
 $x = \frac{5}{3}$

2. $16^{y+2} = 32$
 $(2^4)^{y+2} = 2^5$
 $2^{4y+8} = 2^5$
 $4y + 8 = 5$
 $4y = -3$
 $y = -\frac{3}{4}$

3. $\frac{27^x}{3} = \frac{1}{9^4}$
 $(3^3)^x(3^{-1}) = (3^{-2})^4$
 $3^{3x-1} = 3^{-8}$
 $3x - 1 = -8$
 $3x = -7$
 $x = -\frac{7}{3}$

4. $2^n \times 8^{n+1} = 4$
 $2^n \times (2^3)^{n+1} = 2^2$
 $2^n \times 2^{3n+3} = 2^2$
 $2^{n+3n+3} = 2^2$
 $4n + 3 = 2$
 $4n = -1$
 $n = -\frac{1}{4}$

5. $6^x + 6^{x+1} = 252$
 $6^x + (6^x)(6) = 252$
 $6^x(1+6) = 252$
 $6^x(7) = 252$
 $6^x = 36$
 $6^x = 6^2$
 $x = 2$

6. $3^{x+2} - 2(3^{x+1}) = 1$
 $(3^x)(3^2) - 2(3^x)(3) = 1$
 $3^x(9 - 6) = 1$
 $3^x(3) = 1$
 $3^x = \frac{1}{3} = 3^{-1}$
 $x = -1$

B. Selesaikan persamaan yang berikut.
Solve the following equations.

CONTOH

$4^{x-3} = 7^x$
 $\log_{10} 4^{x-3} = \log_{10} 7^x$
 $(x-3) \log_{10} 4 = x \log_{10} 7$
 $x \log_{10} 4 - 3 \log_{10} 4 = x \log_{10} 7$
 $x \log_{10} 4 - x \log_{10} 7 = 3 \log_{10} 4$
 $x(\log_{10} 4 - \log_{10} 7) = 3 \log_{10} 4$
 $x = \frac{3 \log_{10} 4}{\log_{10} 4 - \log_{10} 7} = -7.432$

Gunakan kalkulator untuk mencari nilai ini.

2. $5^{2x} = 4^{x-1}$
 $\log_{10} 5^{2x} = \log_{10} 4^{x-1}$
 $2x \log_{10} 5 = (x-1) \log_{10} 4$
 $2x \log_{10} 5 = x \log_{10} 4 - \log_{10} 4$
 $x \log_{10} 4 = x \log_{10} 5 + \log_{10} 4$
 $x(\log_{10} 4 - \log_{10} 5) = \log_{10} 4$
 $x = \frac{\log_{10} 4}{\log_{10} 4 - 2 \log_{10} 5} = -0.7565$

1. $6^{x+1} = 9^x$
 $\log_{10} 6^{x+1} = \log_{10} 9^x$
 $(x+1) \log_{10} 6 = x \log_{10} 9$
 $x \log_{10} 6 + \log_{10} 6 = x \log_{10} 9$
 $\log_{10} 6 = x \log_{10} 9 - x \log_{10} 6$
 $\log_{10} 6 = x(\log_{10} 9 - \log_{10} 6)$
 $x = \frac{\log_{10} 6}{\log_{10} 9 - \log_{10} 6} = 4.419$

3. $13^{x-1} - 7^{x+3} = 0$
 $13^{x-1} = 7^{x+3}$
 $\log_{10} 13^{x-1} = \log_{10} 7^{x+3}$
 $(x-1) \log_{10} 13 = (x+3) \log_{10} 7$
 $x \log_{10} 13 - \log_{10} 13 = x \log_{10} 7 + 3 \log_{10} 7$
 $x \log_{10} 13 - x \log_{10} 7 = 3 \log_{10} 7 + \log_{10} 13$
 $x(\log_{10} 13 - \log_{10} 7) = 3 \log_{10} 7 + \log_{10} 13$
 $x = \frac{3 \log_{10} 7 + \log_{10} 13}{\log_{10} 13 - \log_{10} 7} = 13.574$

57

C. Selesaikan persamaan logaritma yang berikut.
Solve the following logarithmic equations.

CONTOH

$\log_5(2x+7) - \log_5(x+2) = 1$
 $\log_5 \frac{2x+7}{x+2} = 1$
 $\frac{2x+7}{x+2} = 5^1$
 $2x+7 = 5(x+2)$
 $2x+7 = 5x+10$
 $3x = -3$
 $x = -1$

2. $\log_4 x + 1 = \log_4(x+9)$
 $\log_4 x + \log_4 4 = \log_4(x+9)$
 $\log_4 4x = \log_4(x+9)$
 $4x = x+9$
 $3x = 9$
 $x = 3$

1. $\log_2(5x+2) = \log_2(x-2) + 3$
 $\log_2(5x+2) = \log_2(x-2) + \log_2 8$
 $\log_2(5x+2) = \log_2(x-2) + \log_2 8$
 $5x+2 = 8(x-2)$
 $5x+2 = 8x-16$
 $3x = 18$
 $x = 6$

3. $\log_5(x-8) + \log_5 x = 2$
 $\log_5(x-8)x = 2$
 $x(x-8) = 3^2$
 $x^2 - 8x = 9$
 $x^2 - 8x - 9 = 0$
 $(x+1)(x-9) = 0$
 Diberikan $x > 0$, maka $x = 9$.

D. Selesaikan persamaan logaritma yang berikut.
Solve the following logarithmic equations.

CONTOH

$\log_3 x - \log_3(2x+15) = 0$
 $\log_3 x = \log_3(2x+15)$
 $\log_3(x+1) \log_3 6 = x \log_3 9$
 $x \log_{10} 6 + \log_{10} 6 = x \log_{10} 9$
 $\log_{10} 6 = x \log_{10} 9 - x \log_{10} 6$
 $\log_{10} 6 = x(\log_{10} 9 - \log_{10} 6)$
 $x = \frac{\log_{10} 6}{\log_{10} 9 - \log_{10} 6} = 4.419$

1. $\log_6(3x+4) - 2 \log_6 x = 1$
 $\log_6(3x+4) - \frac{2 \log_6 x}{\log_6 6} = 1$
 $\log_6(3x+4) - \frac{2 \log_6 x}{2} = 1$
 $\log_6(3x+4) - \log_6 x = 1$
 $\log_6 \frac{3x+4}{x} = 1$
 $\frac{3x+4}{x} = 6^1$
 $3x+4 = 6x$
 $3x = 4$
 $x = \frac{4}{3}$

2. $\log_2(4x+3) - 6 \log_8 x = 2$
 $\log_2(4x+3) - \frac{6 \log_2 x}{\log_2 8} = 2$
 $\log_2(4x+3) - \frac{6 \log_2 x}{\log_2 2^3} = 2$
 $\log_2(4x+3) - \frac{6 \log_2 x}{3} = 2$
 $\log_2(4x+3) - 2 \log_2 x = 2$
 $\log_2(4x+3) - \log_2 x^2 = 2$
 $\log_2 \frac{4x+3}{x^2} = 2$
 $\frac{4x+3}{x^2} = 2^2$
 $4x+3 = 4x^2$
 $4x^2 - 4x - 3 = 0$
 $(2x+1)(2x-3) = 0$
 Diberikan $x > 0$, maka $x = \frac{3}{2}$.

58

PRAKTIS FORMATIF Kertas 1

Jawab semua soalan.
Answer all the questions.

1. Ringkaskan: $\frac{(6x^2y)^2}{4x^2y}$
 Simplify:

$\frac{(6x^2y)^2}{4x^2y} = \frac{36x^4y^2}{4x^2y}$
 $= \frac{36}{4} \times x^{4-2} \times y^{2-1}$
 $= 9x^2y^2$

2. Diberi $2^{3k} = k$, $2^l = h$ dan $2^{l-3k} = 6 + 8^k$. Ungkapkan h dalam sebutan k .
 It is given that $2^{3k} = k$, $2^l = h$ and $2^{l-3k} = 6 + 8^k$. Express h in terms of k .

$2^{l-3k} = 6 + 8^k$
 $\frac{2^l}{2^{3k}} = 6 + (2^3)^k$
 $\frac{h}{k} = 6 + 2^{3k}$
 $\frac{h}{k} = 6 + k$
 $h = k(6+k)$

3. Diberi $a = \frac{1}{x^r}$, cari $\log_4 a$.
 Given $a = \frac{1}{x^r}$, find $\log_4 a$.

(a) $\log_4 a = \log_4 \frac{1}{x^r}$
 $= \log_4 x^{-r}$
 $= -r \log_4 x$
 $= -4r(1)$
 $= -4r$
 (b) $8 \log_4 x = 8(\frac{1}{\log_4 a})$
 $= 8(\frac{1}{-4r})$
 $= -2$

Subtopik	2013	2014	2015	2016
5.1	-	5.6	5.7	-
5.2	5.70	5.70	-	5.140
5.3	5.70	5.70	5.6	5.140
5.4	5.6	-	-	5.15

4. Diberi $\log_2 x = m$ dan $\log_2 y = n$, ungkapkan $\log_2 \frac{x}{y^2}$ dalam sebutan m dan n .

Given $\log_2 x = m$ and $\log_2 y = n$, express $\log_2 \frac{x}{y^2}$ in terms of m and n .
 $\log_2 \frac{x}{y^2} = \log_2 x - \log_2 y^2$
 $= \log_2 x - 2 \log_2 y$
 $= m - 2n$

5. Diberi $\log_4 27 = 3$, cari nilai $\log_{27}(\frac{1}{k})$.

(a) $\log_4 27 = 3$
 $27 = 4^3$
 $3^3 = k^2$
 $k = 3$
 (b) $\log_{27}(\frac{1}{k}) = \log_{27} k^{-1}$
 $= -\log_{27} k$
 $= -\frac{\log_4 k}{\log_4 27}$
 $= -\frac{1}{3}$

6. Diberi $\log_6 6 = m$, ungkapkan setiap yang berikut dalam sebutan m .

(a) $\log_6 36$
 $\log_6 36 = \log_6 6^2$
 $= 2 \log_6 6$
 $= 2m$
 (b) $\log_6 1296a^2 = \log_6 1296 + \log_6 a^2$
 $= \log_6 6^4 + \log_6 a^2$
 $= 4 + \frac{2}{m}$
 $= \frac{4m+2}{m}$

59

7. Diberi $\log_3 3 = x$ dan $\log_3 6 = y$, ungkapkan $\log_6 27p^3$ dalam sebutan x dan y .
 Given $\log_3 3 = x$ and $\log_3 6 = y$, express $\log_6 27p^3$ in terms of x and y .

$\log_6 27p^3 = \log_6 27 + \log_6 p^3$
 $= \frac{\log_3 27}{\log_3 6} + \frac{\log_3 p^3}{\log_3 6}$
 $= \frac{\log_3 3^3}{\log_3 2 \cdot 3} + \frac{3 \log_3 p}{\log_3 2 \cdot 3}$
 $= \frac{3 \log_3 3}{\log_3 2 + \log_3 3} + \frac{3 \log_3 p}{\log_3 2 + \log_3 3}$
 $= \frac{3x}{y} + \frac{3}{y}$

8. Diberi $\log_2 3 = m$ dan $\log_2 5 = n$, ungkapkan $\log_4 75$ dalam sebutan m dan n .

$\log_4 75 = \frac{\log_2 75}{\log_2 4}$
 $= \frac{\log_2(3 \times 5^2)}{2}$
 $= \frac{\log_2 3 + 2 \log_2 5}{2}$
 $= \frac{m + 2n}{2}$

9. Selesaikan persamaan:
 Solve the equation:

$3^{2x} - 3^{2x-2} = 24$
 $3^{2x} - \frac{3^{2x}}{3^2} = 24$
 $3^{2x} - \frac{3^{2x}}{9} = 24$
 $3^{2x} - \frac{3^{2x}}{9} = 24$
 $(1 - \frac{1}{9})3^{2x} = 24$
 $(\frac{8}{9})3^{2x} = 24$
 $3^{2x} = 27$
 $3^{2x} = 3^3$
 $2x = 3$
 $x = \frac{3}{2}$

10. Selesaikan persamaan:
 Solve the equation:

$8(2^{2x-4}) = 1$
 $8(2^{2x-4}) = 1$
 $2^{2x-4} = \frac{1}{8} = 2^{-3}$
 $2x-4 = -3$
 $2x = 1$
 $x = \frac{1}{2}$

11. Selesaikan persamaan:
 Solve the equation:

$1 + \log_3 x = \log_3(x+6)$
 $1 + \log_3 x = \log_3(x+6)$
 $\log_3(x+6) - \log_3 x = 1$
 $\log_3(\frac{x+6}{x}) = 1$
 $\frac{x+6}{x} = 3^1$
 $x+6 = 3x$
 $2x = 6$
 $x = 3$

12. Diberi $2^p = 5^k = 10^r$, ungkapkan p dalam sebutan k dan r .
 Given $2^p = 5^k = 10^r$, express p in terms of k and r .

$2^p = 5^k$
 $\log_2 2^p = \log_2 5^k$
 $p \log_2 2 = k \log_2 5$
 $p = k \log_2 5$
 $\log_2 5 = \frac{p}{k}$
 $2^p = 10^r$
 $\log_2 2^p = \log_2 10^r$
 $p \log_2 2 = r \log_2(2 \times 5)$
 $p = r(\log_2 2 + \log_2 5)$
 $p = r(1 + \frac{p}{k})$
 $p = r(\frac{k+p}{k})$
 $pk = kr + pr$
 $pk - pr = kr$
 $p(k-r) = kr$
 $p = \frac{kr}{k-r}$

60

PRAKTIS FORMATIF Kertas 2

Subtopik	2013	2014	2015	2016
5.1	-	5.49	-	-
5.2	-	-	-	-
5.3	-	5.49	-	-
5.4	-	-	-	-

Jawab semua soalan.
Answer all the questions.

1. Diberi bahawa $p = 3^x$ dan $q = 3^y$.
It is given that $p = 3^x$ and $q = 3^y$.

(a) Ungkapkan $\frac{27^{x+y}}{9^y}$ dalam sebutan p dan q .
Express $\frac{27^{x+y}}{9^y}$ in terms of p and q .

(b) Cari $\log_9 \frac{9q^2}{p}$ dalam sebutan x dan y .
Find $\log_9 \frac{9q^2}{p}$ in terms of x and y .

(a) $\frac{27^{x+y}}{9^y} = \frac{3^{3(x+y)}}{3^{2y}}$
 $= 3^{3x+3y-2y}$
 $= 3^{3x+y}$
 $= (3^x)^3 \times 3^y$
 $= p^3 q$

(b) $p = 3^x$ dan $q = 3^y$
 $\log_3 p = x$ dan $\log_3 q = y$
 $\log_9 \frac{9q^2}{p} = \log_9 9q^2 - \log_9 p$
 $= \log_9 9 + \log_9 q^2 - \frac{\log_3 p}{\log_3 9}$
 $= 1 + \frac{\log_3 q^2}{\log_3 3^2} - \frac{\log_3 p}{\log_3 3^2}$
 $= 1 + \frac{2 \log_3 q}{2} - \frac{\log_3 p}{2}$
 $= 1 + y - \frac{x}{2}$

2. (a) Permudahkan:
Simplify:
 $\log_2 (2x+3) - 3 \log_4 x^2 + 2 \log_2 x$ [4]

(b) Seterusnya, selesaikan persamaan:
Hence, solve the equation:
 $\log_2 (2x+3) - 3 \log_4 x^2 + 2 \log_2 x = 3$ [2]

(a) $\log_2 (2x+3) - 3 \log_4 x^2 + 2 \log_2 x$
 $= \log_2 (2x+3) - 3 \left(\frac{\log_2 x^2}{\log_2 4} \right) + \log_2 x^2$
 $= \log_2 (2x+3) - 3 \left(\frac{\log_2 x^2}{2} \right) + \log_2 x^2$
 $= \log_2 (2x+3) - \frac{3}{2} \log_2 x^2 + \log_2 x^2$
 $= \log_2 (2x+3) - \frac{1}{2} \log_2 x^2$
 $= \log_2 (2x+3) - \log_2 (x^2)^{\frac{1}{2}}$
 $= \log_2 (2x+3) - \log_2 x$
 $= \log_2 \frac{2x+3}{x}$

(b) $\log_2 (2x+3) - 3 \log_4 x^2 + 2 \log_2 x = 3$
 Maka, $\log_2 \frac{2x+3}{x} = 3$
 $\frac{2x+3}{x} = 2^3 = 8$
 $2x+3 = 8x$
 $6x = 3$
 $x = \frac{1}{2}$

FOKUS KBAT

Kemahiran Kognitif: Mengaplikasi dan Membanding
Konteks: Penyelesaian Persamaan Indeks

1. Diberi $2^x = 9^y = 24^z$, ungkapkan y dalam sebutan x dan z .
Given that $2^x = 9^y = 24^z$, express y in terms of x and z .

Info KBAT

Gunakan suatu anu untuk mewakili ketiga-tiga nombor indeks itu. Aplikasi hukum indeks dan selanjutnya bandingkan indeks bagi anu itu. Use an unknown to represent the three index numbers. Apply the law of indices and then compare the indices of the unknown.

Katakan $2^x = 9^y = 24^z = k$
 Maka, $2^x = k$, $9^y = k$, $24^z = k$
 $2 = k^{\frac{1}{x}}$, $9 = k^{\frac{1}{y}}$, $24 = k^{\frac{1}{z}}$

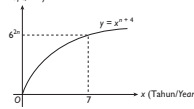
Daripada $3 \times 8 = 24$
 $9^{\frac{1}{y}} \times 2^3 = 24$
 $\left(k^{\frac{1}{y}}\right)^{\frac{1}{y}} \times \left(k^{\frac{1}{x}}\right)^3 = k^{\frac{1}{z}}$
 $k^{\frac{1}{y^2}} \times k^{\frac{3}{x}} = k^{\frac{1}{z}}$
 $k^{\frac{1}{y^2} + \frac{3}{x}} = k^{\frac{1}{z}}$

Bandingkan indeks di kedua-dua belah persamaan.

$\frac{1}{y^2} + \frac{3}{x} = \frac{1}{z}$
 $\frac{1}{2y} = \frac{1}{z} - \frac{3}{x}$
 $\frac{1}{2y} = \frac{x - 3z}{xz}$
 $y = \frac{xz}{2x - 6z}$

Kemahiran Kognitif: Mengaplikasi
Konteks: Penyelesaian Masalah dengan Logaritma

2. Keuntungan, RM_y
Profit, RM_y



Rajah di sebelah ialah graf yang menunjukkan keuntungan bagi sebuah syarikat. Cari keuntungan syarikat itu pada tahun ke-7. Beri jawapan anda kepada puluh ribu ringgit yang terdekat.
The diagram is a graph showing the profits of a company. Find the profits of the company in the 7th year. Give your answer to the nearest ten thousand ringgit.

Info KBAT

Gantikan koordinat (7, 6⁶) ke dalam persamaan $y = x^{n+4}$. Substitute the coordinates of (7, 6⁶) into the equation $y = x^{n+4}$.

Diberi $y = x^{n+4}$

Pada $x = 7$, $y = 6^{6n}$

Maka, $6^{6n} = 7^{n+4}$
 $\log_{10} 6^{6n} = \log_{10} 7^{n+4}$
 $2n \log_{10} 6 = (n+4) \log_{10} 7$
 $2n \log_{10} 6 = n \log_{10} 7 + 4 \log_{10} 7$
 $2n \log_{10} 6 - n \log_{10} 7 = 4 \log_{10} 7$
 $n(2 \log_{10} 6 - \log_{10} 7) = 4 \log_{10} 7$
 $n = \frac{4 \log_{10} 7}{2 \log_{10} 6 - \log_{10} 7}$
 $= 4.753$

Keuntungan syarikat

$= 6^{6(4.753)}$
 $= RM24\ 952\ 024$
 $= RM24\ 950\ 000$

Atau

Keuntungan syarikat
 $= 7^{4(4.753+4)}$
 $= RM24\ 954\ 171$
 $= RM24\ 950\ 000$

BAB 6 GEOMETRI KOORDINAT COORDINATE GEOMETRY

6.1 Jarak di antara Dua Titik

A. Cari jarak di antara setiap pasangan titik yang berikut.
Find the distance between each of the following pairs of points.

CONTOH
 C(3, -1), D(5, 2)
 Jarak CD = $\sqrt{(3-5)^2 + (-1-2)^2}$
 $= \sqrt{(-2)^2 + (-3)^2}$
 $= \sqrt{4+9}$
 $= \sqrt{13}$
 $= 3.606$ unit

FAKTA UTAMA

Jarak PQ Distance of PQ
 $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ atau
 Jarak PQ Distance of PQ
 $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- P(8, 1), Q(-4, 6)
 Jarak PQ = $\sqrt{(8-(-4))^2 + (1-6)^2}$
 $= \sqrt{12^2 + (-5)^2}$
 $= \sqrt{144 + 25}$
 $= \sqrt{169}$
 $= 13$ unit
- M(-5, 13), N(-11, 4)
 Jarak MN = $\sqrt{(-5-(-11))^2 + (13-4)^2}$
 $= \sqrt{6^2 + 9^2}$
 $= \sqrt{36 + 81}$
 $= \sqrt{117}$
 $= 10.817$ unit
- S(1, 14), T(9, 12)
 Jarak ST = $\sqrt{(1-9)^2 + (14-12)^2}$
 $= \sqrt{(-8)^2 + 2^2}$
 $= \sqrt{64 + 4}$
 $= \sqrt{68}$
 $= 8.246$ unit

B. Cari nilai h bagi setiap yang berikut.
Find the values of h for each of the following.

CONTOH
 A(-2, 5), B(h, -10), AB = 17 unit
 $AB = 17$ unit
 $\sqrt{(-2-h)^2 + (5-(-10))^2} = 17$
 $(-2-h)^2 + 15^2 = 289$
 $4 + 4h + h^2 + 225 = 289$
 $h^2 + 4h - 60 = 0$
 $(h-6)(h+10) = 0$
 $h = 6$ atau -10

1. E(3, h), F(-1, 7), EF = 5 unit
 $EF = 5$ unit
 $\sqrt{(3-(-1))^2 + (h-7)^2} = 5$
 $4^2 + (h-7)^2 = 25$
 $16 + h^2 - 14h + 49 = 25$
 $h^2 - 14h + 40 = 0$
 $(h-4)(h-10) = 0$
 $h = 4$ atau 10

2. K(h, 1), L(-3, 6), KL = $\sqrt{41}$ unit
 $KL = \sqrt{41}$ unit
 $\sqrt{(h-(-3))^2 + (1-6)^2} = \sqrt{41}$
 $(h+3)^2 + (-5)^2 = 41$
 $h^2 + 6h + 9 + 25 = 41$
 $h^2 + 6h - 7 = 0$
 $(h-1)(h+7) = 0$
 $h = 1$ atau -7

6.2 Pembahagian Tembereng Garis

A. Cari titik tengah bagi setiap tembereng garis AB.
Find the midpoint of each line segment AB.

CONTOH
 A(6, 11), B(-8, 7)
 Titik tengah AB = $\left(\frac{6+(-8)}{2}, \frac{11+7}{2} \right)$
 $= \left(\frac{-2}{2}, \frac{18}{2} \right)$
 $= (-1, 9)$

FAKTA UTAMA

Titik tengah AB
 Midpoint of AB
 $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

- A(4, -7), B(10, -3)
 Titik tengah AB = $\left(\frac{4+10}{2}, \frac{-7+(-3)}{2} \right)$
 $= \left(\frac{14}{2}, \frac{-10}{2} \right)$
 $= (7, -5)$
- A(-2, -11), B(9, -5)
 Titik tengah AB = $\left(\frac{-2+9}{2}, \frac{-11+(-5)}{2} \right)$
 $= \left(\frac{7}{2}, \frac{-16}{2} \right)$
 $= \left(3\frac{1}{2}, -8 \right)$

B. Cari nilai r bagi setiap yang berikut.
Find the value of r in each of the following.

CONTOH
 A(r, 6), B(1, 8)
 Titik tengah PQ = (5, 4)
 Midpoint of PQ
 $\left(\frac{r+1}{2}, \frac{6+8}{2} \right) = (5, 4)$
 Maka, $\frac{r+1}{2} = 5$
 $r+1 = 10$
 $r = 9$

1. P(11, -9), Q(-1, r)
 Titik tengah PQ = (5, 4)
 Midpoint of PQ
 $\left(\frac{11+(-1)}{2}, \frac{-9+r}{2} \right) = (5, 4)$
 Maka, $\frac{-9+r}{2} = 4$
 $-9+r = 8$
 $r = 17$

C. Bagi setiap yang berikut, ungkapkan h dalam sebutan k .

CONTOH
 F(k, -1), G(4, h)
 Titik tengah FG = (p, 2p)
 Midpoint of FG
 $\left(\frac{k+4}{2}, \frac{-1+h}{2} \right) = (p, 2p)$
 $\frac{k+4}{2} = p$ dan $\frac{-1+h}{2} = 2p$
 Maka, $\frac{-1+h}{2} = 2 \left(\frac{k+4}{2} \right)$
 $-1+h = 2(k+4)$
 $-1+h = 2k+8$
 $h = 2k+9$

1. A(7, h), B(3k, 9)
 Titik tengah AB = (r, 5r)
 Midpoint of AB
 $\left(\frac{7+3k}{2}, \frac{h+9}{2} \right) = (r, 5r)$
 $\frac{7+3k}{2} = r$ dan $\frac{h+9}{2} = 5r$
 Maka, $\frac{h+9}{2} = 5 \left(\frac{7+3k}{2} \right)$
 $h+9 = 35+15k$
 $h = 15k+26$

2. P(2h, k), Q(-6, -2)
 Titik tengah PQ = (3n, n)
 Midpoint of PQ
 $\left(\frac{2h+(-6)}{2}, \frac{k+(-2)}{2} \right) = (3n, n)$
 $\frac{2h-6}{2} = 3n$ dan $\frac{k-2}{2} = n$
 Maka, $\frac{2h-6}{2} = 3 \left(\frac{k-2}{2} \right)$
 $2h-6 = 3k-6$
 $h = \frac{3}{2}k$

D. Cari koordinat titik P yang membahagikan tembereng garis AB.
Find the coordinates of point P which divides the line segment AB.

