JAWAPAN

BAB 1: JANJANG

1.1

A

1. $d_1 = T_2 - T_1$
$= (-2) - (-5)$
$= 3$

$d_2 = T_3 - T_2$
$= 1 - (-2)$
$= 3$

Jujukan ini ialah J.A. sebab beza antara sebarang dua sebutan berturutan adalah sama, iaitu 3.

2. $d_1 = T_2 - T_1$
$= (a - b) - (a + b)$
$= -2b$

$d_2 = T_3 - T_2$
$= (a + 2b) - (a - b)$
$= 3b$

Jujukan ini bukan J.A. sebab beza antara sebarang dua sebutan berturutan adalah tidak sama.

3. $d_1 = T_2 - T_1$
$= \dfrac{3}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}$
$= \dfrac{2}{\sqrt{2}}$

$d_2 = T_3 - T_2$
$= \dfrac{5}{\sqrt{2}} - \dfrac{3}{\sqrt{2}}$
$= \dfrac{2}{\sqrt{2}}$

Jujukan ini ialah J.A. sebab beza antara sebarang dua sebutan berturutan adalah sama, iaitu $\dfrac{2}{\sqrt{2}}$.

4. $d_1 = T_2 - T_1$
$= \log_{10} (mn^2) - \log_{10} (mn)$
$= (\log_{10} m + 2 \log_{10} n) - (\log_{10} m + \log_{10} n)$
$= \log_{10} n$

$d_2 = T_3 - T_2$
$= \log_{10} (mn^3) - \log_{10} (mn^2)$
$= (\log_{10} m + 3 \log_{10} n) - (\log_{10} m + 2 \log_{10} n)$
$= \log_{10} n$

Jujukan ini ialah J.A. sebab beza antara sebarang dua sebutan berturutan adalah sama, iaitu $\log_{10} n$.

B

1. (i) $T_1 =$ Perimeter segi empat tepat pertama
$= 8 + 2x$

$T_2 =$ Perimeter segi empat tepat kedua
$= 8 + 2(x + 3) = 14 + 2x$

$T_3 =$ Perimeter segi empat tepat ketiga
$= 8 + 2(x + 6) = 20 + 2x$

Perimeter: $8 + 2x$, $14 + 2x$, $20 + 2x$, …

$T_2 - T_1 = (14 + 2x) - (8 + 2x) = 6$
$T_3 - T_2 = (20 + 2x) - (14 + 2x) = 6$

Oleh sebab $T_2 - T_1 = T_3 - T_2 = 6$, maka perimeter segi empat tepat di atas membentuk suatu janjang aritmetik.

(ii) $T_1 =$ Luas segi empat tepat pertama $= 4x$
$T_2 =$ Luas segi empat tepat kedua
$= 4(x + 3) = 4x + 12$
$T_3 =$ Luas segi empat tepat ketiga
$= 4(x + 6) = 4x + 24$

Luas: $4x$, $4x + 12$, $4x + 24$, …

$T_2 - T_1 = (4x + 12) - 4x = 12$
$T_3 - T_2 = (4x + 24) - (4x + 12) = 12$

Oleh sebab $T_2 - T_1 = T_3 - T_2 = 12$, maka luas segi empat tepat di atas membentuk suatu janjang aritmetik.

2. (i) $T_1 =$ Perimeter segi empat sama pertama
$= 4x$
$T_2 =$ Perimeter segi empat sama kedua
$= 4(x + 2) = 4x + 8$
$T_3 =$ Perimeter segi empat sama ketiga
$= 4(x + 4) = 4x + 16$

Perimeter: $4x$, $4x + 8$, $4x + 16$, …

$T_2 - T_1 = (4x + 8) - 4x = 8$
$T_3 - T_2 = (4x + 16) - (4x + 8) = 8$

Oleh sebab $T_2 - T_1 = T_3 - T_2 = 8$, maka perimeter segi empat sama di atas membentuk suatu janjang aritmetik.

(ii) $T_1 =$ Luas segi empat sama pertama $= x^2$
$T_2 =$ Luas segi empat sama kedua
$= (x + 2)^2 = x^2 + 4x + 4$
$T_3 =$ Luas segi empat sama ketiga
$= (x + 4)^2 = x^2 + 8x + 16$

Luas: x^2, $x^2 + 4x + 4$, $x^2 + 8x + 16$, …

$T_2 - T_1 = (x^2 + 4x + 4) - x^2 = 4x + 4$
$T_3 - T_2 = (x^2 + 8x + 16) - (x^2 + 4x + 4)$
$= 4x + 12$

Oleh sebab $T_2 - T_1 \neq T_3 - T_2$, maka luas segi empat sama di atas tidak membentuk suatu janjang aritmetik.

C

1. $T_n = 2n^2 - 3$

$T_{n - 1} = 2(n - 1)^2 - 3$
$= 2n^2 - 4n + 2 - 3$
$= 2n^2 - 4n - 1$

$$T_n - T_{n-1} = (2n^2 - 3) - (2n^2 - 4n - 1)$$
$$= 4n - 2$$

Oleh sebab $T_n - T_{n-1}$ bukan satu pemalar, maka jujukan itu bukan janjang aritmetik.

2. (a) $T_{n-1} = 2n + 7$
$$= 2n - 2 + 2 + 7$$
$$= 2(n - 1) + 2 + 7$$
$$T_{n-1} = 2(n - 1) + 9$$
$$T_n = 2n + 9$$

(b) $T_n - T_{n-1} = (2n + 9) - (2n + 7)$
$$= 2$$

Oleh sebab $T_n - T_{n-1}$ ialah satu pemalar, maka jujukan itu ialah janjang aritmetik.

D

1. $a = 18$

$d = 15 - 18$
$= -3$

$T_{15} = 18 + (15 - 1)(-3)$
$= -24$

2. $a = 4mn$

$d = 4m(n - 3) - 4mn$
$= 4mn - 12m - 4mn$
$= -12m$

$T_{10} = 4mn + (10 - 1)(-12m)$
$= 4mn - 108m$

E

1. $a = 4\frac{9}{10}$ $d = 5\frac{2}{5} - 4\frac{9}{10} = \frac{1}{2}$

$$T_n = 8\frac{2}{5}$$

$$\frac{49}{10} + (n - 1)\left(\frac{1}{2}\right) = \frac{42}{5}$$

$$\frac{49}{10} + \frac{1}{2}n - \frac{1