CONTOH

$A(6, -4), B(-1, 10), AP : PB = 3 : 4$

Koordinat titik P = $\left(\frac{4(6) + 3(-1)}{3+4}, \frac{4(-4) + 3(10)}{3+4} \right)$
 $= \left(\frac{24-3}{7}, \frac{-16+30}{7} \right)$
 $= (3, 2)$

FAKTA UTAMA

Koordinat titik P / Coordinates of point P
 $= \left(\frac{mx_1 + nx_2}{m+n}, \frac{my_1 + ny_2}{m+n} \right)$

1. $A(5, 8), B(11, -7), AP : PB = 2 : 1$
 Koordinat titik P = $\left(\frac{1(5) + 2(11)}{2+1}, \frac{1(8) + 2(-7)}{2+1} \right)$
 $= \left(\frac{5+22}{3}, \frac{8-14}{3} \right)$
 $= (9, -2)$

2. $A(-1, -5), B(9, 3), AP : PB = 3 : 7$
 Koordinat titik P = $\left(\frac{7(-1) + 3(9)}{3+7}, \frac{7(-5) + 3(3)}{3+7} \right)$
 $= \left(\frac{-7+27}{10}, \frac{-35+9}{10} \right)$
 $= \left(2, -\frac{13}{5} \right)$

3. $A(-2, 4), B(-7, 19), AB : PB = 5 : 3$
 Diberi $AB : PB = 5 : 3$, maka $AP : PB = 2 : 3$
 Koordinat titik P = $\left(\frac{3(-2) + 2(-7)}{2+3}, \frac{3(4) + 2(19)}{2+3} \right)$
 $= \left(\frac{-6-14}{5}, \frac{12+38}{5} \right)$
 $= (-4, 10)$

4. $A(1, -3), B(-2, -17), 2AP = 5PB$
 Diberi $2AP = 5PB$, maka $AP : PB = 5 : 2$
 Koordinat titik P = $\left(\frac{2(1) + 5(-2)}{5+2}, \frac{2(-3) + 5(-17)}{5+2} \right)$
 $= \left(\frac{2-10}{7}, \frac{-6-85}{7} \right)$
 $= \left(-\frac{8}{7}, -13 \right)$

E. Cari nilai h dan nilai k dengan keadaan titik P membahagikan tembereng garis AB.
Find the values of h and k where point P divides the line segment AB.

CONTOH

$A(h, 9), B(2, k), P(-2, 6), AP : PB = 3 : 1$

$\left(\frac{1(h) + 3(2)}{3+1}, \frac{1(9) + 3(k)}{3+1} \right) = (-2, 6)$
 $\left(\frac{h+6}{4}, \frac{9+3k}{4} \right) = (-2, 6)$
 Maka, $\frac{h+6}{4} = -2$ dan $\frac{9+3k}{4} = 6$
 $h+6 = -8$ $9+3k = 24$
 $h = -14$ $k = 5$

1. $A(0, h), B(2k, 8), P(12, 3), AP : PB = 4 : 1$
 $\left(\frac{1(0) + 4(2k)}{4+1}, \frac{1(h) + 4(8)}{4+1} \right) = (12, 3)$
 $\left(\frac{8k}{5}, \frac{h+32}{5} \right) = (12, 3)$
 Maka, $\frac{8k}{5} = 12$ dan $\frac{h+32}{5} = 3$
 $8k = 60$ $h+32 = 15$
 $k = 7.5$ $h = -17$

2. $A(h, -1), B(5, k), P(7, 4), AP : PB = 1 : 2$
 $\left(\frac{2(h) + 1(5)}{1+2}, \frac{2(-1) + 1(k)}{1+2} \right) = (7, 4)$
 $\left(\frac{2h+5}{3}, \frac{-2+k}{3} \right) = (7, 4)$
 Maka, $\frac{2h+5}{3} = 7$ dan $\frac{-2+k}{3} = 4$
 $2h+5 = 21$ $-2+k = 12$
 $h = 8$ $k = 14$

3. $A(3h, k), B(-4, -3), P(-13, 6), AP : PB = 2 : 9$
 $\left(\frac{9(3h) + 2(-4)}{2+9}, \frac{9(k) + 2(-3)}{2+9} \right) = (-13, 6)$
 $\left(\frac{27h-8}{11}, \frac{9k-6}{11} \right) = (-13, 6)$
 Maka, $\frac{27h-8}{11} = -13$ dan $\frac{9k-6}{11} = 6$
 $27h-8 = -143$ $9k-6 = 66$
 $h = -5$ $k = 8$

6.3 Luas Poligon

A. Hitung luas setiap poligon berdasarkan bucu-bucu yang diberikan.
Calculate the area of each polygon based on the given vertices.

CONTOH

(a) $A(2, -5), B(0, 4), C(-2, 6)$

Luas segi tiga ABC
 $= \frac{1}{2} \begin{vmatrix} 2 & 0 & -2 \\ -5 & 4 & 6 \\ -5 & -5 & -5 \end{vmatrix}$
 $= \frac{1}{2} [(8+0+10) - (0-8+12)]$
 $= \frac{1}{2} [14]$
 $= 7 \text{ unit}^2$

(b) $A(1, 6), B(5, -1), C(2, -3), D(-2, 0)$

Luas sisi empat ABCD
 $= \frac{1}{2} \begin{vmatrix} 1 & 5 & 2 & -2 \\ 6 & -1 & -3 & 0 \\ 6 & -1 & -3 & 0 \\ 0 & 6 & -1 & -3 \end{vmatrix}$ Pastikan semua bucu disusun ikut arah jam atau lawan arah jam.
 $= \frac{1}{2} [(-1-15+0-12) - (30-2+6+0)]$
 $= \frac{1}{2} [-62]$
 $= 31 \text{ unit}^2$ Luas sentiasa bernilai positif.

FAKTA UTAMA

• Luas segi tiga ABC
Area of triangle ABC
 $= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$
 $= \frac{1}{2} [(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)]$

• Luas sisi empat ABCD
Area of quadrilateral ABCD
 $= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}$
 $= \frac{1}{2} [(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)]$

• Nilai positif atau negatif bagi luas yang diperoleh bergantung sama ada terbu bucu disusun ikut arah jam atau lawan arah jam. Apabila luas yang diperoleh bernilai negatif, tukarkannya kepada positif.
The positive or negative value of the area obtained depends on whether the coordinates of vertices are arranged in a clockwise or anticlockwise direction. When the area obtained is negative, convert it to positive.

1. $P(7, 2), Q(-1, -2), R(8, 0)$
 Luas segi tiga PQR
 $= \frac{1}{2} \begin{vmatrix} 7 & -1 & 8 \\ 2 & -2 & 0 \\ 2 & -2 & 0 \end{vmatrix}$
 $= \frac{1}{2} [(-14+0+16) - (-2-16+0)]$
 $= \frac{1}{2} [20]$
 $= 10 \text{ unit}^2$

2. $E(2, -4), F(-5, 6), G(-2, 3)$
 Luas segi tiga EFG
 $= \frac{1}{2} \begin{vmatrix} 2 & -5 & -2 \\ -4 & 6 & 3 \\ -4 & 6 & 3 \end{vmatrix}$
 $= \frac{1}{2} [(12-15+8) - (20-12+6)]$
 $= \frac{1}{2} [-9]$
 $= 4.5 \text{ unit}^2$

3. $K(0, -1), L(4, -3), M(8, -2), N(-1, 10)$
 Luas sisi empat KLMN
 $= \frac{1}{2} \begin{vmatrix} 0 & 4 & 8 & -1 \\ -1 & -3 & -2 & 10 \\ -1 & -3 & -2 & 10 \\ 0 & -1 & -3 & -2 \end{vmatrix}$
 $= \frac{1}{2} [(0-8+80+1) - (-4-24+2+0)]$
 $= \frac{1}{2} [99]$
 $= 49.5 \text{ unit}^2$

4. $P(5, 9), Q(6, 0), R(3, -1), S(-5, -2)$
 Luas sisi empat PQRS
 $= \frac{1}{2} \begin{vmatrix} 5 & 6 & 3 & -5 \\ 9 & 0 & -1 & -2 \\ 9 & 0 & -1 & -2 \\ 5 & 9 & 6 & 0 \end{vmatrix}$
 $= \frac{1}{2} [(0-6-6-45) - (54+0+5-10)]$
 $= \frac{1}{2} [-106]$
 $= 53 \text{ unit}^2$

B. Selesaikan masalah yang berikut.
Solve the following problems.

CONTOH

(a) Diberi titik $A(-7, 1), B(p, 3)$ dan $C(2, -1)$ adalah segaris. Cari nilai p.
Given the points $A(-7, 1), B(p, 3)$ and $C(2, -1)$ are collinear. Find the value of p.

$\frac{1}{-21} = \frac{p-2}{1-2} = \frac{3-1}{3-1}$
 $|-21 - p + 2 = -(p+6+7)| = 0$
 $|-19 - p - (p+13)| = 0$
 $|-2p - 32| = 0$
 $-2p - 32 = 0$
 $-2p = 32$
 $p = -16$

(b) Bucu-bucu sebuah segi tiga XYZ ialah $X(1, h), Y(-2, 5)$ dan $Z(-1, 3)$. Diberi luas segi tiga itu ialah 4 unit^2 . Cari nilai-nilai h yang mungkin.
The vertices of a triangle XYZ are $X(1, h), Y(-2, 5)$ and $Z(-1, 3)$. Given the area of the triangle is 4 unit^2 . Find the possible values of h.

$\frac{1}{2} \begin{vmatrix} 1 & -2 & -1 \\ h & 5 & 3 \\ 1 & 5 & 3 \end{vmatrix} = 4$
 $|(5-6-h) - (-2h-5+3)| = 8$
 $|-1-h-(-2h+2)| = 8$
 $|h+1| = 8$
 $h+1 = 8$ atau $h+1 = -8$
 $h = 7$ $h = -9$

1. Diberi titik $P(5, k), Q(2, 0)$ dan $R(3, -4)$ adalah segaris. Cari nilai k.
Given the points $P(5, k), Q(2, 0)$ and $R(3, -4)$ are collinear. Find the value of k.

$\frac{1}{0-8+3k} = \frac{2-5}{2-0} = \frac{3-5}{3-0}$
 $|0-8+3k - (2k+0-20)| = 0$
 $|-8+3k-2k+20| = 0$
 $|k+12| = 0$
 $k+12 = 0$
 $k = -12$

2. Diberi titik $K(6, -2), L(3, 1)$ dan $M(n, 2)$ adalah segaris. Cari nilai n.
Given the points $K(6, -2), L(3, 1)$ and $M(n, 2)$ are collinear. Find the value of n.

$\frac{1}{6-3} = \frac{3-1}{1-2} = \frac{2-2}{2-n}$
 $|(6+6-2n) - (-6+n+12)| = 0$
 $|12-2n-(n+6)| = 0$
 $|-3n+6| = 0$
 $-3n+6 = 0$
 $3n = 6$
 $n = 2$

3. Bucu-bucu sebuah segi tiga ABC ialah $A(r, 6), B(2, 0)$ dan $C(-1, -2)$. Diberi luas segi tiga itu ialah 7 unit^2 . Cari nilai-nilai r yang mungkin.
The vertices of a triangle ABC are $A(r, 6), B(2, 0)$ and $C(-1, -2)$. Given the area of the triangle is 7 unit^2 . Find the possible values of r.

$\frac{1}{0-4-6} = \frac{2-1}{2-0} = \frac{6-2}{6-(-2)}$
 $|0-4-6 - (12+0-2r)| = 14$
 $|-10 - (12-2r)| = 14$
 $|2r-22| = 14$
 $2r-22 = 14$ atau $2r-22 = -14$
 $2r = 36$ $2r = 8$
 $r = 18$ $r = 4$

4. Bucu-bucu sebuah segi tiga CDE ialah $C(3, -5), D(-1, q)$ dan $E(4, -8)$. Diberi luas segi tiga itu ialah 10 unit^2 . Cari nilai-nilai q yang mungkin.
The vertices of a triangle CDE are $C(3, -5), D(-1, q)$ and $E(4, -8)$. Given the area of the triangle is 10 unit^2 . Find the possible values of q.

$\frac{1}{3-(-1)} = \frac{-1-4}{-1-3} = \frac{-5-8}{-5-q}$
 $|3q+8-20 - (5+4q-24)| = 20$
 $|3q-12-(4q-19)| = 20$
 $|7-q| = 20$
 $7-q = 20$ atau $7-q = -20$
 $q = -13$ $q = 27$

6.4 Persamaan Garis Lurus

A. Cari kecerunan bagi setiap garis lurus yang menyambungkan dua titik yang diberikan.
Find the gradient of each straight line which is connecting two given points.

CONTOH

(a) $A(2, 7)$ dan $B(-6, -3)$

$m_{AB} = \frac{7-(-3)}{2-(-6)}$
 $= \frac{10}{-4}$
 $= -\frac{5}{2}$

(b)

$m_{PQ} = \frac{-4-2}{-6-3} = \frac{2}{3}$

FAKTA UTAMA

• Kecerunan garis lurus PQ ialah
The gradient of the straight line PQ is
 (a) $m = \frac{y_2 - y_1}{x_2 - x_1}$ atau/or
 $m = \frac{y_1 - y_2}{x_1 - x_2}$
 (b) $m = \frac{\text{Pintasan y}}{\text{Pintasan x}}$ atau/or
 $m = \frac{\text{y-intercept}}{\text{x-intercept}}$

• Persamaan garis lurus PQ ialah
The equation of the straight line PQ is
 (a) $y - y_1 = m(x - x_1)$ atau/or $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
 (b) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ atau/or $\frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$
 (c) $\frac{y}{a} + \frac{x}{b} = 1$

1. $A(5, 2)$ dan $B(-3, 8)$
 $m_{AB} = \frac{2-8}{5-(-3)}$
 $= \frac{-6}{8}$
 $= -\frac{3}{4}$

2. $C(-4, 9)$ dan $D(-6, 1)$
 $m_{CD} = \frac{9-1}{-4-(-6)}$
 $= \frac{8}{-2}$
 $= -4$

3. $G(-1, -10)$ dan $H(2, -3)$
 $m_{GH} = \frac{-10-(-3)}{-1-2}$
 $= \frac{-7}{-3}$
 $= \frac{7}{3}$

4. $P(0, 6)$ dan $Q(7, 0)$
 $m_{PQ} = \frac{0-6}{7-0} = -\frac{6}{7}$

5. $K(0, -10)$ dan $L(6, 0)$
 $m_{KL} = \frac{-10-0}{0-6} = \frac{5}{3}$

6. $A(-12, 0)$ dan $B(0, -9)$
 $m_{AB} = \frac{0-(-9)}{-12-0} = -\frac{3}{4}$

B. Cari nilai k bagi setiap yang berikut.
Find the value of k for each of the following.

CONTOH

$A(k, 5), B(-1, 3), m_{AB} = 3$
 $\frac{5-3}{k-(-1)} = 3$
 $\frac{2}{k-1} = 3$
 $2 = 3k-3$
 $3k = 5$
 $k = \frac{5}{3}$

1. $P(4, k), Q(2, 7), m_{PQ} = 2$
 $\frac{k-7}{4-2} = 2$
 $\frac{k-7}{2} = 4$
 $k-7 = 8$
 $k = 15$

2. $X(1, 8), Y(2k, -3), m_{XY} = -1$
 $\frac{8-(-3)}{1-2k} = -1$
 $\frac{11}{1-2k} = -1$
 $11 = -1+2k$
 $12 = 2k$
 $k = 6$

C. Cari persamaan garis lurus yang mempunyai kecerunan m dan/atau melalui titik-titik yang diberikan.
Find the equation of the straight line that has a gradient of m and/or passes through the given point(s).

CONTOH

(a) $m = -2, (3, 5)$
 $y - 5 = -2(x - 3)$
 $y - 5 = -2x + 6$
 $y = -2x + 11$

(b) $(-2, 1), (6, 5)$
 $\frac{y - 1}{x - (-2)} = \frac{5 - 1}{6 - (-2)}$
 $\frac{y - 1}{x + 2} = \frac{1}{2}$
 $y - 1 = \frac{1}{2}(x + 2)$
 $y = \frac{1}{2}x + 1 + 1$
 $y = \frac{1}{2}x + 2$

1. $m = 4, (-1, 3)$
 $y - 3 = 4(x - (-1))$
 $y - 3 = 4(x + 1)$
 $y - 3 = 4x + 4$
 $y = 4x + 7$

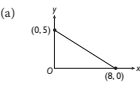
2. $m = -\frac{2}{3}, (7, -6)$
 $y - (-6) = -\frac{2}{3}(x - 7)$
 $y + 6 = -\frac{2}{3}x + \frac{14}{3}$
 $y = -\frac{2}{3}x - \frac{4}{3}$

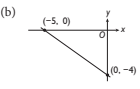
3. $(9, -10), (7, -4)$
 $\frac{y - (-10)}{x - 9} = \frac{-4 - (-10)}{7 - 9}$
 $\frac{y + 10}{x - 9} = -3$
 $y + 10 = -3(x - 9)$
 $y = -3x + 27 - 10$
 $y = -3x + 17$

4. $(-3, -4), (2, -1)$
 $\frac{y - (-4)}{x - (-3)} = \frac{-1 - (-4)}{2 - (-3)}$
 $\frac{y + 4}{x + 3} = \frac{3}{5}$
 $y + 4 = \frac{3}{5}(x + 3)$
 $y = \frac{3}{5}x + \frac{9}{5} - 4$
 $y = \frac{3}{5}x - \frac{11}{5}$

D. Cari persamaan bagi setiap garis lurus yang berikut dalam bentuk pintasan.
Find the equation of each of the following straight lines in the intercept form.

CONTOH

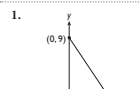
(a)  $\frac{x}{8} + \frac{y}{5} = 1$

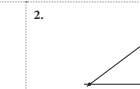
(b)  $\frac{x}{-5} + \frac{y}{-4} = 1$

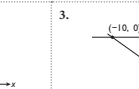
FAKTA UTAMA

Jenis persamaan garis lurus:
Types of equations of straight lines:

- Bentuk am/General form
 $ax + by + c = 0$; a, b, c = pemalar/constants
- Bentuk pintasan/Intercept form
 $\frac{x}{a} + \frac{y}{b} = 1$; a = pintasan-x/x-intercept
b = pintasan-y/y-intercept
- Bentuk kecerunan/Gradient form
 $y = mx + c$; m = kecerunan/gradient
c = pintasan-y/y-intercept

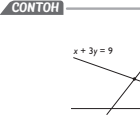
1.  $\frac{x}{4} + \frac{y}{9} = 1$

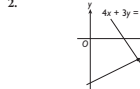
2.  $\frac{x}{-4} + \frac{y}{3} = 1$

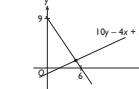
3.  $\frac{x}{-10} + \frac{y}{-7} = 1$

G. Cari koordinat titik persilangan bagi setiap pasangan garis lurus.
Find the coordinates of the point of intersection for each pair of straight lines.

CONTOH

1.  $x + 3y = 9$ ①
 $2x - y + 10 = 0$ ②
 Dari ②: $y = 2x + 10$ ③
 Gantikan ③ ke dalam ①:
 $x + 3(2x + 10) = 9$
 $x + 6x + 30 = 9$
 $7x = -21$
 $x = -3$
 Gantikan $x = -3$ ke dalam ③:
 $y = 2(-3) + 10$
 $y = 4$
 Titik persilangan ialah $(-3, 4)$.

2.  $4x + 3y = 12$ ①
 $\frac{x}{8} + \frac{y}{6} = 1$ ②
 Dari ②: $y = \frac{3}{4}x - 6$ ③
 Gantikan ③ ke dalam ①:
 $4x + 3(\frac{3}{4}x - 6) = 12$
 $4x + \frac{9}{4}x - 18 = 12$
 $\frac{25}{4}x = 30$
 $x = \frac{24}{5}$
 Gantikan $x = \frac{24}{5}$ ke dalam ③:
 $y = \frac{3}{4}(\frac{24}{5}) - 6 = -\frac{12}{5}$
 Titik persilangan ialah $(\frac{24}{5}, -\frac{12}{5})$.

3.  $2x + 3y = 18$ ①
 $x - 2y = 2$ ②
 Dari ②: $x = 2y + 2$ ③
 Gantikan ③ ke dalam ①:
 $2(2y + 2) + 3y = 18$
 $4y + 4 + 3y = 18$
 $7y = 14$
 $y = 2$
 Gantikan $y = 2$ ke dalam ③:
 $x = 2(2) + 2$
 $x = 6$
 Titik persilangan ialah $(6, 2)$.

E. Lengkapkan jadual yang berikut dengan menukarkan persamaan garis lurus kepada bentuk yang dinyatakan.
Complete the following table by changing the equations of straight lines to the stated forms.

Bentuk am General form	Bentuk pintasan Intercept form	Bentuk kecerunan Gradient form
CONTOH $4x - 5y - 20 = 0$	$4x - 5y = 20$ $\frac{4x}{20} - \frac{5y}{20} = 1$ $\frac{x}{5} - \frac{y}{4} = 1$	$-5y = -4x + 20$ $y = -\frac{1}{5}(-4x + 20)$ $y = \frac{4}{5}x - 4$
$3x + 2y - 12 = 0$	$3x + 2y = 12$ $\frac{3x}{12} + \frac{2y}{12} = 1$ $\frac{x}{4} + \frac{y}{6} = 1$	$2y = -3x + 12$ $y = \frac{-3x + 12}{2}$ $y = -\frac{3}{2}x + 6$
$6(-\frac{x}{3} + \frac{y}{6}) = 6$ $-2x + y = 6$ $-2x + y - 6 = 0$	$-\frac{x}{3} + \frac{y}{6} = 1$	$\frac{y}{6} = \frac{x}{3} + 1$ $y = 6(\frac{x}{3} + 1)$ $y = 2x + 6$
$\frac{1}{4}x + y + 3 = 0$ $4(\frac{1}{4}x + y + 3) = 0$ $x + 4y + 12 = 0$	$\frac{1}{4}x + y = -3$ $\frac{x}{4(-3)} + \frac{y}{(-3)} = 1$ $-\frac{x}{12} - \frac{y}{3} = 1$	$y = -\frac{1}{4}x - 3$

F. Cari kecerunan setiap garis lurus berdasarkan persamaan yang diberikan.
Find the gradient of each straight line based on the given equation.

CONTOH

(a) $\frac{x}{6} - \frac{y}{2} = 1$
 Pintasan-x = 6
 Pintasan-y = -2
 Kecerunan, $m = -\frac{(-2)}{6}$
 $= \frac{1}{3}$

1. $\frac{x}{4} + \frac{y}{10} = 1$
 Pintasan-x = 4
 Pintasan-y = 10
 Kecerunan, $m = -\frac{10}{4}$
 $= -\frac{5}{2}$

2. $-\frac{x}{3} + \frac{y}{12} = 1$
 Pintasan-x = -3
 Pintasan-y = 12
 Kecerunan, $m = -\frac{12}{(-3)}$
 $= 4$

(b) $7x + 2y - 5 = 0$
 $2y = -7x + 5$
 $y = mx + c \rightarrow y = -\frac{7}{2}x + \frac{5}{2}$
 Kecerunan, $m = -\frac{7}{2}$

3. $6x - 8y + 3 = 0$
 $-8y = -6x - 3$
 $y = \frac{6x + 3}{8}$
 $y = \frac{3}{4}x + \frac{3}{8}$
 Kecerunan, $m = \frac{3}{4}$

4. $5x - 3y - 6 = 0$
 $-3y = -5x + 6$
 $y = \frac{5x - 6}{3}$
 $y = \frac{5}{3}x - 2$
 Kecerunan, $m = \frac{5}{3}$

6.5 Garis Lurus Selari dan Garis Lurus Serenjang

A. Diberi setiap pasangan garis lurus yang berikut adalah selari. Ungkapkan p dalam sebutan q.
Given each of the following pairs of straight lines are parallel. Express p in terms of q.

CONTOH

$(p + 1)x + 3y = 9, qy - 5x + 7 = 0$
 Bagi $(p + 1)x + 3y = 9$:
 $3y = -(p + 1)x + 9$
 $y = -\frac{p + 1}{3}x + 3$
 $m_1 = -\frac{p + 1}{3}$

Bagi $qy - 5x + 7 = 0$:
 $qy = 5x - 7$
 $y = \frac{5}{q}x - \frac{7}{q}$
 $m_2 = \frac{5}{q}$

Maka, $-\frac{p + 1}{3} = \frac{5}{q}$ ← Bagi garis lurus selari, $m_1 = m_2$
 $p + 1 = -\frac{15}{q}$
 $p = -\frac{15}{q} - 1$

FAKTA UTAMA

- Garis lurus selari
Parallel straight lines
 $m_1 = m_2$
- Garis lurus serenjang
Perpendicular straight lines
 $m_1 m_2 = -1$
atau/or
 $m_2 = -\frac{1}{m_1}$

1. $px - 4y = 5, (q - 2)x + 6y = 3$
 Bagi $px - 4y = 5$:
 $4y = px - 5$
 $y = \frac{p}{4}x - \frac{5}{4}$
 $m_1 = \frac{p}{4}$

Bagi $(q - 2)x + 6y = 3$:
 $6y = -(q - 2)x + 3$
 $y = -\frac{q - 2}{6}x + \frac{1}{2}$
 $m_2 = -\frac{q - 2}{6}$

Maka, $\frac{p}{4} = -\frac{q - 2}{6}$
 $p = -\frac{2q}{3} + \frac{4}{3}$

2. $(2p + 3)x - 8y = 7, 2y - qx - 4 = 0$
 Bagi $(2p + 3)x - 8y = 7$:
 $8y = (2p + 3)x - 7$
 $y = \frac{2p + 3}{8}x - \frac{7}{8}$
 $m_1 = \frac{2p + 3}{8}$

Bagi $2y - qx - 4 = 0$:
 $2y = qx + 4$
 $y = \frac{q}{2}x + 2$
 $m_2 = \frac{q}{2}$

Maka, $\frac{2p + 3}{8} = \frac{q}{2}$
 $p = 2q - \frac{3}{2}$

B. Cari persamaan garis lurus yang melalui titik P dan selari dengan garis lurus yang diberikan.
Find the equation of the straight line that passes through point P and is parallel to the given straight line.

CONTOH

$P(-6, -4), 4x + 3y = 10$
 $3y = -4x + 10$
 $y = -\frac{4}{3}x + \frac{10}{3}$
 Maka, $m = -\frac{4}{3}$

Persamaan garis lurus yang dicari ialah
 $y - (-4) = -\frac{4}{3}[x - (-6)]$
 $y - (-4) = -\frac{4}{3}x - 8$
 $y = -\frac{4}{3}x - 8 - 4$
 $y = -\frac{4}{3}x - 12$

1. $P(5, -4), \frac{x}{3} - \frac{y}{6} = 1$
 $\frac{x}{3} - \frac{y}{6} = 1$
 $-\frac{y}{6} = -\frac{x}{3} + 1$
 $y = 2x - 6$
 Maka, $m = 2$

Persamaan garis lurus yang dicari ialah
 $y - (-4) = 2(x - 5)$
 $y + 4 = 2x - 10$
 $y = 2x - 14$

Cara lain untuk mencari nilai m:
 Diberi $\frac{x}{3} - \frac{y}{6} = 1$:
 $m = -\frac{(-6)}{3} = 2$

2. $P(8, 5)$, $2y - x + 4 = 0$

$$2y - x + 4 = 0$$

$$2y = x - 4$$

$$y = \frac{1}{2}x - 2$$

Maka, $m = \frac{1}{2}$

Persamaan garis lurus yang dicari ialah

$$y - 5 = \frac{1}{2}(x - 8)$$

$$y - 5 = \frac{1}{2}x - 4$$

$$y = \frac{1}{2}x + 1$$

3. $P(-1, -2)$, $5x + 7y = 9$

$$5x + 7y = 9$$

$$7y = -5x + 9$$

$$y = -\frac{5}{7}x + \frac{9}{7}$$

Maka, $m = -\frac{5}{7}$

Persamaan garis lurus yang dicari ialah

$$y - (-2) = -\frac{5}{7}[x - (-1)]$$

$$y + 2 = -\frac{5}{7}x - \frac{5}{7}$$

$$y = -\frac{5}{7}x - \frac{19}{7}$$

C. Cari persamaan garis lurus PQ dalam setiap yang berikut.
Find the equation of the straight line PQ in each of the following.

CONTOH

Bagi $2y - 3x + 5 = 0$:

$$2y - 3x + 5 = 0$$

$$y = \frac{3}{2}x - \frac{5}{2}$$

$$m = \frac{3}{2}$$

$$m_{PQ} = -\frac{2}{3}$$

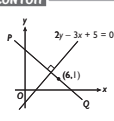
$$m_2 = -\frac{1}{m_1}$$

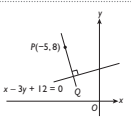
Persamaan garis lurus PQ ialah

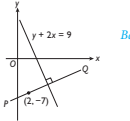
$$y - 1 = -\frac{2}{3}(x - 6)$$

$$y - 1 = -\frac{2}{3}x + 4$$

$$y = -\frac{2}{3}x + 5$$

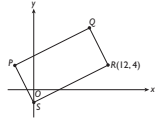
1. 

2. 

3. 

D. Selesaikan masalah yang berikut.
Solve the following problems.

1. Rajah di bawah menunjukkan segi empat tepat PQRS. Titik S terletak pada paksi-y. Persamaan garis lurus PQ ialah $2y - x = 11$.
The diagram shows a rectangle PQRS. Point S lies on the y-axis. The equation of the straight line PQ is $2y - x = 11$.



(a) Cari persamaan garis lurus QR.
Find the equation of the straight line QR.

(b) Cari koordinat titik Q.
Find the coordinates of point Q.

(c) Hitung luas segi empat tepat PQRS.
Calculate the area of the rectangle PQRS.

(a) Persamaan garis lurus PQ: $2y - x = 11$

$$y = \frac{1}{2}x + \frac{11}{2}$$

Maka, $m_{PQ} = \frac{1}{2}$ dan $m_{QR} = -2$.

Persamaan garis lurus QR ialah $y - 4 = -2(x - 2)$

$$y - 4 = -2x + 4$$

$$y = -2x + 8$$

(b) Q ialah titik persilangan PQ dan QR.

$$2y - x = 11 \quad \dots \text{ ①}$$

$$y = -2x + 8 \quad \dots \text{ ②}$$

Gantikan ② ke dalam ①.

$$2(-2x + 8) - x = 11$$

$$-4x + 16 - x = 11$$

$$-5x = -5$$

$$x = 1$$

Gantikan $x = 1$ ke dalam ②.

$$y = -2(1) + 8 = 6$$

Koordinat titik Q ialah (1, 6).

(c) PQ dan SR adalah selari. Maka, $m_{SR} = \frac{1}{2}$

$$\frac{s - 4}{0 - 12} = \frac{1}{2}$$

$$s - 4 = -6$$

$$s = -2$$

Koordinat titik S ialah (0, -2).

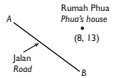
Luas segi empat tepat PQRS = $2 \times$ Luas segi tiga QRS

$$= 2 \times \frac{1}{2} \begin{vmatrix} 0 & 12 & 9 & 0 \\ -2 & 4 & 10 & -2 \end{vmatrix}$$

$$= \frac{1}{2} [(0 + 120 - 18) - (-24 + 36 + 0)]$$

$$= 90 \text{ unit}^2$$

2. Rajah di sebelah menunjukkan sebatang jalan lurus AB yang diwakili oleh persamaan $3y + 2x = 16$. Sebuah stesen petrol terletak di sepanjang jalan AB dan pada jarak terdekat dari rumah Phua.
The diagram shows a straight road AB which is represented by the equation $3y + 2x = 16$. A petrol station is located along the road AB and at the nearest distance from Phua's house.



(a) Cari lokasi stesen petrol itu.
Find the location of the petrol station.

(b) Sebatang lampu isyarat terletak di antara rumah Phua dan stesen petrol itu dengan keadaan jarak rumah Phua dari stesen petrol adalah 3 kali jaraknya dari lampu isyarat itu. Tentukan lokasi lampu isyarat itu.
A traffic light is located in between Phua's house and the petrol station where the distance of Phua's house from the petrol station is 3 times the distance from the traffic light.

(c) Hitung jarak di antara lampu isyarat dan rumah Phua jika 1 unit mewakili 50 m.
Calculate the distance between the traffic light and Phua's house if 1 unit represents 50 m.

(a) Katakan P = Rumah Phua dan S = Stesen petrol

Daripada $3y + 2x = 16 \quad \dots \text{ ①}$

$$y = -\frac{2}{3}x + \frac{16}{3}$$

Maka, $m_{AB} = -\frac{2}{3}$ dan $m_{PS} = \frac{3}{2}$

Persamaan garis lurus PS ialah $y - 13 = \frac{3}{2}(x - 8)$

$$y - 13 = \frac{3}{2}x - 12$$

$$y = \frac{3}{2}x + 1 \quad \dots \text{ ②}$$

Gantikan ② ke dalam ①.

$$3\left(\frac{3}{2}x + 1\right) + 2x = 16$$

$$\frac{13}{2}x + 3 = 16$$

$$\frac{13}{2}x = 13$$

$$x = 2$$

Gantikan $x = 2$ ke dalam ②.

$$y = \frac{3}{2}(2) + 1 = 4$$

Maka, lokasi stesen petrol diwakili oleh koordinat (2, 4).

(b) Katakan T = Lampu isyarat
PS = 3PT, maka PT : TS = 1 : 2

Koordinat titik T = $\left(\frac{2(8) + 1(2)}{2 + 1}, \frac{2(13) + 1(4)}{2 + 1}\right)$

$$= \left(\frac{18}{3}, \frac{30}{3}\right)$$

$$= (6, 10)$$

Maka, lokasi lampu isyarat diwakili oleh koordinat (6, 10).

(c) $PT = \sqrt{(8 - 6)^2 + (13 - 10)^2}$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13}$$

$$= 3.606 \text{ unit}$$

Jarak di antara lampu isyarat dan rumah Phua = $3.606 \times 50 \text{ m}$

$$= 180.3 \text{ m}$$

6.6 Lokus

A. Cari persamaan lokus bagi titik bergerak P(x, y) yang memuaskan syarat yang diberikan.
Find the equation of the locus of a moving point P(x, y) that satisfies the given condition.

CONTOH

(a) Titik P bergerak dengan keadaan jaraknya dari titik Q(5, -1) ialah 6 unit.
A point P moves such that its distance from point Q(5, -1) is 6 units.

$$PQ = 6 \text{ unit}$$

$$\sqrt{(x - 5)^2 + [y - (-1)]^2} = 6$$

$$(x - 5)^2 + (y + 1)^2 = 36$$

$$x^2 - 10x + 25 + y^2 + 2y + 1 = 36$$

$$x^2 + y^2 - 10x + 2y - 10 = 0$$

(b) Titik P bergerak dengan keadaan jaraknya dari titik A(6, 3) dan B(0, -7) adalah sama.
A point P moves such that it is equidistant from the points A(6, 3) and B(0, -7).

$$PA = PB$$

$$\sqrt{(x - 6)^2 + (y - 3)^2} = \sqrt{(x - 0)^2 + [y - (-7)]^2}$$

$$(x - 6)^2 + (y - 3)^2 = x^2 + (y + 7)^2$$

$$x^2 - 12x + 36 + y^2 - 6y + 9 = x^2 + y^2 + 14y + 49$$

$$12x + 20y + 4 = 0$$

$$3x + 5y + 1 = 0$$

Permudahkan.

1. Titik P bergerak dengan keadaan jaraknya dari titik Q(-2, 9) ialah 4 unit.
A point P moves such that its distance from point Q(-2, 9) is 4 units.

$$PQ = 4 \text{ unit}$$

$$\sqrt{[x - (-2)]^2 + (y - 9)^2} = 4$$

$$(x + 2)^2 + (y - 9)^2 = 16$$

$$x^2 + 4x + 4 + y^2 - 18y + 81 = 16$$

$$x^2 + y^2 + 4x - 18y + 69 = 0$$

2. Diberi titik bergerak P dan titik T(1, 4) dengan keadaan PT = 7 unit.
Given a moving point P and a point T(1, 4) is such that PT = 7 units.

$$PT = 7 \text{ unit}$$

$$\sqrt{(x - 1)^2 + (y - 4)^2} = 7$$

$$(x - 1)^2 + (y - 4)^2 = 49$$

$$x^2 - 2x + 1 + y^2 - 8y + 16 = 49$$

$$x^2 + y^2 - 2x - 8y - 32 = 0$$

3. Titik P bergerak dengan keadaan jaraknya dari titik A(-3, 0) dan B(8, 1) adalah sama.
A point P moves such that it is equidistant from the points A(-3, 0) and B(8, 1).

$$PA = PB$$

$$\sqrt{[x - (-3)]^2 + (y - 0)^2} = \sqrt{(x - 8)^2 + (y - 1)^2}$$

$$(x + 3)^2 + y^2 = (x - 8)^2 + (y - 1)^2$$

$$x^2 + 6x + 9 + y^2 = x^2 - 16x + 64 + y^2 - 2y + 1$$

$$22x + 2y - 56 = 0$$

$$11x + y - 28 = 0$$

4. Titik P bergerak dengan keadaan jaraknya dari titik A(-1, -4) dan B(3, -2) adalah sama.
A point P moves such that it is equidistant from the points A(-1, -4) and B(3, -2).

$$PA = PB$$

$$\sqrt{[x - (-1)]^2 + [y - (-4)]^2} = \sqrt{(x - 3)^2 + [y - (-2)]^2}$$

$$(x + 1)^2 + (y + 4)^2 = (x - 3)^2 + (y + 2)^2$$

$$x^2 + 2x + 1 + y^2 + 8y + 16 = x^2 - 6x + 9 + y^2 + 4y + 4$$

$$8x + 4y + 4 = 0$$

$$2x + y + 1 = 0$$

B. Cari persamaan locus bagi titik bergerak P(x, y) yang memuaskan syarat yang diberikan.
Find the equation of the locus of a moving point P(x, y) that satisfies the given condition.

61(B)

CONTOH

Diberi dua titik Q(4, 3) dan R(-1, 0), titik P(x, y) bergerak dengan keadaan PQ : PR = 2 : 1.
Given two points Q(4, 3) and R(-1, 0), a point P(x, y) moves such that PQ : PR = 2 : 1.

$$\frac{PQ}{PR} = \frac{2}{1}$$

$$PQ = 2PR$$

$$\sqrt{(x-4)^2 + (y-3)^2} = 2\sqrt{[x-(-1)]^2 + (y-0)^2}$$

$$(x-4)^2 + (y-3)^2 = 4[(x+1)^2 + y^2]$$

$$x^2 - 8x + 16 + y^2 - 6y + 9 = 4(x^2 + 2x + 1 + y^2)$$

$$x^2 + y^2 - 8x - 6y + 25 = 4x^2 + 4y^2 + 8x + 4$$

$$3x^2 + 3y^2 + 16x + 6y - 21 = 0$$

Persamaan locus bagi titik P ialah $3x^2 + 3y^2 + 16x + 6y - 21 = 0$.

1. Diberi dua titik M(1, 5) dan N(-2, -3), titik P(x, y) bergerak dengan keadaan PM : PN = 3 : 2.
Given two points M(1, 5) and N(-2, -3), a point P(x, y) moves such that PM : PN = 3 : 2.

$$\frac{PM}{PN} = \frac{3}{2}$$

$$2PM = 3PN$$

$$2\sqrt{(x-1)^2 + (y-5)^2} = 3\sqrt{[x-(-2)]^2 + [y-(-3)]^2}$$

$$4[(x-1)^2 + (y-5)^2] = 9[(x+2)^2 + (y+3)^2]$$

$$4(x^2 - 2x + 1 + y^2 - 10y + 25) = 9(x^2 + 4x + 4 + y^2 + 6y + 9)$$

$$4x^2 - 8x + 4 + 4y^2 - 40y + 100 = 9x^2 + 36x + 36 + 9y^2 + 54y + 81$$

$$4x^2 + 4y^2 - 8x - 40y + 104 = 9x^2 + 9y^2 + 36x + 54y + 117$$

$$5x^2 + 5y^2 + 44x + 94y + 13 = 0$$

Persamaan locus bagi titik P ialah $5x^2 + 5y^2 + 44x + 94y + 13 = 0$.

2. Titik P(x, y) bergerak dengan keadaan jaraknya dari titik A(-7, 2) adalah tiga kali jaraknya dari titik B(0, 3).
A point P(x, y) moves such that its distance from point A(-7, 2) is thrice its distance from point B(0, 3).

$$PA = 3PB$$

$$\sqrt{[x-(-7)]^2 + (y-2)^2} = 3\sqrt{(x-0)^2 + (y-3)^2}$$

$$(x+7)^2 + (y-2)^2 = 9[x^2 + (y-3)^2]$$

$$x^2 + 14x + 49 + y^2 - 4y + 4 = 9(x^2 + y^2 - 6y + 9)$$

$$x^2 + y^2 + 14x - 4y + 53 = 9x^2 + 9y^2 - 54y + 81$$

$$8x^2 + 8y^2 - 14x - 50y + 28 = 0$$

$$4x^2 + 4y^2 - 7x - 25y + 14 = 0$$

Persamaan locus bagi titik P ialah $4x^2 + 4y^2 - 7x - 25y + 14 = 0$.

77

C. Selesaikan masalah yang berikut.
Solve the following problems.

62

CONTOH

Suatu titik P bergerak dengan keadaan jaraknya dari titik S(-3, 5) adalah dua kali jaraknya dari titik T(-2, -1).
A point P moves such that its distance from point S(-3, 5) is twice its distance from point T(-2, -1).

(a) Cari persamaan locus bagi titik P.
Find the equation of the locus of point P.

(b) Tentukan sama ada locus bagi titik P menyalang paksi-x atau tidak.
Determine whether the locus of point P intersects the x-axis.

(a) Katakan koordinat titik P ialah (x, y).

(b) Pada paksi-x, y = 0.

$$PS = 2PT$$

$$\sqrt{[x-(-3)]^2 + (y-5)^2} = 2\sqrt{[x-(-2)]^2 + [y-(-1)]^2}$$

$$(x+3)^2 + (y-5)^2 = 4[(x+2)^2 + (y+1)^2]$$

$$x^2 + 6x + 9 + y^2 - 10y + 25 = 4(x^2 + 4x + 4 + y^2 + 2y + 1)$$

$$x^2 + y^2 + 6x - 10y + 34 = 4x^2 + 4y^2 + 16x + 8y + 20$$

$$3x^2 + 3y^2 + 10x + 18y - 14 = 0$$

Gantikan y = 0 ke dalam persamaan locus titik P.
Maka, $3x^2 + 10x - 14 = 0$
 $b^2 - 4ac = 10^2 - 4(3)(-14)$
 $= 268 (> 0)$
Maka, locus bagi titik P menyalang paksi-x pada dua titik yang berlainan.

1. Diberi dua titik E(-7, 4) dan H(1, 6), titik P(x, y) bergerak dengan keadaan lokusnya adalah pembahagi dua sama serenjang bagi garis lurus EH.
Given two points E(-7, 4) and H(1, 6), a point P(x, y) moves such that its locus is the perpendicular bisector of the straight line EH.

(a) Cari persamaan locus bagi titik P.
Find the equation of the locus of point P.

(b) Tentukan sama ada locus bagi titik P selari atau berserenjang dengan garis lurus $4y - x = 5$.
Determine whether the locus of point P is parallel or perpendicular to the straight line $4y - x = 5$.

(a) Locus titik P ialah pembahagi dua sama serenjang bagi EH. Maka,

(b) Bagi $4x + y + 7 = 0$:
 $y = -4x - 7 \rightarrow m_1 = -4$
Bagi $4y - x = 5$:
 $y = \frac{1}{4}x + \frac{5}{4} \rightarrow m_2 = \frac{1}{4}$

$$\frac{PE}{EH} = \frac{PH}{EH}$$

$$\sqrt{[x-(-7)]^2 + (y-4)^2} = \sqrt{(x-1)^2 + (y-6)^2}$$

$$(x+7)^2 + (y-4)^2 = (x-1)^2 + (y-6)^2$$

$$x^2 + 14x + 49 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 - 12y + 36$$

$$16x + 4y + 28 = 0$$

$$4x + y + 7 = 0$$

Maka, locus bagi titik P berserenjang dengan $4y - x = 5$.

2. Diberi dua titik S(4, -7) dan T(8, 1), titik P(x, y) bergerak dengan keadaan $\angle SPT = 90^\circ$.
Given two points S(4, -7) and T(8, 1), a point P(x, y) moves such that $\angle SPT = 90^\circ$.

(a) Cari persamaan locus bagi titik P.
Find the equation of the locus of point P.

(b) Tentukan sama ada locus bagi titik P menyalang paksi-y atau tidak.
Determine whether the locus of point P intersects the y-axis.

(a) Apabila $\angle SPT = 90^\circ$, PS berserenjang dengan PT. Maka, $m_{PS} \times m_{PT} = -1$

(b) Pada paksi-y, x = 0.

$$\frac{y-(-7)}{x-4} \times \frac{y-1}{x-8} = -1$$

$$\frac{(y+7)(y-1)}{(x-4)(x-8)} = -1$$

$$\frac{y^2 + 6y - 7}{x^2 - 12x + 32} = -1$$

$$y^2 + 6y - 7 = -x^2 + 12x - 32$$

$$x^2 + y^2 - 12x + 6y + 25 = 0$$

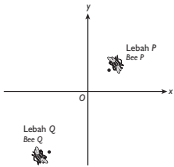
Gantikan x = 0 ke dalam persamaan locus titik P.
Maka, $y^2 + 6y + 25 = 0$
 $b^2 - 4ac = 6^2 - 4(1)(25)$
 $= -64 (< 0)$
Maka, locus bagi titik P tidak menyalang paksi-y.

78

PRAKTIS FORMATIF Kertas 1

Jawab semua soalan.
Answer all the questions.

1. Rajah di bawah menunjukkan kedudukan dua ekor lebah, P dan Q.
The diagram shows the positions of two bees, P and Q.



Koordinat bagi lebah P dan lebah Q masing-masing adalah (4, 4) dan (-8, -12). Kedua-dua lebah itu terbang ke arah satu sama lain pada satu garis lurus dengan halaju berbeza. Halaju lebah Q adalah tiga kali halaju lebah P. Cari jarak lebah Q dari titik asalnya apabila bertemu dengan lebah P.
The coordinates of bee P and bee Q are (4, 4) and (-8, -12) respectively. Both bees fly towards each other on a straight line with different velocities. The velocity of bee Q is three times the velocity of bee P. Find the distance of bee Q from its initial point when it meets with bee P.

Andaikan lebah P dan lebah Q bertemu di titik M. Maka, jarak QM adalah 3 kali jarak PM.



$$M(x, y) = \left(\frac{3(4) + 1(-8)}{3+1}, \frac{3(4) + 1(-12)}{3+1} \right)$$

$$= \left(\frac{12-8}{4}, \frac{12-12}{4} \right)$$

$$= (1, 0)$$

$$\text{Jarak QM} = \sqrt{[1-(-8)]^2 + [0-(-12)]^2}$$

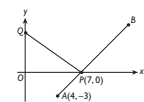
$$= \sqrt{9^2 + 12^2}$$

$$= \sqrt{225}$$

$$= 15 \text{ unit}$$

Subtopik	2012	2014	2015	2016
6.1	-	-	S. 12	-
6.2	S. 136B	S. 120B	-	-
6.3	-	-	-	-
6.4	-	S. 120B	S. 119B	S. 8, 90B
6.5	S. 130B	S. 11	S. 119B	S. 90B
6.6	S. 14	-	-	-

2. Rajah di bawah menunjukkan garis lurus PQ dengan persamaan $\frac{x}{7} + \frac{y}{5} = 1$ bersilang dengan garis lurus AB pada titik P.
The diagram shows a straight line PQ with an equation of $\frac{x}{7} + \frac{y}{5} = 1$ intersects a straight line AB at point P.



(a) Nyatakan pintasan-y bagi PQ.
(b) Cari koordinat bagi titik B jika BP = 2PA.
Find the coordinates of point B if BP = 2PA.

(a) Pintasan-y = 5

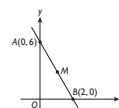
(b) Katakan koordinat titik B ialah (x, y).
 $(7, 0) = \left(\frac{x+2(4)}{1+2}, \frac{y+2(-3)}{1+2} \right)$
 $= \left(\frac{x+8}{3}, \frac{y-6}{3} \right)$
Maka, $\frac{x+8}{3} = 7$ dan $\frac{y-6}{3} = 0$
 $x+8 = 21$ $y-6 = 0$
 $x = 13$ $y = 6$
Koordinat titik B ialah (13, 6).

3. Garis lurus $3y = 5x + h - 6$ menyalang paksi-x pada 4k, dengan keadaan h dan k ialah pemalar. Ungkapkan h dalam sebutan k.
The straight line $3y = 5x + h - 6$ intersects the x-axis at 4k, where h and k are constants. Express h in terms of k.

Pada titik (4k, 0), $3(0) = 5(4k) + h - 6$
 $0 = 20k + h - 6$
 $h = 6 - 20k$

79

4. Rajah di bawah menunjukkan garis lurus AB.
The diagram shows a straight line AB.



Diberi M ialah titik tengah AB. Cari
Given M is the midpoint of AB. Find
(a) Koordinat titik M.
(b) persamaan garis lurus yang berserenjang dengan AB dan melalui M.
the equation of the straight line which is perpendicular to AB and passing through M.

(a) Koordinat titik M = $\left(\frac{0+2}{2}, \frac{6+0}{2} \right)$
 $= (1, 3)$

(b) Kecerunan AB = $\frac{6-0}{0-2} = -3$
Kecerunan garis lurus yang berserenjang dengan AB ialah $\frac{1}{3}$.
Persamaan garis lurus yang berserenjang dengan AB dan melalui M ialah
 $y - 3 = \frac{1}{3}(x - 1)$
 $y - 3 = \frac{1}{3}x - \frac{1}{3}$
 $y = \frac{1}{3}x + \frac{8}{3}$

5. Garis lurus $y = 3x + 8$ adalah selari dengan garis lurus $y = (k - 2)x + 7$, dengan keadaan k ialah pemalar. Tentukan nilai k.
The straight line $y = 3x + 8$ is parallel to the straight line $y = (k - 2)x + 7$, where k is a constant. Determine the value of k.

$k - 2 = 3$
 $k = 5$

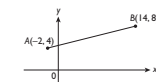
80

6. Persamaan suatu garis lurus diberi oleh $qx - py = 1$, dengan keadaan p dan q ialah pemalar. Cari dalam sebutan p dan q.
The equation of a straight line is given by $qx - py = 1$, where p and q are constants. Find in terms of p and q.
(a) kecerunan garis lurus itu.
(b) kecerunan garis lurus yang berserenjang dengan garis $qx - py = 1$.
the gradient of the straight line which is perpendicular to the line $qx - py = 1$.

(a) $qx - py = 1$
 $py = qx - 1$
 $y = \frac{q}{p}x - \frac{1}{p}$
Maka, kecerunan garis lurus = $\frac{q}{p}$

(b) Kecerunan = $-\frac{q}{p} = -\frac{p}{q}$

7. Rajah di bawah menunjukkan garis lurus AB.
The diagram shows a straight line AB.



Cari
Find
(a) titik tengah AB.
(b) persamaan pembahagi dua sama serenjang bagi garis AB.
the equation of the perpendicular bisector of the line AB.

(a) Titik tengah AB = $\left(\frac{-2+4}{2}, \frac{4+8}{2} \right)$
 $= (1, 6)$

(b) $m_{AB} = \frac{8-4}{4-(-2)} = \frac{4}{6} = \frac{2}{3}$
Kecerunan pembahagi dua sama serenjang bagi garis AB ialah $-\frac{3}{2}$.
Persamaan pembahagi dua sama serenjang bagi garis AB ialah
 $y - 6 = -\frac{3}{2}(x - 1)$
 $y - 6 = -\frac{3}{2}x + \frac{3}{2}$
 $y + 4x = 30$

(a) (i) Luas OPQR

$$= \frac{1}{2} \begin{vmatrix} 0 & 12 & 8 & -5 & 0 \\ 0 & 5 & 14 & 4 & 0 \\ 0 & 0 & 168 & 32 & 0 \end{vmatrix} - (0 + 40 - 70 + 0)$$

$$= \frac{1}{2} |230|$$

$$= 115 \text{ m}^2$$

(ii)

(1, 5) = $\frac{2x + 7}{3}, \frac{2y + 9}{3}$

$$\frac{2x + 7}{3} = 1 \text{ dan } \frac{2y + 9}{3} = 5$$

$$2x + 7 = 3 \quad 2y + 9 = 15$$

$$2x = -4 \quad 2y = 6$$

$$x = -2 \quad y = 3$$

Maka, koordinat titik A ialah (-2, 3).

(b) Persamaan laluan serbuk beledang ialah

$$\sqrt{(x - 7)^2 + (y - 9)^2} = 2$$

$$(x - 7)^2 + (y - 9)^2 = 4$$

$$x^2 - 14x + 49 + y^2 - 18y + 81 = 4$$

$$x^2 + y^2 - 14x - 18y + 126 = 0$$

FOKUS KBAT

Kemahiran Kognitif: Menganalisis
Konteks: Garis Lurus, Lokus

Rajah di sebelah ialah salah Cartes yang menunjukkan kedudukan tiga orang murid. Shaqif berdiri pada suatu kedudukan yang paling dekat kepada Cheng Wei tetapi sama jarak dari Norita dan Ruthra. Cari kedudukan Shaqif.
 The diagram is a Cartesian plane which shows the positions of three students. Shaqif stands at a position nearest to Cheng Wei but equidistant from Norita and Ruthra. Find the position of Shaqif.

Info KBAT
 Titik yang sentiasa sama jarak dari dua titik merupakan lokus pembahagi dua sama serenjang bagi kedua-dua titik itu.
 The point which is always equidistant from two points is the locus of the perpendicular bisector of both points.

Katakan: Kedudukan Norita = N(-4, -2)
 Kedudukan Ruthra = R(2, 10)
 Kedudukan Cheng Wei = C(12, 5)
 Kedudukan Shaqif = S(x, y)

Kecerunan garis lurus NR = $\frac{10 - (-2)}{2 - (-4)}$

Kecerunan pembahagi dua sama serenjang bagi NR = $-\frac{1}{2}$

Titik tengah NR = $\left(\frac{2 + (-4)}{2}, \frac{10 + (-2)}{2}\right) = (-1, 4)$

Persamaan pembahagi dua sama serenjang bagi NR ialah
 $y - 4 = -\frac{1}{2}(x - (-1))$
 $y - 4 = -\frac{1}{2}x - \frac{1}{2}$
 $y = -\frac{1}{2}x + \frac{7}{2}$ ①

Persamaan garis lurus CS ialah
 $y - 5 = 2(x - 12)$
 $y - 5 = 2x - 24$
 $y = 2x - 19$ ②

Kedudukan Shaqif ialah titik persilangan antara garis lurus ① dan garis lurus ②.

Slesaikan ① dan ② secara serentak.

$$y = \frac{12}{6}$$

$$2x - 19 = -\frac{1}{2}x + \frac{7}{2}$$

Maka, $2x = -\frac{1}{2}x + \frac{7}{2}$
 $\frac{5}{2}x = \frac{45}{2}$
 $5x = 45$
 $x = 9$

$y = 2(9) - 19 = -1$
 Kedudukan Shaqif ialah pada titik (9, -1).

BAB 7 STATISTIK STATISTICS

7.1 Sukatan Kecenderungan Memusat

A. Cari min, mod dan median bagi setiap set data yang diberikan. Find the mean, mode and median of each given set of data.

CONTOH

23, 20, 17, 18, 15, 17, 16, 20
 $N = 8$
 $\Sigma x = 23 + 20 + 17 + 18 + 15 + 17 + 16 + 20 = 146$
 $\text{Min}, \bar{x} = \frac{\Sigma x}{N} = \frac{146}{8} = 18.25$
 Mod = 17 dan 20 ← Terdapat dua mod
 Susun semula semua nilai mengikut tertib menaik:
 15, 16, 17, 17, 18, 20, 20, 23
 ↑
 Median
 Median = Min nilai ke-4 dan nilai ke-5
 $= \frac{17 + 18}{2} = 17.5$

1. 2, 5, 7, 6, 10, 7, 9, 8, 4
 $N = 9$
 $\Sigma x = 2 + 5 + 7 + 6 + 10 + 7 + 9 + 8 + 4 = 58$
 $\text{Min}, \bar{x} = \frac{\Sigma x}{N} = \frac{58}{9} = 6.444$
 Mod = 7
 Susun semula semua nilai:
 2, 4, 5, 6, 7, 7, 8, 9, 10
 ↑
 Median
 Median = Nilai ke-5
 $= 7$

2. 8, 2, 11, 9, 4, 5, 13, 9
 $N = 8$
 $\Sigma x = 8 + 2 + 11 + 9 + 4 + 5 + 13 + 9 = 61$
 $\text{Min}, \bar{x} = \frac{\Sigma x}{N} = \frac{61}{8} = 7.625$
 Mod = 9
 Susun semula semua nilai:
 2, 4, 5, 8, 9, 9, 11, 13
 ↑
 Median
 Median = Min nilai ke-4 dan nilai ke-5
 $= \frac{8 + 9}{2} = 8.5$

3. 4.6, 3.1, 4.3, 3.1, 4.3, 2.8, 2.1
 $N = 7$
 $\Sigma x = 4.6 + 3.1 + 4.3 + 3.1 + 4.3 + 2.8 + 2.1 = 24.3$
 $\text{Min}, \bar{x} = \frac{\Sigma x}{N} = \frac{24.3}{7} = 3.471$
 Mod = 3.1 dan 4.3
 Susun semula semua nilai:
 2.1, 2.8, 3.1, 3.1, 4.3, 4.3, 4.6
 ↑
 Median
 Median = Nilai ke-4
 $= 3.1$

B. Cari min, mod dan median bagi setiap set data yang diberikan. Find the mean, mode and median of each given set of data.

CONTOH

Panjang (cm)	9	10	11	12	13
Length (cm)	9	10	11	12	13
Kekerapan	4	5	3	2	1

$\Sigma fx = 4(9) + 5(10) + 3(11) + 2(12) + 1(13) = 156$
 $\Sigma f = 4 + 5 + 3 + 2 + 1 = 15$
 $\text{Min}, \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{156}{15} = 10.4 \text{ cm}$
 Mod = 10 cm
 Median = Nilai ke $\left(\frac{15 + 1}{2}\right) = \text{Nilai ke-8} = 10 \text{ cm}$

1.

Jisim (kg)	35	40	45	50	55
Mass (kg)	35	40	45	50	55
Bilangan murid	3	4	2	7	6

 $\Sigma fx = 3(35) + 4(40) + 2(45) + 7(50) + 6(55) = 1035$
 $\Sigma f = 3 + 4 + 2 + 7 + 6 = 22$
 $\text{Min}, \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1035}{22} = 47.05 \text{ kg}$
 Mod = 50 kg
 Median = Min nilai ke-11 dan nilai ke-12 = 50 kg

2.

Diameter (cm)	13	14	15	16	17	18
Diameter (cm)	13	14	15	16	17	18
Bilangan silinder	5	6	8	7	3	2

 $\Sigma fx = 5(13) + 6(14) + 8(15) + 7(16) + 3(17) + 2(18) = 468$
 $\Sigma f = 5 + 6 + 8 + 7 + 3 + 2 = 31$
 $\text{Min}, \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{468}{31} = 15.10 \text{ cm}$
 Mod = 15 cm
 Median = Nilai ke-16 = 15 cm

C. Selesaikan masalah yang berikut. Solve the following problems.

CONTOH

Diberi set data 3, 6, m, 7, n, dengan mod = min = 6 dan $m \neq n$.
 Given a set of data 3, 6, m, 7, n, where mode = mean = 6 and $m \neq n$.
 (a) Cari nilai m dan nilai n. Find the values of m and n.
 (b) Nyatakan mediannya. State the median.
 (a) Diberi mod = 6. Maka, $m = 6$ atau $n = 6$. Jika $m = 6$, maka $\frac{3 + 6 + 6 + 7 + n}{5} = 6$
 $22 + n = 30$
 $n = 8$
 Dengan cara yang sama, jika $n = 6$, $m = 8$.

Diberi set data 3, 5, 2, 10, 12, 8, m, n, dengan mod = min = 8 dan $m \neq n$.
 Given a set of data 3, 5, 2, 10, 12, 8, m, n, where mode = mean = 8 and $m \neq n$.
 (a) Cari nilai m dan nilai n. Find the values of m and n.
 (b) Nyatakan mediannya. State the median.
 (a) Diberi mod = 8. Maka, $m = 8$ atau $n = 8$. Jika $m = 8$, maka $\frac{3 + 5 + 2 + 10 + 12 + 8 + m}{8} = 8$
 $48 + n = 64$
 $n = 16$
 Dengan cara yang sama, jika $n = 8$, $m = 16$.
 (b) Set data ialah 2, 3, 5, 8, 8, 10, 12, 16
 $\text{Median} = \frac{8 + 8}{2} = 8$

Diberi set data 8, 5, 12, 9, 4, m, 1, 7 mempunyai min 7.
 Given a set of data 8, 5, 12, 9, 4, m, 1, 7, has a mean of 7. Find
 (a) nilai m.
 (b) mod dan median bagi set data itu.
 (a) Diberi min = 7. $\frac{8 + 5 + 12 + 9 + 4 + m + 1 + 7}{8} = 7$
 $46 + m = 56$
 $m = 10$
 (b) Susun semula set data mengikut tertib menaik: 1, 4, 5, 7, 8, 9, 10, 12
 Set data ini tiada mod sebab tiada nilai yang berulang.
 $\text{Median} = \frac{7 + 8}{2} = 7.5$

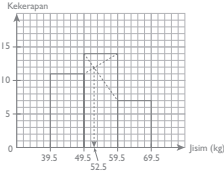
D. Bagi setiap set data terkumpul, tentukan kelas mod dan nilai modnya.
For each set of grouped data, determine the modal class and the mode.

1.47.15

CONTOH

Jisim (kg) Mass (kg)	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
Kekerapan Frequency	8	11	14	7	5

Kelas mod = 50 - 59 kg

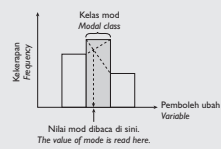


Mod = 52.5 kg

FAKTA UTAMA

• Bagi set data terkumpul, kelas mod ialah kelas yang mempunyai kekerapan tertinggi.
For a set of grouped data, the modal class is the class with the highest frequency.

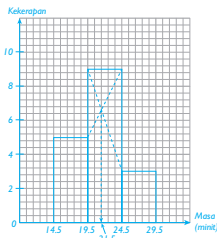
• Mod bagi set data terkumpul boleh dianggarkan dengan melukis sebuah histogram.
The mode of a set of grouped data can be estimated by drawing a histogram.



1.

Masa (minit) Time (minute)	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34
Kekerapan Frequency	4	5	9	3	2

Kelas mod = 20 - 24 minit

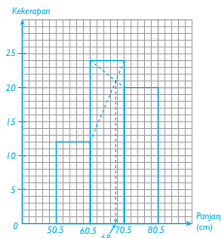


Mod = 21.5 minit

2.

Panjang (cm) Length (cm)	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90
Kekerapan Frequency	6	12	24	20	17

Kelas mod = 61 - 70 cm



Mod = 68 cm

89

E. Cari min bagi setiap set data terkumpul yang diberikan.
Find the mean of each given set of grouped data.

1.8

CONTOH

Jarak (km) Distance (km)	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34
Kekerapan Frequency	8	17	24	20	11

Jarak Distance (km)	Titik tengah Midpoint x	Kekerapan Frequency f	fx
10 - 14	12	8	96
15 - 19	17	17	289
20 - 24	22	24	528
25 - 29	27	20	540
30 - 34	32	11	352
		$\Sigma f = 80$	$\Sigma fx = 1\ 805$

$$\text{Min, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1\ 805}{80} = 22.56 \text{ km}$$

1.

Upah (RM) Wage (RM)	51 - 55	56 - 60	61 - 65	66 - 70	71 - 75
Kekerapan Frequency	8	8	12	7	5

Upah Wage (RM)	Titik tengah Midpoint x	Kekerapan Frequency f	fx
51 - 55	53	8	424
56 - 60	58	8	464
61 - 65	63	12	756
66 - 70	68	7	476
71 - 75	73	5	365
		$\Sigma f = 40$	$\Sigma fx = 2\ 485$

$$\text{Min, } \bar{x} = \frac{2\ 485}{40} = \text{RM}62.13$$

2.

Markah Marks	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59
Bilangan murid Number of students	8	19	50	45	28

Markah Marks	Titik tengah Midpoint x	Kekerapan Frequency f	fx
10 - 19	14.5	8	116
20 - 29	24.5	19	465.5
30 - 39	34.5	50	1\ 725
40 - 49	44.5	45	2\ 002.5
50 - 59	54.5	28	1\ 526
		$\Sigma f = 150$	$\Sigma fx = 5\ 835$

$$\text{Min, } \bar{x} = \frac{5\ 835}{150} = 38.9 \text{ markah}$$

3.

Masa (saat) Time (second)	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25
Kekerapan Frequency	1	5	15	17	12

Masa (saat) Time (second)	Titik tengah Midpoint x	Kekerapan Frequency f	fx
1 - 5	3	1	3
6 - 10	8	5	40
11 - 15	13	15	195
16 - 20	18	17	306
21 - 25	23	12	276
		$\Sigma f = 50$	$\Sigma fx = 820$

$$\text{Min, } \bar{x} = \frac{820}{50} = 16.4 \text{ saat}$$

90

F. Bagi setiap set data, cari median dengan menggunakan (a) rumus untuk median, (b) ogif.
For each set of data, find the median by using (a) the formula for median, (b) an ogive.

1.73.18

1.

Jisim (kg) Mass (kg)	31 - 35	36 - 40	41 - 45	46 - 50	51 - 55	56 - 60
Bilangan murid Number of students	3	8	10	18	9	2

Jisim (kg) Mass (kg)	Kekerapan Frequency	Kekerapan longgokan Cumulative frequency	Sempadan atas Upper boundary
31 - 35	3	3	35.5
36 - 40	8	11	40.5
41 - 45	10	21 F	45.5
46 - 50	18 f _m	39	50.5
51 - 55	9	48	55.5
56 - 60	2	50	60.5

(a) Median, $m = L + \left(\frac{\frac{N}{2} - F}{f_m} \right) C$

Jumlah kekerapan, $N = 50$

dan $\frac{N}{2} = 25$

Kelas median = 46 - 50 kg

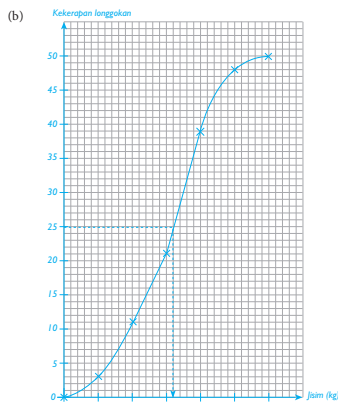
$L = 45.5$

$F = 21$

$f_m = 18$

$C = 5$

$$\text{Median} = 45.5 + \left(\frac{25 - 21}{18} \right) (5) = 46.6 \text{ kg}$$



Berdasarkan ogif, median = 46.5 kg

91

2.

Tinggi (cm) Height (cm)	110 - 119	120 - 129	130 - 139	140 - 149	150 - 159	160 - 169
Kekerapan Frequency	7	11	16	26	13	7

Tinggi (cm) Height (cm)	Kekerapan Frequency	Kekerapan longgokan Cumulative frequency	Sempadan atas Upper boundary
110 - 119	7	7	119.5
120 - 129	11	18	129.5
130 - 139	16	34 F	139.5
140 - 149	26 f _m	60	149.5
150 - 159	13	73	159.5
160 - 169	7	80	169.5

(a) $N = 80$

$\frac{N}{2} = 40$

Kelas median = 140 - 149 cm

$L = 139.5$

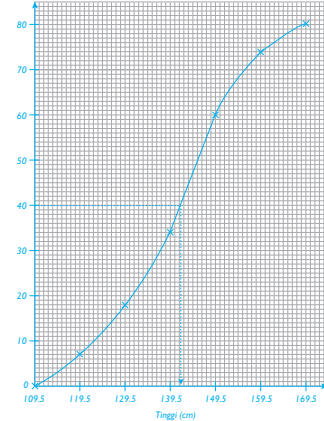
$F = 34$

$f_m = 26$

$C = 10$

$$\text{Median} = 139.5 + \left(\frac{40 - 34}{26} \right) (10) = 141.8 \text{ cm}$$

(b) Kekerapan longgokan



Berdasarkan ogif, median = 141.5 cm

92

G. Selesaikan masalah yang berikut.
Solve the following problems.

18

CONTOH

Mod, median dan min bagi satu set nombor masing-masing ialah 9, 10.8 dan 11.5. Cari mod, median dan min yang baharu jika setiap nombor dalam set itu dikurangkan dengan 3 dan kemudian didarab dengan 2.

(a) Mod baharu = $(9 - 3) \times 2 = 12$
Median baharu = $(10.8 - 3) \times 2 = 15.6$
Min baharu = $(11.5 - 3) \times 2 = 17$

(b) didarab dengan 4 dan kemudian ditolak dengan 7.
Mod baharu = $9 \times 4 - 7 = 29$
Median baharu = $10.8 \times 4 - 7 = 36.2$
Min baharu = $11.5 \times 4 - 7 = 39$

1. Mod, median dan min bagi satu set nombor masing-masing ialah 20, 22.5 dan 21.2.

The mode, median and mean of a set of numbers are 20, 22.5 and 21.2 respectively.

(a) Cari mod, median dan min yang baharu jika setiap nombor dalam set itu ditambah dengan 2.1 dan kemudian dibahagi dengan 2.

(b) Jika satu nombor u ditolak daripada setiap nombor dalam set itu dan kemudian bezanya didarab dengan 3, mediannya menjadi 60. Cari nilai u .
If a number u is subtracted from each number in the set and the difference is multiplied by 3, the median becomes 60. Find the value of u .

(a) Mod baharu = $(20 + 2.1) \div 2 = 11.05$
Median baharu = $(22.5 + 2.1) \div 2 = 12.3$
Min baharu = $(21.2 + 2.1) \div 2 = 11.65$

(b) $(22.5 - u) \times 3 = 60$
 $22.5 - u = 20$
 $u = 2.5$

2. Diberi satu set yang terdiri daripada tujuh nombor, $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, mempunyai min 17. Jika x_4 dikeluarkan daripada set itu, nilai minnya berkurang sebanyak 2. Tentukan nilai x_4 .
Given a set of seven numbers, $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, has a mean of 17. If x_4 is removed from the set, the mean decreases by 2. Determine the value of x_4 .

$\frac{\sum x}{7} = 17$
 $\sum x = 119$
Apabila x_4 dikeluarkan, min = 15.
 $\frac{\sum x - x_4}{6} = 15$
 $119 - x_4 = 90$
 $x_4 = 119 - 90 = 29$

93

7.2 Sukatan Serakan

21/22

A. Cari julat dan julat antara kuartil bagi setiap set data yang diberikan.
Find the range and interquartile range of each given set of data.

21/22

CONTOH

42, 40, 45, 32, 28

Julat = Nilai terbesar - Nilai terkecil
= $45 - 28 = 17$

Susun semua nilai mengikut tertib menaik:

28 32 40 42 45
↓ ↓ ↓
 Q_1 Median Q_3

$Q_1 = \frac{28 + 32}{2} = 30$
 $Q_3 = \frac{42 + 45}{2} = 43.5$

Julat antara kuartil
= $Q_3 - Q_1$
= $43.5 - 30 = 13.5$

1. 50, 40, 46, 34, 41, 45, 39

Julat = $50 - 34 = 16$

Susun semula semua nilai:

34, 39, 40, 41, 45, 46, 50
↓ ↓ ↓
 Q_1 Median Q_3

Julat antara kuartil
= $Q_3 - Q_1$
= $46 - 39 = 7$

2. 20, 15, 11, 13, 14, 18, 9, 22

Julat = $22 - 9 = 13$

Susun semula semua nilai:

9, 11, 13, 14, 15, 18, 20, 22
↓ ↓ ↓
 Q_1 Median Q_3

$Q_1 = \frac{11 + 13}{2} = 12$
 $Q_3 = \frac{18 + 20}{2} = 19$

Julat antara kuartil
= $Q_3 - Q_1$
= $19 - 12 = 7$

B. Cari julat antara kuartil bagi setiap set data yang diberikan.
Find the interquartile range of each given set of data.

22

CONTOH

Jisim (kg) Mass (kg)	51	52	53	54	55	56
Kekerapan Frequency	5	7	10	17	13	6

Jisim (kg) Mass (kg)	Kekerapan Frequency	Kekerapan longgokan Cumulative frequency
51	5	5
52	7	12
$Q_1 \rightarrow 53$	10	22
54	17	39
$Q_3 \rightarrow 55$	13	52
56	6	58

$Q_1 =$ Nilai ke- $(\frac{1}{4} \times 58)$
= Nilai ke-14.5
= Nilai ke-15
= 53 kg

$Q_3 =$ Nilai ke- $(\frac{3}{4} \times 58)$
= Nilai ke-43.5
= Nilai ke-44
= 55 kg

Julat antara kuartil
= $Q_3 - Q_1$
= $55 - 53 = 2$ kg

94

Skor Score	0	1	2	3	4	5
Bilangan peserta Number of participants	15	17	23	29	24	12

Skor Score	Kekerapan Frequency	Kekerapan longgokan Cumulative frequency
0	15	15
1	17	32
$Q_1 \rightarrow 2$	23	55
3	29	84
$Q_3 \rightarrow 4$	24	108
5	12	120

$Q_1 =$ Nilai ke- $(\frac{1}{4} \times 120)$
= Nilai ke-30
= 2

$Q_3 =$ Nilai ke- $(\frac{3}{4} \times 120)$
= Nilai ke-90
= 4

Julat antara kuartil
= $4 - 2 = 2$

Umur (tahun) Age (years)	40	41	42	43	44	45
Bilangan ahli Number of members	12	13	20	30	18	17

Umur (tahun) Age (years)	Kekerapan Frequency	Kekerapan longgokan Cumulative frequency
40	12	12
41	13	25
$Q_1 \rightarrow 42$	20	45
43	30	75
$Q_3 \rightarrow 44$	18	93
45	17	110

$Q_1 =$ Nilai ke- $(\frac{1}{4} \times 110)$
= Nilai ke-27.5
= Nilai ke-28
= 42 tahun

$Q_3 =$ Nilai ke- $(\frac{3}{4} \times 110)$
= Nilai ke-82.5
= Nilai ke-83
= 44 tahun

Julat antara kuartil
= $44 - 42 = 2$ tahun

95

C. Bagi set data terkumpul yang berikut, cari (a) julat dan (b) julat antara kuartil secara (i) penghitungan, (ii) melukis ogif.

For the following set of grouped data, find the (a) range and (b) interquartile range by (i) calculation, (ii) plotting an ogive.

23/25

Panjang (mm) Length (mm)	10 - 13	14 - 17	18 - 21	22 - 25	26 - 29	30 - 33
Kekerapan Frequency	8	15	49	73	32	3

Panjang (mm) Length (mm)	Kekerapan Frequency	Kekerapan longgokan Cumulative frequency	Sempadan atas Upper boundary
10 - 13	8	8	13.5
14 - 17	15	23	17.5
$Q_1 \rightarrow 18 - 21$	49	72	21.5
$Q_3 \rightarrow 22 - 25$	73	145	25.5
26 - 29	32	177	29.5
30 - 33	3	180	33.5

(a) Julat = Titik tengah kelas tertinggi - Titik tengah kelas terendah

= $(\frac{30 + 33}{2}) - (\frac{10 + 13}{2})$
= $31.5 - 11.5$
= 20 mm

(b) (i) $N = 180$

$\frac{N}{4} = \frac{180}{4} = 45$

Q_1 berada dalam kelas 18 - 21 mm

$Q_1 = L_1 + \left(\frac{\frac{N}{4} - F_1}{f_1} \right) C$
= $17.5 + \left(\frac{45 - 23}{49} \right) (4)$
= $17.5 + 1.80$
= 19.3 mm

Julat antara kuartil = $Q_3 - Q_1$
= $25.0 - 19.3$
= 5.7 mm

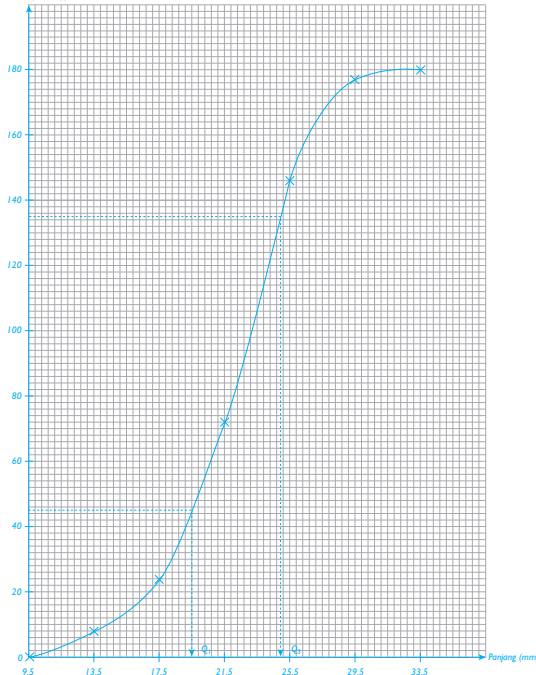
$\frac{3N}{4} = \frac{3 \times 180}{4} = 135$

Q_3 berada dalam kelas 22 - 25 mm

$Q_3 = L_3 + \left(\frac{\frac{3N}{4} - F_3}{f_3} \right) C$
= $21.5 + \left(\frac{135 - 72}{73} \right) (4)$
= $21.5 + 3.45$
= 25.0 mm

96

(b) (ii) Kekerapan loggekkan



Berdasarkan ogif, $Q_1 = 19.5 \text{ mm}$
 $Q_3 = 24.9 \text{ mm}$
 Julat antara kuartil = $Q_3 - Q_1$
 $= 24.9 - 19.5$
 $= 5.4 \text{ mm}$

97

D. Cari min, varians dan sisihan piawai bagi setiap set data yang diberikan.
 Find the mean, variance and standard deviation of each given set of data.

26(a); 27(a)

CONTOH

5, 6, 7, 2, 3

$$\text{Min, } \bar{x} = \frac{\sum x}{N} = \frac{5 + 6 + 7 + 2 + 3}{5} = 4.6$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{\sum x^2}{N} - \bar{x}^2 \\ &= \frac{5^2 + 6^2 + 7^2 + 2^2 + 3^2}{5} - 4.6^2 \\ &= 3.44 \end{aligned}$$

$$\text{Sisihan piawai, } \sigma = \sqrt{3.44} = 1.855$$

1. 19, 14, 16, 18, 10, 12

$$\text{Min, } \bar{x} = \frac{19 + 14 + 16 + 18 + 10 + 12}{6} = 14.83$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{19^2 + 14^2 + 16^2 + 18^2 + 10^2 + 12^2}{6} - 14.83^2 \\ &= \frac{1381}{6} - 14.83^2 \\ &= 10.24 \end{aligned}$$

$$\text{Sisihan piawai, } \sigma = \sqrt{10.24} = 3.2$$

2. 101, 112, 124, 131, 98

$$\text{Min, } \bar{x} = \frac{101 + 112 + 124 + 131 + 98}{5} = 113.2$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{101^2 + 112^2 + 124^2 + 131^2 + 98^2}{5} - 113.2^2 \\ &= \frac{64886}{5} - 113.2^2 \\ &= 162.96 \end{aligned}$$

$$\text{Sisihan piawai, } \sigma = \sqrt{162.96} = 12.77$$

3. 1.3, 2.5, 4.6, 3.0, 3.4, 4.1, 2.8

$$\text{Min, } \bar{x} = \frac{1.3 + 2.5 + 4.6 + 3.0 + 3.4 + 4.1 + 2.8}{7} = 3.1$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{1.3^2 + 2.5^2 + 4.6^2 + 3.0^2 + 3.4^2 + 4.1^2 + 2.8^2}{7} - 3.1^2 \\ &= \frac{74.31}{7} - 3.1^2 \\ &= 1.006 \end{aligned}$$

$$\text{Sisihan piawai, } \sigma = \sqrt{1.006} = 1.003$$

E. Cari min, varians dan sisihan piawai bagi setiap set data yang diberikan.
 Find the mean, variance and standard deviation of each given set of data.

26(b); 27(b)

CONTOH

Wang saku (RM) Pocket money (RM)	2	3	4	5	6	7
Bilangan murid Number of students	12	16	15	14	12	11

x	f	fx	x ²	fx ²
2	12	24	4	48
3	16	48	9	144
4	15	60	16	240
5	14	70	25	350
6	12	72	36	432
7	11	77	49	539
	$\Sigma f = 80$	$\Sigma fx = 351$		$\Sigma fx^2 = 1753$

$$\text{Min, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{351}{80} = 4.388$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2 \\ &= \frac{1753}{80} - 4.388^2 \\ &= 2.658 \end{aligned}$$

$$\text{Sisihan piawai} = \sqrt{2.658} = \text{RM}1.63$$

98

1.

Markah Marks	55	60	65	70	75
Bilangan murid Number of students	5	10	20	5	10

x	f	fx	x ²	fx ²
55	5	275	3 025	15 125
60	10	600	3 600	36 000
65	20	1 300	4 225	84 500
70	5	350	4 900	24 500
75	10	750	5 625	56 250
	$\Sigma f = 50$	$\Sigma fx = 3 275$		$\Sigma fx^2 = 216 375$

$$\text{Min, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{3 275}{50} = 65.5$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2 = \frac{216 375}{50} - 65.5^2 \\ &= 37.25 \end{aligned}$$

$$\text{Sisihan piawai, } \sigma = \sqrt{37.25} = 6.103$$

2.

Jisim (g) Mass (g)	16	17	18	19	20
Bilangan cili Number of chillies	16	20	22	27	15

x	f	fx	x ²	fx ²
16	16	256	256	4 096
17	20	340	289	5 780
18	22	396	324	7 128
19	27	513	361	9 747
20	15	300	400	6 000
	$\Sigma f = 100$	$\Sigma fx = 1 805$		$\Sigma fx^2 = 32 751$

$$\text{Min, } \bar{x} = \frac{1 805}{100} = 18.05$$

$$\text{Varians, } \sigma^2 = \frac{32 751}{100} - 18.05^2 = 1.708$$

$$\text{Sisihan piawai, } \sigma = \sqrt{1.708} = 1.307$$

F. Cari min, varians dan sisihan piawai bagi setiap set data yang diberikan.
 Find the mean, variance and standard deviation of each given set of data.

24(b); 27(b)

CONTOH

Umur (tahun) Age (years)	25 - 28	29 - 32	33 - 36	37 - 40	41 - 44	45 - 48
Bilangan pekerja Number of staff	3	5	9	10	7	6

Umur (tahun) Age (years)	Titik tengah, x Midpoint, x	f	fx	fx ²
25 - 28	26.5	3	79.5	2 106.75
29 - 32	30.5	5	152.5	4 651.25
33 - 36	34.5	9	310.5	10 712.25
37 - 40	38.5	10	385.0	14 822.50
41 - 44	42.5	7	297.5	12 643.75
45 - 48	46.5	6	279.0	12 973.50
		$\Sigma f = 40$	$\Sigma fx = 1 504$	$\Sigma fx^2 = 57 910$

$$\begin{aligned} \text{Min, } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{1 504}{40} \\ &= 37.6 \end{aligned}$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2 \\ &= \frac{57 910}{40} - 37.6^2 \\ &= 33.99 \end{aligned}$$

$$\begin{aligned} \text{Sisihan piawai, } \sigma &= \sqrt{33.99} \\ &= 5.830 \text{ tahun} \end{aligned}$$

99

1.

Bilangan durian Number of durians	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34
Kekerapan Frequency	18	30	32	26	14

Bilangan durian Number of durians	Titik tengah, x Midpoint, x	f	fx	fx ²
10 - 14	12	18	216	2 592
15 - 19	17	30	510	8 670
20 - 24	22	32	704	15 488
25 - 29	27	26	702	18 954
30 - 34	32	14	448	14 336
		$\Sigma f = 120$	$\Sigma fx = 2 580$	$\Sigma fx^2 = 60 040$

$$\begin{aligned} \text{Min, } \bar{x} &= \frac{2 580}{120} \\ &= 21.5 \end{aligned}$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{60 040}{120} - 21.5^2 \\ &= 38.08 \end{aligned}$$

$$\begin{aligned} \text{Sisihan piawai, } \sigma &= \sqrt{38.08} \\ &= 6.171 \end{aligned}$$

2.

Harga (RM) Price (RM)	20 - 23	24 - 27	28 - 31	32 - 35	36 - 39	40 - 43
Kekerapan Frequency	7	8	10	16	17	6

Harga (RM) Price (RM)	Titik tengah, x Midpoint, x	f	fx	fx ²
20 - 23	21.5	7	150.5	3 235.75
24 - 27	25.5	8	204.0	5 202.00
28 - 31	29.5	10	295.0	8 702.50
32 - 35	33.5	16	536.0	17 956.00
36 - 39	37.5	17	637.5	23 906.25
40 - 43	41.5	6	249.0	10 333.50
		$\Sigma f = 64$	$\Sigma fx = 2 072$	$\Sigma fx^2 = 69 336$

$$\begin{aligned} \text{Min, } \bar{x} &= \frac{2 072}{64} \\ &= \text{RM}32.38 \end{aligned}$$

$$\begin{aligned} \text{Varians, } \sigma^2 &= \frac{69 336}{64} - 32.375^2 \\ &= 35.23 \end{aligned}$$

$$\begin{aligned} \text{Sisihan piawai, } \sigma &= \sqrt{35.23} \\ &= \text{RM}5.94 \end{aligned}$$

100

G. Selesaikan masalah yang berikut.
Solve the following problems.

CONTOH

Julat, julat antara kuartil dan varians bagi satu set data masing-masing ialah 21, 9 dan 12. Cari julat, julat antara kuartil dan varians yang baharu jika setiap nilai dalam set itu

- The range, interquartile range and variance of a set of data are 21, 9 and 12 respectively. Find the new range, new interquartile range and new variance if each value in the set.
- didarab dengan 4 dan kemudian 5 ditolak daripadanya. *is multiplied by 4 and then 5 is subtracted from it.*
 - ditambah dengan 6 dan kemudian dibahagi dengan 3. *is added by 6 and then divided by 3.*

(a) Julat baharu = $21 \times 4 = 84$
Julat antara kuartil baharu = $9 \times 4 = 36$
Varians baharu = $12 \times 4^2 = 192$

(b) Julat baharu = $21 + 3 = 24$
Julat antara kuartil baharu = $9 + 3 = 12$
Varians baharu = $12 + 3^2 = 33$

FAKTA UTAMA

Apabila setiap nilai dalam satu set data *When each value in a set of data*

- ditambah atau ditolak dengan suatu pemalar, sukatan serakan bagi set data baharu kekal tidak berubah. *is added by a constant or a constant is subtracted from it, the measures of dispersion of the new set of data remain unchanged.*
- didarab dengan pemalar k , *is multiplied by a constant k:*
 - Julat baharu = $k \times$ Julat asal
New range = $k \times$ Original range
 - Julat antara kuartil baharu = $k \times$ Julat antara kuartil asal
New interquartile range = $k \times$ Original interquartile range
 - Sisihan piawai baharu = $k \times$ Sisihan piawai asal
New standard deviation = $k \times$ Original standard deviation
 - Varians baharu = $k^2 \times$ Varians asal
New variance = $k^2 \times$ Original variance

1. Satu set data mempunyai julat 10, julat antara kuartil 6 dan sisihan piawai 1.8. Setiap nilai dalam set itu dibahagi dengan 4 dan kemudian 2 ditolak daripadanya. Cari

A set of data has a range of 10, an interquartile range of 6 and a standard deviation of 1.8. Each value in the set is divided by 4 and then 2 is subtracted from it. Find

- Julat baharu. *the new range.*
- Julat antara kuartil baharu. *the new interquartile range.*
- Sisihan piawai baharu. *the new standard deviation.*

(a) Julat baharu = $\frac{10 + 4}{4} = 2.5$

(b) Julat antara kuartil baharu = $\frac{6 + 4}{4} = 1.5$

(c) Sisihan piawai baharu = $\frac{1.8 + 4}{4} = 0.45$

2. Satu set data terdiri daripada 4, 6, 8, 9, 12 dan 15.

A set of data consists of 4, 6, 8, 9, 12 and 15.

- Cari min, julat dan varians bagi set data ini. *Find the mean, range and variance of this set of data.*
- Setiap nilai dalam set itu didarab dengan 2 dan kemudian ditambah dengan 5. Cari min, julat dan varians bagi set data baharu. *Each value in the set is multiplied by 2 and then added by 5. Find the mean, range and variance of the new set of data.*

(a) Min, $\bar{x} = \frac{4 + 6 + 8 + 9 + 12 + 15}{6} = 9$
Julat = $15 - 4 = 11$
Varians, $\sigma^2 = \frac{4^2 + 6^2 + 8^2 + 9^2 + 12^2 + 15^2}{6} - 9^2 = 13.33$

(b) Min baharu = $9 \times 2 + 5 = 23$
Julat baharu = $11 \times 2 = 22$
Varians baharu = $2^2 \times 13.33 = 53.33$

4. Satu set enam nombor mempunyai min 10.

A set of six numbers has a mean of 10.

- Cari Σx . *Find Σx .*
- Apabila satu nombor m ditambah kepada set ini, min baharu ialah 12. Cari nilai m . *When a number m is added to this set, the new mean is 12. Find the value of m .*

(a) $\frac{\Sigma x}{6} = 10$
 $\Sigma x = 60$

(b) $\frac{\Sigma x + m}{7} = 12$
 $\frac{60 + m}{7} = 12$
 $60 + m = 84$
 $m = 24$

5. Satu set data 12 nombor, x_1, x_2, \dots, x_{12} , mempunyai varians 30 dan diberi bahawa $\Sigma x^2 = 840$. Cari

A set of 12 numbers, x_1, x_2, \dots, x_{12} , has a variance of 30 and it is given that $\Sigma x^2 = 840$. Find

- min, \bar{x} , *the mean, \bar{x} .*
- nilai bagi Σx . *the value of Σx .*

$N = 12, \sigma^2 = 30, \Sigma x^2 = 840$

(a) $\sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2$
 $30 = \frac{840}{12} - \bar{x}^2$
 $30 = 70 - \bar{x}^2$
 $\bar{x}^2 = 40$
 $\bar{x} = 6.325$

(b) $\frac{\Sigma x}{12} = 6.325$
 $\Sigma x = 12 \times 6.325 = 75.90$

6. Sekumpulan 8 orang murid mempunyai jumlah umur 120 tahun. Hasil tambah kuasa dua umur-umur mereka ialah 1 816 tahun. Cari

A group of 8 students has a total age of 120 years. The sum of the squares of their ages is 1 816 years. Find

- min umur 8 orang murid itu. *the mean of the ages of the 8 students.*
- sisihan piawai. *the standard deviation.*

$N = 8, \Sigma x = 120, \Sigma x^2 = 1 816$

(a) Min umur, $\bar{x} = \frac{120}{8} = 15$ tahun

(b) Sisihan piawai, $\sigma = \sqrt{\frac{\Sigma x^2}{N} - \bar{x}^2} = \sqrt{\frac{1 816}{8} - 15^2} = \sqrt{2} = 1.414$ tahun

7. Jisim satu kumpulan 8 orang murid mempunyai min 50 kg dan sisihan piawai 4 kg. Cari

The masses of a group of 8 students has a mean of 50 kg and a standard deviation of 4 kg. Find

- hasil tambah jisim murid-murid itu. *the sum of the masses of the students.*
- hasil tambah kuasa dua jisim murid-murid itu. *the sum of the squares of the masses of the students.*

(a) Hasil tambah jisim murid-murid = 8×50 kg = 400 kg

(b) $\sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2$
 $4^2 = \frac{\Sigma x^2}{8} - 50^2$
 $\frac{\Sigma x^2}{8} = 16 + 2 500$
 $\Sigma x^2 = 8 \times 2 516 = 20 128$
Hasil tambah kuasa dua jisim murid-murid itu ialah 20 128 kg².

PRAKTIS FORMALIF Kertas 1

ANALISIS SOALAN SPM				
Sisuba	2013	2014	2015	2016
7.1	—	S.21.2D	S.20	—
7.2	S.22	—	S.19	S.1

Jawab semua soalan.
Answer all the questions.

1. Satu set data terdiri daripada 7, 0, 5, $x^2 - 3$ dan 2. Diberi min ialah 4, cari

A set of data consists of 7, 0, 5, $x^2 - 3$ and 2. Given the mean is 4, find

- nilai positif bagi x .
- median dengan menggunakan nilai x di (a).

(a) Min = $\frac{7 + 0 + 5 + (x^2 - 3) + 2}{5} = 4$
 $11 + x^2 = 20$
 $x^2 = 9$
 $x = 3$

(b) Susun semula data mengikut tertib menaik: 0, 2, 5, 6, 7
Maka, median = 5

2. Jadual di bawah menunjukkan skor bagi suatu kuiz yang diperoleh sekumpulan murid.

The table shows the scores of a quiz obtained by a group of students.

Skor Score	1	2	3	4	5
Kekerapan Frequency	4	k	12	9	8

- Cari nilai maksimum bagi k , jika mod ialah 3. *Find the maximum value of k , if the mode is 3.*
- Cari julat nilai k , jika median ialah 3. *Find the range of values of k , if the median is 3.*

(a) Jika mod ialah 3, maka nilai $k < 12$.
Jadi, nilai maksimum $k = 11$.

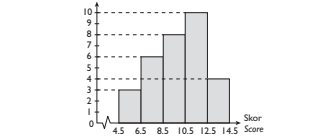
(b) Kes 1: Jika median ialah skor 3 yang pertama, maka
 $4 + k = (12 - 1) + 9 + 8$
 $4 + k = 28$
 $k = 24$

Kes 2: Jika median ialah skor 3 yang terakhir, maka
 $4 + k + (12 - 1) = 9 + 8$
 $k + 15 = 17$
 $k = 2$

Julat nilai k ialah $2 \leq k \leq 24$, dengan keadaan k ialah integer.

3. Rajah di bawah menunjukkan satu histogram bagi taburan skor yang diperoleh sekumpulan peserta dalam satu kuiz.

The diagram shows a histogram for the distribution of scores obtained by a group of participants in a quiz.



- Cari jumlah bilangan peserta. *Find the total number of participants.*
- Cari skor min. *Find the mean score.*

(a) Jumlah bilangan peserta = $3 + 6 + 8 + 10 + 4 = 31$ orang

Titik tengah	f	fx
5.5	3	16.5
7.5	6	45
9.5	8	76
11.5	10	115
13.5	4	54
	$\Sigma f = 31$	$\Sigma fx = 306.5$

Skor min = $\frac{\Sigma fx}{\Sigma f} = \frac{306.5}{31} = 9.89$

PRAKTIS FORMATIF Kertas 2

ANALISIS SOALAN SPM				
Subtopik	2013	2014	2015	2016
7.1	-	-	-	S.1
7.2	S.6	-	-	-

Jawab semua soalan. Answer all the questions.

1. Jadual di bawah menunjukkan taburan kekerapan bagi gaji mingguan sekumpulan pekerja. The table shows the frequency distribution of the weekly wages of a group of workers.

Gaji (RM) Wages (RM)	Bilangan pekerja Number of workers
300 – 399	12
400 – 499	13
500 – 599	20
600 – 699	m
700 – 799	10

Diberi bahawa min gaji mingguan bagi pekerja itu ialah RM546.60.

It is given that the mean of the weekly wages of the workers is RM546.60.

- (a) Cari nilai m . Find the value of m .

- (b) Seterusnya, tanpa menggunakan ogif, hitung median bagi gaji mingguan pekerja itu. Hence, without using an ogive, calculate the median of the weekly wages of the workers.

(a) Min gaji mingguan = RM546.60

$$\frac{12(349.50) + 13(449.50) + 20(549.50) + m(649.50) + 10(749.50)}{12 + 13 + 20 + m + 10} = 546.60$$

$$\frac{28\ 522.5 + 649.5m}{55 + m} = 546.6$$

$$28\ 522.5 + 649.5m = 30\ 063 + 546.6m$$

$$649.5m - 546.6m = 30\ 063 - 28\ 522.5$$

$$102.9m = 1\ 540.5$$

$$m = 15$$

(b) Jumlah bilangan pekerja = $55 + 15 = 70$ orang
Kelas median = $500 - 599$
Median gaji mingguan = $499.5 + \frac{1}{2} \left(\frac{700 - 25}{20} \right) (100)$
= RM524.50

2. Min bagi suatu set nombor 1, y , 7 , $2y + 2$, 13 dan 16 ialah 9.

The mean of a set of numbers 1, y , 7 , $2y + 2$, 13 and 16 is 9.

- (a) Cari (i) nilai y , the value of y . (ii) varians, the variance.

- (b) Setiap nombor dalam set itu didarab dengan 2 dan kemudian ditambah dengan 3. Bagi set nombor ini, cari

Each number in the set is multiplied by 2 and then 3 is added to it. For this set of numbers, find (i) min, the mean. (ii) sisihan piawai, the standard deviation.

(a) (i)
$$\frac{1 + y + 7 + 2y + 2 + 13 + 16}{6} = 9$$
$$3y + 39 = 54$$
$$3y = 15$$
$$y = 5$$

(ii) Varians = $\frac{1^2 + 5^2 + 7^2 + 12^2 + 13^2 + 16^2}{6} - 9^2$
$$= \frac{644}{6} - 81$$
$$= 26 \frac{1}{3}$$

(b) (i) Min baharu = $\frac{9(2) + 3}{21} = 21$

(ii) Sisihan piawai asal = $\sqrt{26 \frac{1}{3}}$
$$= 5.1316$$

Sisihan piawai baharu = $5.1316 \times 2 = 10.26$

3. Jadual di bawah menunjukkan hasil tambah dan hasil tambah kuasa dua bagi x , dengan keadaan x ialah pendapatan bulanan, dalam RM, bagi Encik Mazlan untuk 8 bulan pertama tahun 2017. The table shows the sum and the sum of the squares of x , where x is the monthly income, in RM, of Encik Mazlan for the first 8 months in the year 2017.

Σx	16 800
Σx^2	35 385 000

- (a) Cari sisihan piawai bagi pendapatan bulanan. Find the standard deviation of his monthly income.

- (b) Jika anak lelaki Encik Mazlan memberi RM300 setiap bulan dalam tempoh masa itu, cari min baharu dan sisihan piawai baharu bagi pendapatan bulanan. If Encik Mazlan's son gives him RM300 every month during that period, find the new mean and new standard deviation of his monthly income.

(a)
$$\bar{x} = \frac{\Sigma x}{N} = \frac{16\ 800}{8} = 2\ 100$$
$$\sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2 = \frac{35\ 385\ 000}{8} - 2\ 100^2 = 13\ 125$$
$$\sigma = 114.56$$

Maka, sisihan piawai bagi pendapatan bulanan ialah RM114.56.

- (b) Min baharu = $\frac{RM2\ 100 + RM300}{8} = RM2\ 400$
Tiada perubahan pada sisihan piawai.
Maka, sisihan piawai baharu = RM114.56

4. Bilangan buku yang dibaca oleh setiap murid lelaki dalam Kelas 4 Cerdik diberi oleh $x_1, x_2, x_3, \dots, x_{15}$. Min bagi bilangan buku yang dibaca ialah 6 dan sisihan piawainya ialah 3. Cari

The number of books read by each boy in Class 4 Cerdik is given by $x_1, x_2, x_3, \dots, x_{15}$. The mean of the number of books read is 6 and the standard deviation is 3. Find (i) jumlah bilangan buku yang dibaca, Σx , the total number of books read, Σx . (ii) hasil tambah kuasa dua bagi bilangan buku yang dibaca, Σx^2 , the sum of the squares of the numbers of books read, Σx^2 .

- (b) Min bagi bilangan buku yang dibaca oleh murid perempuan dalam Kelas 4 Cerdik ialah 7 dan hasil tambah kuasa dua bilangan buku yang dibaca ialah 720. Diberi bahawa jumlah bilangan buku yang dibaca oleh murid perempuan itu ialah 105. Cari varians bagi bilangan buku yang dibaca oleh semua murid dalam Kelas 4 Cerdik. The mean of the number of books read by the girls in Class 4 Cerdik is 7 and the sum of the squares of the number of books read is 720. It is given that the total number of books read by the girls is 105. Find the variance of the number of books read by all the students in Class 4 Cerdik.

(a) (i)
$$\frac{\Sigma x}{15} = 6$$
$$\Sigma x = 6 \times 15 = 90$$

(ii) Varians = $\frac{\Sigma x^2}{15} - 6^2 = 3^2$
$$\Sigma x^2 = (9 + 36) \times 15 = 675$$

(b) Jumlah bilangan buku yang dibaca oleh murid perempuan = 105
Maka, $\Sigma x = 105$
Diberi min = $\frac{\Sigma x}{N} = 7$.
Maka, $\frac{105}{N} = 7$
 $N = \frac{105}{7} = 15$

Jumlah bilangan murid perempuan ialah 15 orang.
Apabila murid perempuan dan murid lelaki diambil kira,
 $N = 15 + 15 = 30$
 $\Sigma x = 105 + 90 = 195$
 $\Sigma x^2 = 720 + 675 = 1\ 395$
Min bilangan buku yang dibaca oleh semua murid = $\frac{195}{30}$

Varians bagi bilangan buku yang dibaca oleh semua murid = $\frac{\Sigma x^2}{N} - \bar{x}^2 = \frac{1\ 395}{30} - \left(\frac{195}{30}\right)^2 = 4.25$

FOKUS KBAT

Kemahiran Kognitif: Menganalisis
Konteks: Nilai Min dan Perubahan Min

Dalam sebuah syarikat, gaji min bagi semua pekerja ialah RM2 700. Apabila seorang pekerja baharu dengan gaji sebanyak RM3 350 menyertai syarikat itu, gaji min bagi semua pekerja meningkat sebanyak RM50. Berapakah jumlah bilangan pekerja dalam syarikat itu sekarang?

In a company, the mean salary of all the employees was RM2 700. When a new employee with a salary of RM3 350 joined the company, the mean salary of all the employees was increased by RM50. What is the total number of employees in the company now?

Min asal = 2 700
$$\frac{\Sigma x}{N} = 2\ 700$$
$$\Sigma x = 2\ 700N \dots\dots \textcircled{1}$$

Setelah seorang pekerja baharu menyertai syarikat itu:

Min baharu = 2 700 + 50 = 2 750
$$\frac{\Sigma x + 3\ 350}{N + 1} = 2\ 750 \dots\dots \textcircled{2}$$

Gantikan $\textcircled{1}$ ke dalam $\textcircled{2}$.
$$\frac{2\ 700N + 3\ 350}{N + 1} = 2\ 750$$
$$2\ 700N + 3\ 350 = 2\ 750N + 2\ 750$$
$$3\ 350 - 2\ 750 = 2\ 750N - 2\ 700N$$
$$600 = 50N$$
$$N = 12$$

Jumlah bilangan pekerja sekarang = $12 + 1 = 13$ orang

Info KBAT

Pertimbangkan jumlah bilangan pekerja semasa menggunakan formula min. Consider the total number of employees while using the formula for the mean.

BAB 8

SUKATAN MEMBULAT
CIRCULAR MEASURES

8.1 Radian

- A. Tukarkan setiap yang berikut kepada darjah dan minit. Convert each of the following to degrees and minutes.

CONTOH

FAKTA UTAMA

(a) $\frac{\pi}{6}$ rad = $\frac{\pi}{6} \times \frac{180^\circ}{\pi} = 30^\circ$

(b) 1.5 rad = $1.5 \times \frac{180^\circ}{\pi} = 85^\circ 57'$

Daripada radian kepada darjah: From radians to degrees:
 θ rad = $\theta \times \frac{180^\circ}{\pi}$

1. $\frac{\pi}{4}$ rad = $\frac{\pi}{4} \times \frac{180^\circ}{\pi} = 45^\circ$

2. 0.45 rad = $0.45 \times \frac{180^\circ}{\pi} = 25^\circ 47'$

3. 2.12 rad = $2.12 \times \frac{180^\circ}{\pi} = 121^\circ 28'$

- B. Tukarkan setiap yang berikut kepada radian dalam sebutan π . Convert each of the following to radians in terms of π .

CONTOH

FAKTA UTAMA

(a) $60^\circ = 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3}$ rad

(b) $135^\circ = 135^\circ \times \frac{\pi}{180^\circ} = \frac{3}{4}\pi$ rad

Daripada darjah kepada radian: From degrees to radians:
 $\theta^\circ = \theta^\circ \times \frac{\pi}{180^\circ}$ rad

1. $90^\circ = 90^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{2}$ rad

2. $150^\circ = 150^\circ \times \frac{\pi}{180^\circ} = \frac{5}{6}\pi$ rad

3. $270^\circ = 270^\circ \times \frac{\pi}{180^\circ} = \frac{3}{2}\pi$ rad

- C. Tukarkan setiap yang berikut kepada radian. Convert each of the following to radians.

CONTOH



1. $55^\circ = 55^\circ \times \frac{\pi}{180^\circ} = 0.960$ rad

2. $88.3^\circ = 88.3^\circ \times \frac{\pi}{180^\circ} = 1.541$ rad

3. $115^\circ 21' = 115.35^\circ \times \frac{\pi}{180^\circ} = 2.013$ rad

4. $283^\circ 25' = 283.4167^\circ \times \frac{\pi}{180^\circ} = 4.947$ rad

8.2 Panjang Lengkuk Bulatan

A. Cari panjang lengkok, s , bagi setiap bulatan yang berikut.
Find the length of arc, s , in each of the following circles.

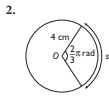
CONTOH



$$\begin{aligned} s &= r\theta \\ &= 5 \times 45^\circ \times \frac{\pi}{180^\circ} \\ &= 3.927 \text{ cm} \end{aligned}$$



$$\begin{aligned} s &= r\theta \\ &= 10 \times 33^\circ \times \frac{\pi}{180^\circ} \\ &= 5.760 \text{ cm} \end{aligned}$$



$$\begin{aligned} s &= r\theta \\ &= 4 \times \frac{2}{3}\pi \\ &= 8.378 \text{ cm} \end{aligned}$$



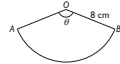
$$\begin{aligned} s &= r\theta \\ &= 8 \times 300^\circ \times \frac{\pi}{180^\circ} \\ &= 41.89 \text{ cm} \end{aligned}$$

B. Selesaikan masalah yang berikut.

Solve the following problems.

[Guna/Use $\pi = 3.142$]

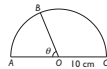
1. Rajah di bawah menunjukkan sektor AOB.
The diagram shows a sector AOB.



- (a) Cari panjang lengkok AB jika nilai θ ialah
Find the length of arc AB if the value of θ is
(i) 0.4 rad. (ii) 0.8 rad. (iii) 0.65 rad.
(b) Cari nilai θ jika panjang lengkok AB ialah
Find the value of θ if the length of arc AB is
(i) 10 cm. (ii) 18 cm. (iii) 15 cm.

- (a) (i) Panjang lengkok AB = $8 \times 0.4 = 3.2$ cm
(ii) Panjang lengkok AB = $8 \times 0.8 = 6.4$ cm
(iii) Panjang lengkok AB = $8 \times 0.65 = 5.2$ cm
(b) (i) Panjang lengkok AB = j
 $10 = 8\theta$
 $\theta = \frac{10}{8} = 1.25$ rad
(ii) $18 = 8\theta$
 $\theta = \frac{18}{8} = 2.25$ rad
(iii) $15 = 8\theta$
 $\theta = \frac{15}{8} = 1.875$ rad

2. Rajah di bawah menunjukkan semibulatan OABC yang berpusat O.
The diagram shows a semicircle OABC with centre O.



- (a) Cari panjang lengkok AB dan lengkok BC jika $\theta = 0.78$ rad.
Find the lengths of arc AB and arc BC if $\theta = 0.78$ rad.
(b) Jika panjang lengkok AB ialah 12 cm, cari nilai θ , dalam radian. Seterusnya, cari $\angle BOC$, dalam darjah.
If the length of arc AB is 12 cm, find the value of θ , in radians. Hence, find $\angle BOC$, in degrees.

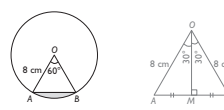
- (a) Panjang lengkok AB = $10 \times 0.78 = 7.8$ cm
 $\angle BOC = 3.142 - 0.78 = 2.362$ rad
Panjang lengkok BC = $10 \times 2.362 = 23.62$ cm
(b) Panjang lengkok AB = 12 cm
 $12 = 10\theta$
 $\theta = \frac{12}{10} = 1.2$ rad
 $\angle BOC = 3.142 - 1.2 = 1.942$ rad
 $= \frac{1.942 \times 180^\circ}{3.142} = 111.25^\circ$

109

C. Cari perimeter bagi setiap tembereng berlorek yang berikut.
Find the perimeter of each of the following shaded segments.

22

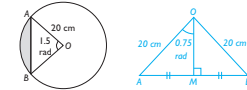
CONTOH



Dalam $\triangle OAM$, $\sin 30^\circ = \frac{AM}{8}$
 $AM = 8 \sin 30^\circ$
 $AB = 2 \times 8 \sin 30^\circ = 8$ cm
Panjang lengkok AB = $8 \times 60^\circ \times \frac{\pi}{180^\circ} = 8.378$ cm

Perimeter tembereng berlorek
 $= 8 + 8.378$
 $= 16.378$ cm

1.



$$0.75 \text{ rad} = 0.75 \times \frac{180^\circ}{\pi} = 42.97^\circ$$

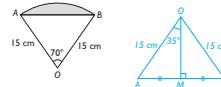
Dalam $\triangle OAM$, $\sin 42.97^\circ = \frac{AM}{20}$
 $AM = 20 \sin 42.97^\circ$

$$AB = 2 \times 20 \sin 42.97^\circ = 27.26 \text{ cm}$$

Panjang lengkok AB = $20 \times 1.5 = 30$ cm

Perimeter tembereng berlorek
 $= 27.26 + 30 = 57.26$ cm

2.



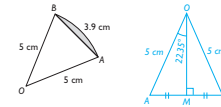
Dalam $\triangle OAM$, $\sin 35^\circ = \frac{AM}{15}$
 $AM = 15 \sin 35^\circ$

$$AB = 2 \times 15 \sin 35^\circ = 17.21 \text{ cm}$$

Panjang lengkok AB = $15 \times 70^\circ \times \frac{\pi}{180^\circ} = 18.33$ cm

Perimeter tembereng berlorek
 $= 17.21 + 18.33 = 35.54$ cm

3.



$$\angle AOB = \frac{3.9}{5} = 0.78 \text{ rad} = 44.69^\circ$$

Dalam $\triangle OAM$, $\sin 22.35^\circ = \frac{AM}{5}$
 $AM = 5 \sin 22.35^\circ$

$$AB = 2 \times 5 \sin 22.35^\circ = 3.803 \text{ cm}$$

Perimeter tembereng berlorek
 $= 3.803 + 3.9 = 7.703$ cm

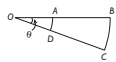
110

D. Selesaikan masalah yang berikut.

Solve the following problems.

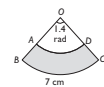
23

1. Rajah di bawah menunjukkan dua sektor, OAD dan OBC, berpusat O. Diberi AB = 2OA dan panjang OA adalah sama dengan panjang lengkok BC.
The diagram shows two sectors, OAD and OBC, with centre O. Given AB = 2OA and the length of OA is equal to the length of arc BC.



- (a) Tentukan nilai θ , dalam radian.
Determine the value of θ , in radians.
(b) Cari panjang lengkok AD jika AB = 18 cm.
Find the length of arc AD if AB = 18 cm.
- (a) Katakan OA = j , maka AB = $2j$, OB = $3j$ dan panjang lengkok BC = j .
Panjang lengkok BC = $OB \times \theta$
 $j = 3j \times \theta$
 $\theta = \frac{j}{3}$ rad
- (b) Jika AB = 18 cm, maka OA = 9 cm.
Panjang lengkok AD = $9 \times \frac{j}{3} = 3$ cm

2. Rajah di bawah menunjukkan dua sektor, OAD dan OBC, berpusat O. Diberi OA : AB = 3 : 2, cari
The diagram shows two sectors, OAD and OBC, with centre O. Given OA : AB = 3 : 2, find



- (a) panjang OA dan panjang AB, the lengths of OA and AB.
(b) perimeter kawasan berlorek, the perimeter of the shaded region.
- (a) $OB = \frac{7}{1.4} = 5$ cm
 $\frac{OA}{OB} = \frac{3}{5}$
 $OA = \frac{3}{5} \times 5 = 3$ cm
Maka, AB = $5 - 3 = 2$ cm
- (b) Panjang lengkok AD = $3 \times 1.4 = 4.2$ cm
Perimeter kawasan berlorek = $7 + 4.2 + 2 = 15.2$ cm

3. Rajah di bawah menunjukkan satu sektor berpusat O. Jika $\triangle OAB$ ialah segi tiga sama sisi, cari
The diagram shows a sector with centre O. If $\triangle OAB$ is an equilateral triangle, find



- (a) $\angle AOB$, dalam radian, $\angle AOB$, in radians.
(b) perimeter tembereng berlorek, the perimeter of the shaded segment. (Berikan jawapan dalam sebutan π .) (Give the answers in terms of π .)
- (a) OAB ialah segi tiga sama sisi. Maka,
 $\angle AOB = 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3}$ rad
- (b) AB = 12 cm
Panjang lengkok AB = $12 \times \frac{\pi}{3} = 4\pi$ cm
Perimeter tembereng berlorek = $(12 + 4\pi)$ cm

111

8.3 Luas Sektor Bulatan

A. Cari luas setiap sektor berlorek yang berikut.
Find the area of each of the following shaded sectors.

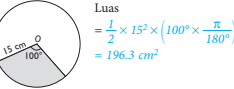
31

CONTOH



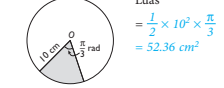
$$\begin{aligned} \text{Luas} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 8^2 \times 1.22 \\ &= 39.04 \text{ cm}^2 \end{aligned}$$

1.



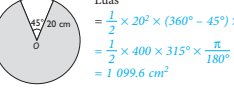
$$\begin{aligned} \text{Luas} &= \frac{1}{2} \times 15^2 \times \left(100^\circ \times \frac{\pi}{180^\circ}\right) \\ &= 196.3 \text{ cm}^2 \end{aligned}$$

2.



$$\begin{aligned} \text{Luas} &= \frac{1}{2} \times 10^2 \times \frac{\pi}{3} \\ &= 52.36 \text{ cm}^2 \end{aligned}$$

3.



$$\begin{aligned} \text{Luas} &= \frac{1}{2} \times 20^2 \times (360^\circ - 45^\circ) \times \frac{\pi}{180^\circ} \\ &= \frac{1}{2} \times 400 \times 315^\circ \times \frac{\pi}{180^\circ} \\ &= 1\,099.6 \text{ cm}^2 \end{aligned}$$

B. Selesaikan masalah yang berikut.

Solve the following problems.

[Guna/Use $\pi = 3.142$]

32

1. Rajah di bawah menunjukkan sektor OAB.
The diagram shows a sector OAB.



- (a) Cari luas sektor OAB jika
Find the area of sector OAB if
(i) $\theta = 0.5$ rad.
(ii) $\theta = 1.1$ rad.

- (b) Cari nilai θ jika luas sektor OAB ialah
Find the value of θ if the area of sector OAB is
(i) 61.5 cm². (ii) 67.5 cm².

(a) (i) Luas sektor OAB = $\frac{1}{2} \times 10^2 \times 0.5 = 25$ cm²

(ii) Luas sektor OAB = $\frac{1}{2} \times 10^2 \times 1.1 = 55$ cm²

(b) (i) $\frac{1}{2} r^2 \theta = 61.5$

$$\begin{aligned} \frac{1}{2} \times 10^2 \times \theta &= 61.5 \\ \theta &= \frac{61.5}{50} \\ &= 1.23 \text{ rad} \end{aligned}$$

(ii) $\frac{1}{2} r^2 \theta = 67.5$

$$\begin{aligned} \frac{1}{2} \times 10^2 \times \theta &= 67.5 \\ \theta &= \frac{67.5}{50} \\ &= 1.35 \text{ rad} \end{aligned}$$

2. Rajah di bawah menunjukkan sektor POQ.
The diagram shows a sector POQ.



- (a) Cari luas sektor POQ jika
Find the area of sector POQ if
(i) $j = 8$ cm.
(ii) $j = 18$ cm.

- (b) Cari jejari sektor POQ jika luas sektor ialah
Find the radius of sector POQ if the area of the sector is
(i) 42.525 cm². (ii) 134.4 cm².

(a) (i) Luas sektor POQ = $\frac{1}{2} \times 8^2 \times 1.05 = 33.6$ cm²

(ii) Luas sektor POQ = $\frac{1}{2} \times 18^2 \times 1.05 = 170.1$ cm²

(b) (i) $\frac{1}{2} j^2 \times 1.05 = 42.525$

$$\begin{aligned} j^2 &= \frac{42.525 \times 2}{1.05} \\ &= 81 \\ j &= 9 \text{ cm} \end{aligned}$$

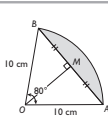
(ii) $\frac{1}{2} j^2 \times 1.05 = 134.4$

$$\begin{aligned} j^2 &= \frac{134.4 \times 2}{1.05} \\ &= 256 \\ j &= 16 \text{ cm} \end{aligned}$$

112

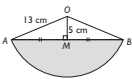
C. Cari luas bagi setiap tembereng berlorek yang berikut.
Find the area of each of the following shaded segments.

CONTOH



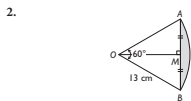
Dalam $\triangle OAM$,
 $\sin 40^\circ = \frac{AM}{10}$ $\cos 40^\circ = \frac{OM}{10}$
 $AM = 10 \sin 40^\circ$ $OM = 10 \cos 40^\circ$
 $AB = 2 \times 10 \sin 40^\circ$
 $= 12.86 \text{ cm}$

Luas tembereng berlorek
 $=$ Luas sektor OAB - Luas $\triangle OAB$
 $= \frac{1}{2}r^2\theta - \frac{1}{2}(AB)(OM)$
 $= \frac{1}{2}(10^2)\left(80 \times \frac{\pi}{180}\right) - \frac{1}{2}(12.86)(7.660)$
 $= 69.81 - 49.25$
 $= 20.56 \text{ cm}^2$



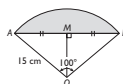
Dalam $\triangle OAM$,
 $AM^2 = 13^2 - 5^2$
 $= 144$
 $AM = 12 \text{ cm}$
 $AB = 2 \times 12$
 $= 24 \text{ cm}$
 $\cos \angle AOM = \frac{5}{13}$
 $\angle AOM = 67.38^\circ$
 $\angle AOB = 2 \times 67.38^\circ$
 $= 134.76^\circ$

Luas tembereng berlorek
 $= \frac{1}{2}r^2\theta - \frac{1}{2}(AB)(OM)$
 $= \frac{1}{2}(13^2)\left(134.76^\circ \times \frac{\pi}{180^\circ}\right) - \frac{1}{2}(24)(5)$
 $= 138.74 \text{ cm}^2$



Dalam $\triangle OAM$,
 $\sin 30^\circ = \frac{AM}{13}$
 $AM = 13 \sin 30^\circ$
 $AB = 2 \times 13 \sin 30^\circ$
 $= 13 \text{ cm}$
 $\cos 30^\circ = \frac{OM}{13}$
 $OM = 13 \cos 30^\circ$
 $= 11.258 \text{ cm}$

Luas tembereng berlorek
 $= \frac{1}{2}(13^2)\left(60^\circ \times \frac{\pi}{180^\circ}\right) - \frac{1}{2}(13)(11.258)$
 $= 15.31 \text{ cm}^2$

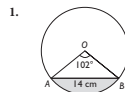


Dalam $\triangle OAM$,
 $\sin 50^\circ = \frac{AM}{15}$
 $AM = 15 \sin 50^\circ$
 $AB = 2 \times 15 \sin 50^\circ$
 $= 22.98 \text{ cm}$
 $\cos 50^\circ = \frac{OM}{15}$
 $OM = 15 \cos 50^\circ$
 $= 9.642 \text{ cm}$

Luas tembereng berlorek
 $= \frac{1}{2}(15^2)\left(100^\circ \times \frac{\pi}{180^\circ}\right) - \frac{1}{2}(22.98)(9.642)$
 $= 85.56 \text{ cm}^2$

113

D. Selesaikan masalah yang berikut.
Solve the following problems.



Rajah di sebelah menunjukkan satu bulatan berpusat O. Diberi luas sektor OAB ialah 72.11 cm² dan luas tembereng berlorek ialah 32.49 cm².
 The diagram shows a circle with centre O. Given the area of sector OAB is 72.11 cm² and the area of the shaded segment is 32.49 cm².
 (a) Cari panjang OA.
 Find the length of OA.
 (b) Hitung tinggi bagi $\triangle AOB$, dengan AB ialah tapaknya.
 Calculate the height of $\triangle AOB$, which has AB as its base.

(a) Luas sektor OAB = 72.11 cm²
 $\frac{1}{2}(OA^2)\left(102^\circ \times \frac{\pi}{180^\circ}\right) = 72.11$
 $OA^2 = \frac{72.11 \times 180 \times 2}{102 \times \pi}$
 $OA = \frac{81.012}{9.001} \text{ cm}$
 $= 9.001 \text{ cm}$
 (b) Luas $\triangle OAB$
 $=$ Luas sektor OAB - Luas tembereng berlorek
 $= 72.11 - 32.49$
 $= 39.62 \text{ cm}^2$
 $\frac{1}{2}(AB)(\text{Tinggi}) = 39.62$
 $\text{Tinggi} = \frac{2 \times 39.62}{14} = 5.66 \text{ cm}$



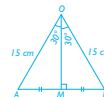
Rajah di sebelah menunjukkan satu sektor AOB yang berpusat O. Diberi panjang lengkok AB ialah 14.4 cm. Cari luas tembereng berlorek.
 The diagram shows a sector AOB with centre O. Given the length of arc AB is 14.4 cm. Find the area of the shaded segment.

Katakan $\angle AOB = \theta$
 $12 \times \theta = 14.4$
 $\theta = 1.2 \text{ rad}$
 Dalam $\triangle OAM$, $\frac{AM}{12} = \sin 0.6 \text{ rad}$
 $AM = 12 \sin 0.6 \text{ rad}$
 $= 6.778 \text{ cm}$
 $AB = 2 \times AM = 13.556 \text{ cm}$
 $OM = \sqrt{12^2 - 6.778^2} = 9.902 \text{ cm}$
 Luas tembereng berlorek
 $= \frac{1}{2}(12^2)(1.2) - \frac{1}{2}(13.556)(9.902)$
 $= 19.28 \text{ cm}^2$



Rajah di sebelah menunjukkan satu bulatan berpusat O. Diberi jejari bulatan itu ialah 15 cm. Cari
 The diagram shows a circle with centre O. Given the radius of the circle is 15 cm. Find
 (a) luas sektor OAB.
 the area of sector OAB.
 (b) luas tembereng berlorek.
 the area of the shaded segment.

(a) $\angle AOB = 2 \times 30^\circ = 60^\circ$
 Luas sektor OAB
 $= \frac{1}{2}(15^2)\left(60^\circ \times \frac{\pi}{180^\circ}\right)$
 $= 117.8 \text{ cm}^2$
 (b) Dalam $\triangle OAB$,
 $AM = 15 \sin 30^\circ$
 $AB = 2 \times 15 \sin 30^\circ = 15 \text{ cm}$
 $OM = 15 \cos 30^\circ = 12.99 \text{ cm}$
 Luas tembereng berlorek
 $= 117.8 - \frac{1}{2}(15)(12.99)$
 $= 20.38 \text{ cm}^2$



114

PRAKTIS FORMATIF Kertas 1

Subtopik	ANALISIS SOALAN SPM			
	2013	2014	2015	2016
8.1	S. 179i	-	S. 190i	-
8.2	S. 179ii	-	-	S. 190ii
8.3	-	S. 20	S. 190i	S. 190ii

Jawab semua soalan.
 Answer all the questions.

1. Rajah di bawah menunjukkan sektor POQ bagi satu bulatan berpusat O.
 The diagram shows a sector POQ of a circle with centre O.



Diberi bahawa OR = 16 cm dan OP = 20 cm. Cari
 It is given that OR = 16 cm and OP = 20 cm. Find
 [Guna/Use $\pi = 3.142$]
 (a) nilai θ , dalam radian.
 the value of θ , in radian.
 (b) perimeter, dalam cm, kawasan berlorek.
 the perimeter, in cm, of the shaded region.

(a) $\cos \theta = \frac{16}{20} = 0.8$
 $\theta = 0.6435 \text{ rad}$
 (b) Panjang lengkok PQ
 $= 20(0.6435)$
 $= 12.87 \text{ cm}$
 Dalam $\triangle OPR$,
 $PR = \sqrt{20^2 - 16^2} = 12 \text{ cm}^2$
 Perimeter kawasan berlorek
 $= PR + RQ + \text{Lengkok PQ}$
 $= 12 + 4 + 12.87$
 $= 28.87 \text{ cm}$

2. Rajah di bawah menunjukkan sektor OPQ dengan pusat O.
 The diagram shows a sector OPQ with centre O.



(a) Cari $\angle POQ$, dalam sebutan π radian.
 Find $\angle POQ$, in terms of π radians.
 (b) Cari perimeter, dalam cm, sektor OPQ.
 Find the perimeter, in cm, of the sector OPQ.
 [Guna/Use $\pi = 3.142$]

(a) $\angle POQ = 150^\circ \times \frac{\pi}{180^\circ}$
 $= \frac{5}{6}\pi \text{ rad}$
 (b) Perimeter sektor OPQ
 $= 9 + 9 + 9\left(\frac{5}{6}\pi\right)$
 $= 41.57 \text{ cm}$

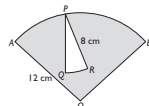
3. Rajah di bawah menunjukkan sektor KOL bagi satu bulatan berpusat O.
 The diagram shows a sector KOL of a circle with centre O.



Diberi bahawa OJ = JK = OM = ML = 4 cm. Cari
 It is given that OJ = JK = OM = ML = 4 cm. Find
 (a) panjang, dalam cm, lengkok KL.
 the length, in cm, of arc KL.
 (b) luas, dalam cm², kawasan berlorek.
 the area, in cm², of the shaded region.

(a) Panjang lengkok KL = 8×1.62
 $= 12.96 \text{ cm}$
 (b) Luas kawasan berlorek
 $=$ Luas sektor KOL - Luas $\triangle OJM$
 $= \frac{1}{2}(8^2)(1.62) - \frac{1}{2}(4^2) \sin 1.62 \text{ rad}$
 $= 43.85 \text{ cm}^2$

4. Rajah di bawah menunjukkan sektor OAB bagi sebuah bulatan berpusat O dan sektor PQR bagi sebuah bulatan berpusat P.
 The diagram shows a sector OAB of a circle with centre O and a sector PQR of a circle with centre P.



Diberi $\angle AOB = 1.6$ radian dan $\angle QPR = 0.45$ radian. Cari luas, dalam cm², kawasan berlorek.
 Given $\angle AOB = 1.6$ radians and $\angle QPR = 0.45$ radians. Find the area, in cm², of the shaded region.

Luas kawasan berlorek
 $=$ Luas sektor OAB - Luas sektor PQR
 $= \frac{1}{2}(12^2)(1.6) - \frac{1}{2}(8^2)(0.45)$
 $= 100.8 \text{ cm}^2$

115

5. Rajah di bawah menunjukkan sektor OQR dan sektor OPS dengan pusat O.
 The diagram shows the sectors OQR and OPS with centre O.



Diberi bahawa OP = 6 cm, nisbah OP : PQ = 3 : 2 dan luas kawasan berlorek ialah 43.64 cm². Cari
 It is given that OP = 6 cm, the ratio of OP : PQ = 3 : 2 and the area of the shaded region is 43.64 cm². Find
 (a) panjang OQ, dalam cm.
 the length of OQ, in cm.
 (b) nilai θ , dalam radian.
 the value of θ , in radian.

(a) OP : PQ = 3 : 2
 $OQ = \frac{5}{3} \times 6 \text{ cm} = 10 \text{ cm}$
 (b) Luas kawasan berlorek
 $=$ Luas sektor OQR - Luas sektor OPS
 $43.64 = \frac{1}{2}(10^2)\theta - \frac{1}{2}(6^2)\theta$
 $43.64 = 50\theta - 18\theta$
 $32\theta = 43.64$
 $\theta = 1.364 \text{ rad}$

6. Rajah di bawah menunjukkan pandangan hadapan sebahagian lukisan mural berbentuk segi empat sama pada sebuah dinding koridor. PT adalah lengkok bulatan berpusat P dan QT adalah lengkok bulatan berpusat Q.
 The diagram shows part of the front view of a square-shaped mural art on a wall of a corridor. PT is an arc of a circle with centre P and QT is an arc of a circle with centre Q.



Kawasan berlorek merupakan bahagian yang perlu dicat semula. Kira luas, dalam m², kawasan itu.
 The shaded region shows the part that needs to be repainted. Calculate the area, in m², of that region.

Luas segi empat sama PQRS = $8 \times 8 = 64 \text{ m}^2$
 Luas sektor PQT = $\frac{1}{2} \times 8 \times 8 \times \frac{\pi}{3} = 33.5 \text{ m}^2$
 Luas tembereng TQ = $33.5 - \text{Luas } \triangle PQT$
 $= 33.5 - \frac{1}{2} \times 8 \times 8 \times \sin 60^\circ$
 $= 5.80 \text{ m}^2$
 Luas kawasan yang perlu dicat semula = $64 - 33.5 - 5.80$
 $= 24.69 \text{ m}^2$

116

7. Rajah di bawah menunjukkan sebuah sektor OAB dengan pusat O dan berjajari 10 cm.
 The diagram shows a sector OAB with centre O and a radius of 10 cm.



Diberi OA = OB dan $\angle OAB = 50^\circ$, cari
 Given that OA = OB and $\angle OAB = 50^\circ$, find
 [Guna/Use $\pi = 3.142$]
 (a) nilai θ , dalam radian.
 the value of θ , in radians.
 (b) luas kawasan berlorek.
 the area of the shaded region.

(a) OAB ialah \triangle sama kaki.
 $\theta = 180^\circ - 50^\circ - 50^\circ$
 $= 80^\circ$
 $= \frac{80}{180} \times 3.142$
 $= 1.396 \text{ rad}$
 (b) Luas kawasan berlorek
 $=$ Luas sektor OAB - Luas $\triangle OAB$
 $= \frac{1}{2}(10^2)(1.396) - \frac{1}{2}(10)(10) \sin 80^\circ$
 $= 20.56 \text{ cm}^2$

8. Rajah di bawah menunjukkan sebuah bulatan dengan pusat O dan jejari 12 cm.
 The diagram shows a circle with centre O and a radius of 12 cm.



Diberi panjang lengkok minor AB ialah 20 cm.
 Given the length of the minor arc AB is 20 cm.
 [Guna/Use $\pi = 3.142$]

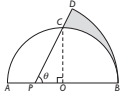
(a) Nyatakan nilai θ , dalam radian.
 State the value of θ , in radians.
 (b) Cari luas sektor major OAB, dalam cm², betul kepada empat angka bererti.
 Find the area of the major sector OAB, in cm², correct to four significant figures.

(a) $120 = 20$
 $\theta = \frac{20}{12} = 1.667 \text{ rad}$
 (b) Luas sektor major OAB
 $= \frac{1}{2} \times 12^2 \times (2\pi - 1.667)$
 $= 332.4 \text{ cm}^2$ (4 a.b.)

PRAKTIK FORMATIF Kertas 2

Jawab semua soalan.
Answer all the questions.

1. Dalam rajah di bawah, $AOBC$ ialah satu semibulatan berpusat O dengan jejari 6 cm. BPD ialah sektor satu bulatan berpusat P dengan jejari 9 cm. In the diagram, $AOBC$ is a semicircle with centre O and a radius of 6 cm. BPD is a sector of a circle with centre P and a radius of 9 cm.



Diberi bahawa OC berserenjang dengan AOB . Hitung It is given that OC is perpendicular to AOB . Calculate [Guna/Use $\pi = 3.142$]

- (a) nilai θ , dalam radian. [2]
- (b) perimeter, dalam cm, rantau berlorek. [4]
- (c) luas, dalam cm^2 , rantau berlorek. [4]

(a) $OP = PB - OB = 9 - 6 = 3$ cm
 $OC = 6$ cm
 $\tan \theta = \frac{OC}{OP} = \frac{6}{3} = 2$
 $\theta = 1.107$ rad

(b) Panjang lengkok BC
 $= 6 \times \frac{\pi}{2}$
 $= 9.426$ cm

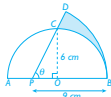
Panjang lengkok $BD = 9 \times 1.107$
 $= 9.963$ cm

Dalam $\triangle OPC$, $PC = \sqrt{6^2 + 3^2}$
 $= 6.708$ cm

$CD = PD - PC = 9 - 6.708$
 $= 2.292$ cm

Perimeter rantau berlorek
 $= \text{Lengkok } BC + \text{Lengkok } BD + CD$
 $= 9.426 + 9.963 + 2.292$
 $= 21.681$ cm

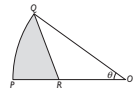
(c) Luas rantau berlorek
 $= \text{Luas sektor } BPD - \text{Luas sukuan bulatan } OBC - \text{Luas } \triangle OPC$
 $= \frac{1}{2}(9^2)(1.107) - \frac{1}{2}(6^2)\left(\frac{\pi}{2}\right) - \frac{1}{2}(3)(6)$
 $= 7.556$ cm^2



117

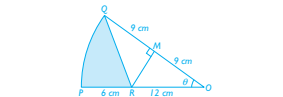
ANALISIS SOALAN SPM					
Selangor	2013	2014	2015	2016	
8.1	5.1161	5.1161	5.1161	5.789	
8.2	5.1161	5.1161	5.661	5.789	
8.3	5.1161	5.1161	5.661	5.789	

2. Rajah di bawah menunjukkan sektor OPQ bagi satu bulatan berpusat O dan berjejari 18 cm. Titik R terletak pada OP . The diagram shows a sector OPQ of a circle, centre O with a radius of 18 cm. Point R lies on OP .



Diberi bahawa $OR = RQ = 12$ cm. Hitung It is given that $OR = RQ = 12$ cm. Calculate [Guna/Use $\pi = 3.142$]

- (a) nilai θ , dalam radian, betul kepada tiga tempat perpuluhan. [2]
- (b) perimeter, dalam cm, kawasan berlorek. [3]
- (c) luas, dalam cm^2 , kawasan berlorek. [5]



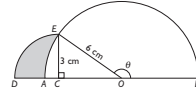
Lukis garis lurus MR yang berserenjang dengan OQ .

(a) Dalam $\triangle OMR$,
 $\cos \theta = \frac{OR}{OQ} = \frac{12}{18} = 0.667$
 $\theta = 0.7227$ rad
 $= 0.723$ rad

(b) Perimeter kawasan berlorek
 $= PR + RQ + \text{Lengkok } PQ$
 $= 6 + 12 + 18(0.723)$
 $= 31.01$ cm

(c) Luas kawasan berlorek
 $= \text{Luas sektor } OPQ - \text{Luas } \triangle OPR$
 $= \frac{1}{2}(18^2)(0.723) - \frac{1}{2}(18)(12) = 9^2$
 $= 81$ cm^2

3. Rajah di bawah menunjukkan semibulatan AEB , dengan pusat O dan sukuan bulatan DCE , dengan pusat C . The diagram shows a semicircle AEB , with centre O and the quadrant of a circle DCE , with centre C .



- Hitung Calculate [Guna/Use $\pi = 3.142$]
- (a) nilai θ , dalam radian. [2]
 - (b) perimeter, dalam cm, seluruh rajah itu. [4]
 - (c) luas, dalam cm^2 , kawasan berlorek itu. [4]

(a) $\angle EOC = \sin^{-1}\left(\frac{3}{6}\right)$
 $= 30^\circ$
 $= \frac{\pi}{6}$ rad
 $\theta = \pi - \frac{\pi}{6} = 2.618$ rad

(b) Panjang $OC = 6 \cos 30^\circ$
 $= 5.196$ cm

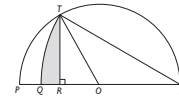
Perimeter seluruh rajah
 $= \text{Lengkok } BE + \text{Lengkok } DE + CD + OC + OB$
 $= 6(2.618) + \frac{1}{4}(2)(3.142)(3) + 3 + 5.196 + 6$
 $= 34.62$ cm

(c) Luas tembereng EAC
 $= \text{Luas sektor } OAE - \text{Luas segi tiga } OCE$
 $= \frac{1}{2}(6^2)\left(\frac{\pi}{6}\right) - \frac{1}{2}(5.196)(3)$
 $= 1.632$ cm^2

Luas kawasan berlorek
 $= \text{Luas sukuan bulatan } DCE - \text{Luas tembereng } EAC$
 $= \frac{1}{4}(3.142)(3^2) - 1.632$
 $= 5.438$ cm^2

118

4. Rajah di bawah menunjukkan semibulatan $OPTS$ dengan pusat O dan berjejari 10 cm. QST ialah sektor sebuah bulatan berpusat S dan R ialah titik tengah OP . The diagram shows a semicircle $OPTS$ with centre O and a radius of 10 cm. QST is a sector of a circle with centre S and R is the midpoint of OP .



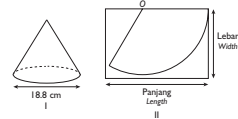
- Dengan menggunakan $\pi = 3.142$, hitung By using $\pi = 3.142$, calculate
- (a) $\angle TOR$, dalam radian. [2]
 - (b) panjang, dalam cm, lengkok TQ . [4]
 - (c) luas, dalam cm^2 , kawasan berlorek. [4]

(a) Dalam $\triangle TOR$,
 $\cos \angle TOR = \frac{OR}{OT} = \frac{5}{10} = \frac{1}{2}$
 $\angle TOR = 60^\circ$
 $= 60 \times \frac{3.142}{180}$
 $= 1.047$ rad

(b) TOS ialah segi tiga sama kaki dan $\angle TOS = 120^\circ$.
Maka, $\angle TSO = 30^\circ$
 $= 0.5237$ rad
Panjang lengkok $TQ = 10 \times 0.5237$
 $= 5.237$ cm

(c) Luas kawasan berlorek
 $= \text{Luas sektor } QST - \text{Luas } \triangle TRS$
Dalam $\triangle TOR$, $TR^2 = 10^2 - 5^2$
 $= 75$
 $TR = 8.660$ cm
Luas $\triangle TRS = \frac{1}{2} \times 8.660 \times 15$
 $= 64.95$ cm^2
Dalam $\triangle TOS$, $TS^2 = 75 + 15^2$
 $= 300$
Luas sektor $QST = \frac{1}{2} \times TS^2 \times 0.5237$
 $= \frac{1}{2} \times 300 \times 0.5237$
 $= 78.56$ cm^2
Luas kawasan berlorek $= 78.56 - 64.95$
 $= 13.61$ cm^2

5. Zahara ingin membuat topi berbentuk kon seperti yang ditunjukkan dalam Rajah I. Tinggi kon itu ialah 26 cm. Zahara wants to make a cap in the shape of a cone as shown in Diagram I. The height of the cone is 26 cm.



Rajah II menunjukkan bentangan kon yang berbentuk sektor bagi sebuah bulatan dengan pusat O yang dilukis di atas sekeping kad berbentuk segi empat tepat. Diagram II shows the net of the cone in the form of a sector of a circle with centre O drawn on a rectangular card.

- (a) Hitung nilai minimum, dalam cm, bagi panjang dan lebar kad itu kepada integer terdekat. Calculate the minimum value, in cm, of length and of width of the card to the nearest integer. [5]
- (b) Sterusnya, cari luas, dalam cm^2 , kad yang tidak digunakan. Hence, find the area, in cm^2 , of the card unused. [3]

(a) Panjang lengkok AB
 $= \text{Lilitan tapak kon}$
 $= 2\pi r$
 $= 2\pi \left(\frac{18.8}{2}\right)$
 $= 59.07$ cm

Panjang sendang kon
 $= \sqrt{26^2 + 9.4^2}$
 $= 27.65$ cm

Maka, $OA = 27.65$ cm

Panjang lengkok $AB = \theta r$
Maka, $27.65\theta = 59.07$
 $\theta = 2.136$ rad

$\angle BOC = \pi - 2.136 = 1.006$ rad $= 57.63^\circ$

$\cos 57.63^\circ = \frac{OC}{27.65}$
 $OC = 14.80$ cm

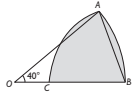
Maka, panjang minimum kad
 $= 27.65 + 14.80$
 $= 42$ cm [kepada integer terdekat]

Lebar minimum kad $= 28$ cm [kepada integer terdekat]

(b) Luas kad yang tidak digunakan
 $= (42 \times 28) - \text{Luas sektor } OAB$
 $= 1176 - \frac{1}{2}(27.65^2)(2.136)$
 $= 359.5$ cm^2

119

6. Rajah di bawah menunjukkan sektor AOB dengan pusat O dan sektor ABC dengan pusat B . The diagram shows a sector AOB with centre O and sector ABC with centre B .



Diberi bahawa $OA = 18$ cm dan $\angle AOB = 40^\circ$. Cari It is given that $OA = 18$ cm and $\angle AOB = 40^\circ$. Find [Guna/Use $\pi = 3.142$]

- (a) panjang AB , dalam cm, betul kepada satu tempat perpuluhan. [2]
- (b) perimeter, dalam cm, sektor ABC . [3]
- (c) luas, dalam cm^2 , rantau berlorek. [5]

(a) $\triangle AOB$ ialah segi tiga sama kaki dengan $OA = OB$.
 $\frac{1}{2}AB = 18 \sin 20^\circ$
 $AB = 2 \times 18 \sin 20^\circ$
 $= 12.313$ cm
 $= 12.3$ cm

(b) $\angle ABC = (180^\circ - 40^\circ) + 2$
 $= 70^\circ$
 $= 1.222$ radian

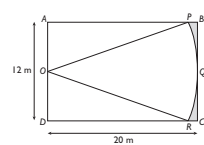
Perimeter sektor ABC
 $= AB + BC + \text{Lengkok } AC$
 $= 12.3 + 12.3 + 12.3 \times 1.222$
 $= 39.63$ cm

(c) Luas rantau berlorek
 $= \text{Luas sektor } ABC + \text{Luas tembereng yang dibatasi oleh lengkok } AB \text{ dan perentas } AB$
 $= \frac{1}{2}(12.3^2)(1.222) + (\text{Luas sektor } AOB - \text{Luas segi tiga } AOB)$
 $= 92.44 + \left[\frac{1}{2}(18^2)\left(40^\circ \times \frac{\pi}{180^\circ}\right) - \frac{1}{2}(12.3)(18 \cos 20^\circ)\right]$
 $= 92.44 + (113.11 - 104.02)$
 $= 101.5$ cm^2

FOKUS KBAT

Kemahiran Kognitif: Mengaplikasi
Konteks: Luas Sektor, Panjang Lengkok

Dalam rajah di bawah, $ABCD$ mewakili sebidang tanah yang berbentuk segi empat tepat. $OPQR$ ialah sebuah sektor berpusat O . In the diagram, $ABCD$ represents a piece of rectangular land. $OPQR$ is a sector with centre O .



Lengkok PQR menyentuh tepi BC di Q . Serene ingin memagari kawasan yang berlorek dan menanam pokok bunga di situ. Hitung Arc PQR is touching the edge BC at Q . Serene intends to fence off the shaded area and plant some flowers there. Calculate

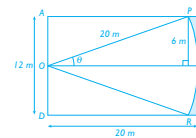
- (a) luas, dalam m^2 , tanah yang akan ditanami dengan pokok bunga. the area, in m^2 , of the land to be planted with flowers.
- (b) jumlah kos untuk pagar, jika kos untuk memagari ialah RM50 per m. the total cost for fencing, if the cost for fencing is RM50 per m.

Info KBAT

BC ialah tangen kepada lengkok PQR . O dan Q masing-masing ialah titik tengah bagi AD dan BC . BC is the tangent to the arc PQR . O and Q are the midpoints of AD and BC respectively.

(a) $OP = OQ = 20$ m
 $\tan \theta = \frac{6}{20}$
 $\theta = 0.2915$ rad
 $AP = \sqrt{20^2 - 6^2} = 19.079$ m
Luas kawasan berlorek
 $= 2(\text{Luas } ABQO - \text{Luas } \triangle OAP - \text{Luas sektor } OPQ)$
 $= 2\left[20(6) - \frac{1}{2}(6)(19.079) - \frac{1}{2}(20^2)(0.2915)\right]$
 $= 8.926$ m^2

(b) $\angle POR = 2(0.2915) = 0.583$ rad
 $PB = RC = 20 - 19.079 = 0.921$ m
Perimeter kawasan berlorek $= 20(0.583) + 2(0.921) + 12$
 $= 25.502$ m
Jumlah kos pagar $= 25.502 \times \text{RM50}$
 $= \text{RM1 275.10}$



120

9.1 Pembezaan dengan Menggunakan Prinsip Pertama

A. Cari nilai bagi setiap yang berikut.
Find the value of each of the following.

CONTOH

$$\begin{aligned} \text{had } \frac{7x+5}{x} &= \text{had } \left(\frac{7x}{x} + \frac{5}{x} \right) \\ &= \text{had } \left(7 + \frac{5}{x} \right) \\ &= 7 + 0 \\ &= 7 \end{aligned}$$

1. had $(2x - 7)$
 $x \rightarrow 0$
 $= 0 - 7$
 $= -7$

2. had $\left(\frac{2}{x+6} \right)$
 $x \rightarrow 2$
 $= \frac{2}{2+6}$
 $= \frac{2}{8}$
 $= \frac{1}{4}$

3. had $\left(\frac{3x+2x^2}{x} \right)$
 $x \rightarrow 0$
 $= \text{had } \left(\frac{3x}{x} + \frac{2x^2}{x} \right)$
 $= \text{had } (3 + 2x)$
 $= 3$

4. had $(x^2 + 2x + 1)$
 $x \rightarrow 3$
 $= 16 + 8 + 1$
 $= 25$

5. had $\left(\frac{x^2 - x - 6}{x - 3} \right)$
 $x \rightarrow 3$
 $= \text{had } \frac{(x-3)(x+2)}{x-3}$
 $= \text{had } (x+2)$
 $= 3 + 2$
 $= 5$

B. Tentukan terbitan pertama bagi setiap yang berikut dengan menggunakan prinsip pertama.
Determine the first derivative of each of the following by using the first principle.

CONTOH

$$\begin{aligned} y &= 3x^2 \\ y + \delta y &= 3(x + \delta x)^2 \\ &= 3[x^2 + 2x\delta x + (\delta x)^2] \\ &= 3x^2 + 6x\delta x + 3(\delta x)^2 \end{aligned}$$

② - ①: $\delta y = 6x\delta x + 3(\delta x)^2$

$$\frac{\delta y}{\delta x} = 6x + 3\delta x$$

Bahagikan kedua-dua
bahagi persamaan
dengan δx .

$$\frac{d y}{d x} = \text{had } \frac{\delta y}{\delta x}$$

$$= \text{had } (6x + 3\delta x)$$

$$= 6x$$

δx = Tokokan kecil dalam x
 δy = Tokokan kecil dalam y

1. $y = 5x$

$$\begin{aligned} y &= 5x \\ y + \delta y &= 5(x + \delta x) \\ &= 5x + 5\delta x \end{aligned}$$

② - ①: $\delta y = 5\delta x$

$$\frac{\delta y}{\delta x} = 5$$

$$\frac{d y}{d x} = \text{had } \frac{\delta y}{\delta x}$$

$$= \text{had } 5$$

$$= 5$$

B. Cari nilai $\frac{dy}{dx}$ atau $f'(x)$ untuk nilai x yang diberikan.

Find the value of $\frac{dy}{dx}$ or $f'(x)$ for the given value of x .

CONTOH

1. $y = 3x^2; x = 4$

$$\frac{dy}{dx} = 3(2x^{2-1}) = 6x$$

Apabila $x = 4$,

$$\frac{dy}{dx} = 6(4) = 24$$

2. $y = 2x^3; x = 2$

$$\frac{dy}{dx} = 2(3x^{3-1}) = 6x^2$$

Apabila $x = 2$,

$$\frac{dy}{dx} = 6(2)^2 = 24$$

3. $y = -3x^2; x = 5$

$$\frac{dy}{dx} = -3(2x^{2-1}) = -6x$$

Apabila $x = 5$,

$$\frac{dy}{dx} = -6(5) = -30$$

3. $f(x) = -x^3; x = 6$

$$f'(x) = -3x^{3-1} = -3x^2$$

$$f'(6) = -3(6)^2 = -108$$

4. $f(x) = \frac{8}{x^2}; x = -2$

$$f(x) = \frac{8}{x^2} = 8x^{-2}$$

$$f'(x) = -16x^{-3} = -\frac{16}{x^3}$$

$$f'(-2) = -\frac{16}{(-2)^3} = 2$$

5. $f(x) = \frac{2}{3x^3}; x = 1$

$$f(x) = \frac{2}{3x^3} = \frac{2}{3}x^{-3}$$

$$f'(x) = \frac{2}{3}(-3x^{-4}) = -\frac{2}{x^4}$$

$$f'(1) = -\frac{2}{1^4} = -2$$

C. Cari nilai $f'(x)$ untuk nilai x yang diberikan.

Find the value of $f'(x)$ for the given value of x .

CONTOH

Diberi $f(x) = x^4$, cari $f'(2)$.

Given $f(x) = x^4$, find $f'(2)$.

$$f'(x) = 4x^{4-1} = 4x^3$$

$$f'(2) = 4(2^3) = 4(8) = 32$$

1. Diberi $f(x) = -4x^2$, cari $f'(3)$.

Given $f(x) = -4x^2$, find $f'(3)$.

$$f'(x) = -8x$$

$$f'(3) = -8(3) = -24$$

2. Diberi $f(x) = 5x^3$, cari $f'(-2)$.

Given $f(x) = 5x^3$, find $f'(-2)$.

$$f'(x) = 15x^2$$

$$f'(-2) = 15(-2)^2 = 60$$

3. Diberi $f(x) = \frac{2}{3}x^3$, cari $f'(4)$.

Given $f(x) = \frac{2}{3}x^3$, find $f'(4)$.

$$f'(x) = \frac{2}{3}(3x^{3-1}) = 2x^2$$

$$f'(4) = 2(4)^2 = 32$$

4. Diberi $f(x) = \frac{4}{x^3}$, cari $f'(2)$.

Given $f(x) = \frac{4}{x^3}$, find $f'(2)$.

$$f(x) = \frac{4}{x^3} = 4x^{-3}$$

$$f'(x) = -16x^{-4} = -\frac{16}{x^4}$$

$$f'(2) = -\frac{16}{2^4} = -\frac{1}{2}$$

5. Diberi $f(x) = \frac{3}{4x^2}$, cari $f'(1)$.

Given $f(x) = \frac{3}{4x^2}$, find $f'(1)$.

$$f(x) = \frac{3}{4x^2} = \frac{3}{4}x^{-2}$$

$$f'(x) = -\frac{3}{2}x^{-3} = -\frac{3}{2x^3}$$

$$f'(1) = -\frac{3}{2(1)^3} = -\frac{3}{2}$$

2. $y = 4x^2 + x$

$$\begin{aligned} y &= 4x^2 + x \\ y + \delta y &= 4(x + \delta x)^2 + (x + \delta x) \\ &= 4x^2 + 8x\delta x + 4(\delta x)^2 + x + \delta x \end{aligned}$$

② - ①: $\delta y = 8x\delta x + 4(\delta x)^2 + \delta x$

$$\frac{\delta y}{\delta x} = 8x + 4\delta x + 1$$

$$\frac{d y}{d x} = \text{had } \frac{\delta y}{\delta x}$$

$$= \text{had } (8x + 4\delta x + 1)$$

$$= 8x + 1$$

3. $y = 5 - 4x^2$

$$\begin{aligned} y &= 5 - 4x^2 \\ y + \delta y &= 5 - 4(x + \delta x)^2 \\ &= 5 - 4[x^2 + 2x\delta x + (\delta x)^2] \\ &= 5 - 4x^2 - 8x\delta x - 4(\delta x)^2 \end{aligned}$$

② - ①: $\delta y = -8x\delta x - 4(\delta x)^2$

$$\frac{\delta y}{\delta x} = -8x - 4\delta x$$

$$\frac{d y}{d x} = \text{had } \frac{\delta y}{\delta x}$$

$$= \text{had } (-8x - 4\delta x)$$

$$= -8x$$

9.2 Terbitan Pertama Fungsi Polinomial

A. Cari $\frac{dy}{dx}$ atau $f'(x)$ bagi setiap fungsi yang berikut.

Find $\frac{dy}{dx}$ or $f'(x)$ for each of the following functions.

CONTOH

(a) $y = 2x^4$

$$\frac{dy}{dx} = 4(2x^{4-1}) = 4(2x^3) = 8x^3$$

(b) $f(x) = -\frac{3}{x^4} = -3x^{-4}$

$$f'(x) = -4(-3x^{-4-1}) = 12x^{-5} = \frac{12}{x^5}$$

FAKTA UTAMA

- $y = k, \frac{dy}{dx} = 0$
- $y = ax^n, \frac{dy}{dx} = nax^{n-1}$
- $y = a\sin x + c, \frac{dy}{dx} = a\cos x + c(-1)^n$

1. $y = 15x$

$$\frac{dy}{dx} = 15x^{1-1} = 15x^0 = 15$$

2. $y = -4x^2$

$$\frac{dy}{dx} = 2(-4x^{2-1}) = 2(-4x) = -8x$$

3. $y = -16x^3$

$$\frac{dy}{dx} = 3(-16x^{3-1}) = 3(-16x^2) = -48x^2$$

4. $f(x) = 18$

$$f'(x) = 0$$

5. $f(x) = \frac{5}{x^2} = 5x^{-2}$

$$f'(x) = -2(5x^{-2-1}) = -10x^{-3} = -\frac{10}{x^3}$$

6. $f(x) = \frac{-2}{5x^3} = -\frac{2}{5}x^{-3}$

$$f'(x) = -3\left(-\frac{2}{5}x^{-3-1}\right) = \frac{6}{5}x^{-4} = \frac{6}{5x^4}$$

D. Bezakan setiap fungsi algebra yang berikut terhadap x .

Differentiate each of the following algebraic functions with respect to x .

CONTOH

(a) $3x^4 + x^3 + \frac{1}{2}x^2$

$$\frac{d}{dx} \left(3x^4 + x^3 + \frac{1}{2}x^2 \right) = \frac{d}{dx} (3x^4) + \frac{d}{dx} (x^3) + \frac{d}{dx} \left(\frac{1}{2}x^2 \right)$$

$$= 12x^3 + 3x^2 + x$$

(b) $\frac{4x^3 - 2x^2}{x}$

$$\frac{d}{dx} \left(\frac{4x^3 - 2x^2}{x} \right) = \frac{d}{dx} (4x^2 - 2x) = \frac{d}{dx} (4x^2) - \frac{d}{dx} (2x)$$

$$= 8x - 2$$

FAKTA UTAMA

- $y = f(x) \pm g(x)$
 $\frac{dy}{dx} = f'(x) \pm g'(x)$
- $y = f(x)g(x)$
 $\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$

1. $-2x^3 + \frac{2}{x^2} + 6x$

$$\frac{d}{dx} \left(-2x^3 + \frac{2}{x^2} + 6x \right) = \frac{d}{dx} (-2x^3) + \frac{d}{dx} (2x^{-2}) + \frac{d}{dx} (6x)$$

$$= -6x^2 - \frac{4}{x^3} + 6$$

2. $\frac{x^3 - 2x + 1}{x^2}$

$$\frac{d}{dx} \left(\frac{x^3 - 2x + 1}{x^2} \right) = \frac{d}{dx} \left(x - \frac{2}{x} + \frac{1}{x^2} \right) = \frac{d}{dx} \left(x - 2x^{-1} + x^{-2} \right)$$

$$= 1 + \frac{2}{x^2} - \frac{2}{x^3}$$

3. $\frac{3}{x^3} - 4x$

$$\frac{d}{dx} \left(\frac{3}{x^3} - 4x \right) = \frac{d}{dx} (3x^{-3} - 4x)$$

$$= -\frac{9}{x^4} - 4$$

E. Cari $\frac{dy}{dx}$ bagi setiap fungsi yang berikut.

Find $\frac{dy}{dx}$ for each of the following functions.

CONTOH

1. $y = x^2(x + 2)$

$$y = x^2(x + 2) = x^3 + 2x^2$$

$$\frac{dy}{dx} = 3x^2 + 4x$$

2. $y = (x + 3)^2$

$$= x^2 + 6x + 9$$

$$\frac{dy}{dx} = 2x + 6$$

3. $y = (x - 3)(5 - 2x)$

$$= 11x - 2x^2 - 15$$

$$\frac{dy}{dx} = 11 - 4x$$

4. $y = \left(2x + \frac{3}{x} \right)^2$

$$= 4x^2 + 12 + \frac{9}{x^2}$$

$$\frac{dy}{dx} = 8x - \frac{18}{x^3}$$

5. $y = x^3 \left(6x - \frac{1}{x} \right)$

$$= 6x^4 - x^2$$

$$\frac{dy}{dx} = 24x^3 - 2x$$

F. Bezakan fungsi algebra yang berikut dengan menggunakan petua hasil darab. Differentiate the following algebraic functions by using the product rule.

CONTOH

$$y = (x^2 - 2)(x^3 + 1)$$

Katakan $u = x^2 - 2$ dan $v = x^3 + 1$

$$\begin{aligned} \frac{du}{dx} &= 2x & \frac{dv}{dx} &= 3x^2 \\ \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= (x^2 - 2)(3x^2) + (x^3 + 1)(2x) \\ &= 3x^4 - 6x^2 + 2x^4 + 2x \\ &= 5x^4 - 6x^2 + 2x \end{aligned}$$

2. $y = (1 - 2x^3)(x + 2)$

u = 1 - 2x³ dan v = x + 2
 $\frac{du}{dx} = -6x^2$ dan $\frac{dv}{dx} = 1$

$$\begin{aligned} \frac{dy}{dx} &= (1 - 2x^3)(1) + (x + 2)(-6x^2) \\ &= 1 - 2x^3 - 6x^3 - 12x^2 \\ &= -8x^3 - 12x^2 + 1 \end{aligned}$$

1. $y = (3x^2 + 6)(3x^2 + 4)$

u = 3x² + 6 dan v = 3x² + 4
 $\frac{du}{dx} = 6x$ dan $\frac{dv}{dx} = 6x$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= (3x^2 + 6)(6x) + (3x^2 + 4)(6x) \\ &= 3x^2 \cdot 6x + 6 \cdot 6x + 3x^2 \cdot 6x + 4 \cdot 6x \\ &= 9x^3 + 24x + 6x^3 + 24x \\ &= 12x^3 + 48x \end{aligned}$$

3. $y = (x^2 - x)(3x^2 + 2x)$

u = x² - x dan v = 3x² + 2x
 $\frac{du}{dx} = 2x - 1$ dan $\frac{dv}{dx} = 6x + 2$

$$\begin{aligned} \frac{dy}{dx} &= (x^2 - x)(6x + 2) + (3x^2 + 2x)(2x - 1) \\ &= 6x^3 - 6x^2 + 2x^2 - 2x + 6x^3 + 4x^2 - 3x^2 - 2x \\ &= 12x^3 - 3x^2 - 4x \end{aligned}$$

G. Bezakan fungsi algebra yang berikut dengan menggunakan petua hasil bahagi. Differentiate the following algebraic functions by using the quotient rule.

CONTOH

(a) $f(x) = \frac{x^2 + 3}{x - 2}$

Katakan $u = x^2 + 3$ dan $v = x - 2$
 $\frac{du}{dx} = 2x$ dan $\frac{dv}{dx} = 1$

$$\begin{aligned} f'(x) &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(x - 2)(2x) - (x^2 + 3)(1)}{(x - 2)^2} \\ &= \frac{2x^2 - 4x - x^2 - 3}{(x - 2)^2} \\ &= \frac{x^2 - 4x - 3}{(x - 2)^2} \end{aligned}$$

(b) $f(x) = \frac{x + 4}{x^2 + 1}$

Katakan $u = x + 4$ dan $v = x^2 + 1$
 $\frac{du}{dx} = 1$ dan $\frac{dv}{dx} = 2x$

$$\begin{aligned} f'(x) &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(x^2 + 1)(1) - (x + 4)(2x)}{(x^2 + 1)^2} \\ &= \frac{x^2 + 1 - 2x^2 - 8x}{(x^2 + 1)^2} \\ &= \frac{-x^2 - 8x + 1}{(x^2 + 1)^2} \end{aligned}$$

H. Cari $\frac{dy}{dx}$ bagi setiap fungsi yang berikut.

Find $\frac{dy}{dx}$ for each of the following functions.

CONTOH

(a) $y = (x^3 + 2x)^2$

$$\begin{aligned} \frac{dy}{dx} &= 2(x^3 + 2x)^2 \cdot \frac{d}{dx}(x^3 + 2x) \\ &= 2(x^3 + 2x)(3x^2 + 2) \\ &= 2x(x^2 + 2)(3x^2 + 2) \\ &= 30x^2(2x^3 - 6)^4 \end{aligned}$$

(b) $y = (2x^3 - 6)^5$

$$\begin{aligned} \frac{dy}{dx} &= 5(2x^3 - 6)^4 \cdot \frac{d}{dx}(2x^3 - 6) \\ &= 5(2x^3 - 6)^4(6x^2) \\ &= 5(2x^3 - 6)^4(6x^2) \\ &= 30x^2(2x^3 - 6)^4 \end{aligned}$$

FAKTA UTAMA

• Petua rantai:
 Chain rule:
 $y = a(bx + c)^n$
 $\frac{dy}{dx} = an(bx + c)^{n-1} \cdot \frac{d}{dx}(bx + c)$
 $= an(bx + c)^{n-1} \cdot b$

1. $y = (4x - 5)^3$

$$\frac{dy}{dx} = 3(4x - 5)^2(4) = 12(4x - 5)^2$$

2. $y = (2x + 3)^3$

$$\frac{dy}{dx} = 3(2x + 3)^2(2) = 6(2x + 3)^2$$

3. $y = (3x + x^2)^4$

$$\frac{dy}{dx} = 4(3x + x^2)^3(3 + 2x) = 4(3 + 2x)(3x + x^2)^3$$

4. $y = (x^2 - 3x + 2)^3$

$$\frac{dy}{dx} = 3(x^2 - 3x + 2)^2(2x - 3) = 3(2x - 3)(x^2 - 3x + 2)^2$$

5. $y = (4x^3 - 3x)^3$

$$\frac{dy}{dx} = 3(4x^3 - 3x)^2(12x^2 - 3) = 9x^2(4x^3 - 3)^2(4x^2 - 1)$$

6. $y = (x^3 - x)^4$

$$\frac{dy}{dx} = 4(x^3 - x)^3(3x^2 - 1) = 4(3x^2 - 1)(x^3 - x)^3$$

7. $y = (x^2 - 2x + 4)^3$

$$\frac{dy}{dx} = 3(x^2 - 2x + 4)^2(2x - 2) = 3(x^2 - 2x + 4)^2(x - 1) = 6(x - 1)(x^2 - 2x + 4)^2$$

8. $y = (x^2 + 3x + 1)^3$

$$\frac{dy}{dx} = 3(x^2 + 3x + 1)^2(2x + 3)$$

1. $f(x) = \frac{3x - 1}{2x + 6}$

u = 3x - 1 dan v = 2x + 6
 $\frac{du}{dx} = 3$ dan $\frac{dv}{dx} = 2$

$$\begin{aligned} f'(x) &= \frac{(2x + 6)(3) - (3x - 1)(2)}{(2x + 6)^2} \\ &= \frac{6x + 18 - 6x + 2}{(2x + 6)^2} \\ &= \frac{20}{(2x + 6)^2} \end{aligned}$$

2. $f(x) = \frac{x^2 + 3}{x - 1}$

u = x² + 3 dan v = x - 1
 $\frac{du}{dx} = 2x$ dan $\frac{dv}{dx} = 1$

$$\begin{aligned} f'(x) &= \frac{(x - 1)(2x) - (x^2 + 3)(1)}{(x - 1)^2} \\ &= \frac{x^2 - 2x - 3}{(x - 1)^2} \\ &= \frac{(x - 3)(x + 1)}{(x - 1)^2} \end{aligned}$$

3. $f(x) = \frac{3x}{2x^2 + 2}$

u = 3x dan v = 2x² + 2
 $\frac{du}{dx} = 3$ dan $\frac{dv}{dx} = 4x$

$$\begin{aligned} f'(x) &= \frac{(2x^2 + 2)(3) - 3x(4x)}{(2x^2 + 2)^2} \\ &= \frac{6x^2 + 6 - 12x^2}{(2x^2 + 2)^2} \\ &= \frac{6 - 6x^2}{(2x^2 + 2)^2} \\ &= \frac{6(1 - x^2)}{4(x^2 + 1)^2} \\ &= \frac{3(1 - x^2)}{2(x^2 + 1)^2} \end{aligned}$$

4. $f(x) = \frac{3x - 4}{5 - 2x}$

u = 3x - 4 dan v = 5 - 2x
 $\frac{du}{dx} = 3$ dan $\frac{dv}{dx} = -2$

$$\begin{aligned} f'(x) &= \frac{(5 - 2x)(3) - (3x - 4)(-2)}{(5 - 2x)^2} \\ &= \frac{15 - 6x + 6x - 8}{(5 - 2x)^2} \\ &= \frac{7}{(5 - 2x)^2} \end{aligned}$$

5. $f(x) = \frac{x^3}{x^2 + 1}$

u = x³ dan v = x² + 1
 $\frac{du}{dx} = 3x^2$ dan $\frac{dv}{dx} = 2x$

$$\begin{aligned} f'(x) &= \frac{(x^2 + 1)(3x^2) - x^3(2x)}{(x^2 + 1)^2} \\ &= \frac{3x^4 + 3x^2 - 2x^4}{(x^2 + 1)^2} \\ &= \frac{x^4 + 3x^2}{(x^2 + 1)^2} \end{aligned}$$

6. $f(x) = \frac{4x}{x^3 + x}$

$\frac{du}{dx} = 4$ dan $\frac{dv}{dx} = 2x$
 $\frac{du}{dx} = 0$ dan $\frac{dv}{dx} = 2x$

$$\begin{aligned} f'(x) &= \frac{(x^2 + 1)(0) - 4(2x)}{(x^2 + 1)^2} \\ &= \frac{-8x}{(x^2 + 1)^2} \end{aligned}$$

I. Cari (i) kecerunan, (ii) persamaan, bagi tangen dan normal kepada lengkung pada titik yang diberikan. Find (i) the gradients, (ii) the equations, of the tangent and the normal to the given curve at the given point.

27/28/29

Cecerunan tangen (m_1) dan normal (m_2) Gradients of tangent (m_1) and normal (m_2)	Persamaan tangen Equation of the tangent	Persamaan normal Equation of the normal
<p>CONTOH</p> <p>$y = 3x^2 - 4x + 1; (1, 0)$</p> <p>$\frac{dy}{dx} = 6x - 4$</p> <p>Pada $x = 1$, $\frac{dy}{dx} = 6(1) - 4 = 2$</p> <p>Maka, $m_1 = 2$ dan $m_2 = -\frac{1}{2}$.</p>	<p>$y - 0 = 2(x - 1)$</p> <p>$y = 2x - 2$</p>	<p>$y - 0 = \frac{1}{2}(x - 1)$</p> <p>$y = \frac{1}{2}x + \frac{1}{2}$</p>
<p>$y = (4x - x^2)^2; (3, 9)$</p> <p>$\frac{dy}{dx} = 2(4x - x^2)(4 - 2x)$</p> <p>Pada $x = 3$, $\frac{dy}{dx} = 2[4(3) - 3^2][4 - 2(3)] = -12$</p> <p>Maka, $m_1 = -12$ dan $m_2 = \frac{1}{12}$.</p>	<p>$y - 9 = -12(x - 3)$</p> <p>$y - 9 = -12x + 36$</p> <p>$y = -12x + 45$</p>	<p>$y - 9 = \frac{1}{12}(x - 3)$</p> <p>$y - 9 = \frac{1}{12}x - \frac{1}{4}$</p> <p>$y = \frac{1}{12}x + \frac{35}{4}$</p>
<p>$y = \frac{3}{(x^2 + x)^2}; (1, \frac{3}{4})$</p> <p>$\frac{dy}{dx} = -6(x^2 + x)^{-3}(2x + 1) = \frac{-6(2x + 1)}{(x^2 + x)^3}$</p> <p>Pada $x = 1$, $\frac{dy}{dx} = \frac{-6(2(1) + 1)}{(1^2 + 1)^3} = -\frac{9}{4}$</p> <p>Maka, $m_1 = -\frac{9}{4}$ dan $m_2 = \frac{4}{9}$.</p>	<p>$y - \frac{3}{4} = -\frac{9}{4}(x - 1)$</p> <p>$y - \frac{3}{4} = -\frac{9}{4}x + \frac{9}{4}$</p> <p>$y = -\frac{9}{4}x + 3$</p>	<p>$y - \frac{3}{4} = \frac{4}{9}(x - 1)$</p> <p>$y - \frac{3}{4} = \frac{4}{9}x - \frac{4}{9}$</p> <p>$y = \frac{4}{9}x + \frac{11}{36}$</p>
<p>$y = (x^2 + 1)(3x - 4); (2, 10)$</p> <p>$\frac{dy}{dx} = (x^2 + 1)(3) + (3x - 4)(2x)$</p> <p>Pada $x = 2$, $\frac{dy}{dx} = (2^2 + 1)(3) + [3(2) - 4][2(2)] = 23$</p> <p>Maka, $m_1 = 23$ dan $m_2 = -\frac{1}{23}$.</p>	<p>$y - 10 = 23(x - 2)$</p> <p>$y - 10 = 23x - 46$</p> <p>$y = 23x - 36$</p>	<p>$y - 10 = -\frac{1}{23}(x - 2)$</p> <p>$y - 10 = -\frac{1}{23}x + \frac{2}{23}$</p> <p>$y = -\frac{1}{23}x + \frac{232}{23}$</p>
<p>$y = \frac{x}{x^2 + 1}; (3, \frac{3}{10})$</p> <p>$\frac{dy}{dx} = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2}$</p> <p>Pada $x = 3$, $\frac{dy}{dx} = \frac{(3^2 + 1) - 3(2(3))}{(3^2 + 1)^2} = -\frac{8}{100} = -\frac{2}{25}$</p> <p>Maka, $m_1 = -\frac{2}{25}$ dan $m_2 = \frac{25}{2}$.</p>	<p>$y - \frac{3}{10} = -\frac{2}{25}(x - 3)$</p> <p>$y - \frac{3}{10} = -\frac{2}{25}x + \frac{6}{25}$</p> <p>$y = -\frac{2}{25}x + \frac{27}{50}$</p>	<p>$y - \frac{3}{10} = \frac{25}{2}(x - 3)$</p> <p>$y - \frac{3}{10} = \frac{25}{2}x - \frac{75}{2}$</p> <p>$y = \frac{25}{2}x + \frac{186}{5}$</p>

J. Selesaikan masalah yang berikut.
Solve the following problems.

27/28/29

1. Diberi kecerunan tangen kepada lengkung $y = 3x^2 + 6x + 4$ ialah 12 apabila $x = p$, dengan keadaan p ialah pemalar. Cari nilai p .
Given the gradient of the tangent to the curve $y = 3x^2 + 6x + 4$ is 12 when $x = p$, where p is a constant. Find the value of p .

$$y = 3x^2 + 6x + 4$$

$$\frac{dy}{dx} = 6x + 6$$

Kecerunan tangen ialah 12 apabila $x = p$.

$$6p + 6 = 12$$

$$6p = 6$$

$$p = 1$$

2. Diberi $y = mx + n$ ialah persamaan tangen kepada lengkung $y = 3x^2 - 5x + 2$ pada titik (2, 4). Cari nilai m dan nilai n .
Given $y = mx + n$ is the equation of the tangent to the curve $y = 3x^2 - 5x + 2$ at the point (2, 4). Find the values of m and n .

$$y = 3x^2 - 5x + 2$$

$$\frac{dy}{dx} = 6x - 5$$

Apabila $x = 2$, $\frac{dy}{dx} = 6(2) - 5 = 7$

Persamaan tangen ialah

$$y - 4 = 7(x - 2)$$

$$y - 4 = 7x - 14$$

$$y = 7x - 10$$

Maka, $m = 7$ dan $n = -10$.

3. Diberi lengkung $y = (x^2 - 4)^2$ dan $\frac{dy}{dx} = mx(x^2 - 4)$, dengan keadaan m ialah pemalar. Cari

Given a curve $y = (x^2 - 4)^2$ and $\frac{dy}{dx} = mx(x^2 - 4)$, where m is a constant. Find

(a) nilai m . (b) persamaan tangen kepada lengkung itu pada titik yang koordinat- x ialah 1. the equation of the tangent to the curve at the point where the x -coordinate is 1.

(a) $y = (x^2 - 4)^2$

$$\frac{dy}{dx} = 2(x^2 - 4)(2x)$$

$$= 4x(x^2 - 4)$$

Bandingkan $mx(x^2 - 4)$ dengan $4x(x^2 - 4)$.

Maka, $m = 4$

(b) Apabila $x = 1$,

$$\frac{dy}{dx} = 4(1)(1^2 - 4) = -12$$

dan $y = (1^2 - 4)^2 = 9$

Persamaan tangen ialah

$$y - 9 = -12(x - 1)$$

$$y - 9 = -12x + 12$$

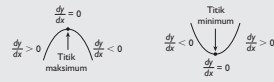
$$y = -12x + 21$$

9.3 Nilai Maksimum dan Nilai Minimum

30/31/32/33/34/35

FAKTA UTAMA

- Nilai maksimum atau nilai minimum diperolehi apabila $\frac{dy}{dx} = 0$.
The maximum or the minimum value is obtained when $\frac{dy}{dx} = 0$.
- Apabila nilai $\frac{dy}{dx}$ berubah daripada nilai positif kepada sifar dan kemudian kepada nilai negatif, nilai maksimum diperolehi.
When the value of $\frac{dy}{dx}$ changes from positive to zero and then to negative, a maximum value is obtained.
- Apabila nilai $\frac{dy}{dx}$ berubah daripada nilai negatif kepada sifar dan kemudian kepada nilai positif, nilai minimum diperolehi.
When the value of $\frac{dy}{dx}$ changes from negative to zero and then to positive, a minimum value is obtained.
- Titik maksimum atau titik minimum juga disebut titik pusingan. A maximum or a minimum point is also called a turning point.



129

Cari titik pusingan bagi setiap lengkung yang berikut. Seterusnya, tentukan sama ada setiap titik pusingan itu ialah titik maksimum atau titik minimum.
Find the turning point(s) of each of the following curves. Hence, determine whether each turning point is a maximum or a minimum point.

31/32

CONTOH

$y = \frac{9}{x} + x$

$$\frac{dy}{dx} = -\frac{9}{x^2} + 1$$

Pada titik pusingan, $\frac{dy}{dx} = 0$.

Maka, $-\frac{9}{x^2} + 1 = 0 \Rightarrow \frac{9}{x^2} = 1$

$$x^2 = 9$$

$$x = 3 \text{ atau } -3$$

Apabila $x = 3$,

$$y = \frac{9}{3} + 3 = 6$$

Apabila $x = -3$,

$$y = \frac{9}{-3} + (-3) = -6$$

Maka, titik pusingan ialah (3, 6) dan (-3, -6).

Nilai x	Pada titik (3, 6)			Pada titik (-3, -6)		
	$x = 2$	$x = 3$	$x = 4$	$x = -4$	$x = -3$	$x = -1$
Nilai $\frac{dy}{dx}$	$-\frac{5}{4}$	0	$\frac{7}{16}$	$\frac{7}{16}$	0	-8
Lakaran tangen	\swarrow	—	\nearrow	\swarrow	—	\swarrow

Maka, (3, 6) ialah titik minimum. Maka, (-3, -6) ialah titik maksimum.

1. $y = 2x^2 + 8x + 3$

$$\frac{dy}{dx} = 4x + 8$$

Pada titik pusingan, $\frac{dy}{dx} = 0$.

$$4x + 8 = 0$$

$$x = -2$$

Apabila $x = -2$,

$$y = 2(-2)^2 + 8(-2) + 3 = -5$$

Maka, titik pusingan ialah (-2, -5).

Nilai x	-3	-2	-1
Nilai $\frac{dy}{dx}$	-4	0	4
Lakaran tangen	\swarrow	—	\nearrow

Maka, (-2, -5) ialah titik minimum.

2. $y = 8x - x^2$

$$\frac{dy}{dx} = 8 - 2x$$

Pada titik pusingan, $\frac{dy}{dx} = 0$.

$$8 - 2x = 0$$

$$x = 4$$

Apabila $x = 4$,

$$y = 8(4) - 4^2 = 16$$

Maka, titik pusingan ialah (4, 16).

Nilai x	3	4	5
Nilai $\frac{dy}{dx}$	2	0	-2
Lakaran tangen	\swarrow	—	\swarrow

Maka, (4, 16) ialah titik maksimum.

3. $y = \frac{1}{3}x^3 - x^2 - 3x$

$$\frac{dy}{dx} = x^2 - 2x - 3$$

Pada titik pusingan, $\frac{dy}{dx} = 0$.

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \text{ atau } 3$$

Apabila $x = -1$,

$$y = \frac{1}{3}(-1)^3 - (-1)^2 - 3(-1) = \frac{5}{3}$$

Apabila $x = 3$, $y = \frac{1}{3}(3)^3 - 3^2 - 3(3) = -9$

Maka, titik pusingan ialah $(-1, \frac{5}{3})$ dan (3, -9).

Nilai x	-2	-1	0	2	3	4
Nilai $\frac{dy}{dx}$	5	0	-3	0	5	
Lakaran tangen	\swarrow	—	\swarrow	—	\swarrow	\swarrow

Maka, $(-1, \frac{5}{3})$ ialah titik maksimum dan (3, -9) ialah titik minimum.

130

9.4 Kadar Perubahan

36/37/38/39

Selesaikan masalah yang berikut.
Solve the following problems.

41

CONTOH

Jejari satu bulatan berkurang pada kadar 0.2 cm s^{-1} . Cari kadar perubahan luas bulatan itu apabila jejari ialah 3 cm.
The radius of a circle decreases at a rate of 0.2 cm s^{-1} . Find the rate of change in the area of the circle when the radius is 3 cm.

$$\frac{dr}{dt} = -0.2$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

Apabila $r = 3$ dan $\frac{dr}{dt} = -0.2$,

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

Petua rantai

$$= 2\pi r \times (-0.2)$$

$$= 2\pi(3) \times (-0.2)$$

$$= -1.2\pi \text{ cm}^2 \text{ s}^{-1}$$

1. Isi padu sebuah silinder bertambah pada kadar $324 \text{ cm}^3 \text{ s}^{-1}$. Tinggi, h cm, silinder itu adalah dua kali jejarinya, r cm. Cari kadar perubahan jejari apabila jejarinya ialah 6 cm.
The volume of a cylinder increases at a rate of $324 \text{ cm}^3 \text{ s}^{-1}$. The height, h cm, of the cylinder is twice the radius, r cm. Find the rate of change in the radius when the radius is 6 cm.

$$\frac{dV}{dt} = 324 \text{ cm}^3 \text{ s}^{-1}$$

$$V = \pi r^2 h = \pi r^2(2r) = 2\pi r^3$$

$$\frac{dV}{dr} = 6\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

Apabila $r = 6$,

$$324 = 6\pi(6)^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{324}{216\pi}$$

$$= \frac{3}{2\pi} \text{ cm s}^{-1}$$

2. Jejari satu bulatan bertambah pada kadar 0.5 cm s^{-1} . Cari kadar perubahan luas bulatan itu ketika luasnya ialah $25\pi \text{ cm}^2$.
The radius of a circle increases at a rate of 0.5 cm s^{-1} . Find the rate of change in the area of the circle when the area is $25\pi \text{ cm}^2$.

$$\frac{dA}{dt} = \frac{dA}{dj} \times \frac{dj}{dt}$$

$$A = \pi j^2$$

$$25\pi = \pi j^2$$

$$j^2 = 25$$

$$j = 5$$

$$\frac{dA}{dj} = 2\pi j$$

Apabila $j = 5$, $\frac{dA}{dj} = 2\pi(5) = 10\pi$

Diberi $\frac{dj}{dt} = 0.5$

Maka, $\frac{dA}{dt} = 10\pi \times 0.5 = 5\pi \text{ cm}^2 \text{ s}^{-1}$

3. Ukuran sebuah kuboid ialah $y \text{ cm} \times y \text{ cm} \times 2y \text{ cm}$. Isi padu kuboid itu bertambah pada kadar $3.6 \text{ cm}^3 \text{ s}^{-1}$ ketika isi padunya ialah 432 cm^3 . Cari kadar perubahan y pada ketika itu.
The measurements of a cuboid is $y \text{ cm} \times y \text{ cm} \times 2y \text{ cm}$. The volume of the cuboid increases at a rate of $3.6 \text{ cm}^3 \text{ s}^{-1}$ when the volume is 432 cm^3 . Find the rate of change in y at that instant.

Isi padu kuboid, $V = y \times y \times 2y$

$$= 2y^3$$

Maka, $2y^3 = 432$

$$y^3 = 216$$

$$y = 6$$

$$\frac{dV}{dy} = 6y^2$$

$$\frac{dV}{dt} = \frac{dV}{dy} \times \frac{dy}{dt}$$

$$3.6 = 6y^2 \times \frac{dy}{dt}$$

Apabila $y = 6$,

$$\frac{dy}{dt} = \frac{3.6}{6(6^2)}$$

$$= \frac{1}{60} \text{ s}^{-1}$$

131

9.5 Perubahan Kecil dan Penghampiran

Selesaikan masalah yang berikut.
Solve the following problems.

51/52

CONTOH

Diberi $y = x^2 + 3x - 8$, cari perubahan kecil dalam y apabila x bertambah daripada 2 kepada 2.01. Seterusnya, cari nilai hampir bagi y selepas perubahan itu berlaku pada x .
Given $y = x^2 + 3x - 8$, find the approximate change in y when x increases from 2 to 2.01. Hence, find the approximate value of y after the change has occurred in x .

$$y = x^2 + 3x - 8$$

$$\frac{dy}{dx} = 2x + 3$$

$$\delta x = 2.01 - 2 = 0.01$$

Apabila $x = 2$, $\delta y = \frac{dy}{dx} \times \delta x$

$$= (2x + 3)(0.01)$$

$$= (2 \times 2 + 3)(0.01)$$

$$= 0.07$$

Nilai hampir bagi $y = y + \delta y$

$$= 2^2 + 3(2) - 8 + 0.07$$

$$= 2.07$$

1. Diberi $y = 4x^2 + 5x - 4$, cari perubahan kecil dalam y apabila x bertambah daripada 3 kepada 3.1. Seterusnya, cari nilai hampir bagi y selepas perubahan itu berlaku pada x .
Given $y = 4x^2 + 5x - 4$, find the approximate change in y when x increases from 3 to 3.1. Hence, find the approximate value of y after the change has occurred in x .

$$y = 4x^2 + 5x - 4$$

$$\frac{dy}{dx} = 8x + 5$$

$$\delta x = 3.1 - 3 = 0.1$$

Apabila $x = 3$, $\delta y = \frac{dy}{dx} \times \delta x$

$$= (8x + 5)(0.1)$$

$$= (8 \times 3 + 5)(0.1)$$

$$= 2.9$$

Nilai hampir bagi $y = y + \delta y$

$$= 4(3)^2 + 5(3) - 4 + 2.9$$

$$= 49.9$$

2. Diberi $y = x^3$, dengan menggunakan pembezaan, cari nilai hampir bagi 2.1^3 .
Given $y = x^3$, by using differentiation, find the approximate value of 2.1^3 .

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

Andaikan x berubah daripada 2 kepada 2.1.

$$\delta x = 2.1 - 2 = 0.1$$

Apabila $x = 2$, $\delta y = \frac{dy}{dx} \times \delta x$

$$= 3x^2 \times \delta x$$

$$= 3(2)^2 \times 0.1$$

$$= 1.2$$

Maka, $2.1^3 = y + \delta y$

$$= 2^3 + 1.2$$

$$= 9.2$$

3. Diberi $y = \frac{5}{x^2}$, cari nilai $\frac{dy}{dx}$ apabila $x = 4$. Seterusnya, cari nilai hampir bagi $\frac{5}{(4.01)^2}$.

Given $y = \frac{5}{x^2}$, find the value of $\frac{dy}{dx}$ when $x = 4$. Hence, find the approximate value of $\frac{5}{(4.01)^2}$.

$$y = \frac{5}{x^2} = 5x^{-2}$$

$$\frac{dy}{dx} = -10x^{-3} = -\frac{10}{x^3}$$

Apabila $x = 4$, $\frac{dy}{dx} = -\frac{10}{4^3} = -0.1563$

Andaikan x berubah daripada 4 kepada 4.01.

$$\delta x = 4.01 - 4 = 0.01$$

$\delta y = \frac{dy}{dx} \times \delta x$

$$= -0.1563 \times 0.01$$

$$= -0.001563$$

Nilai hampir bagi $\frac{5}{(4.01)^2} = y + \delta y$

$$= \frac{5}{4^2} - 0.001563$$

$$= 0.3109$$

132

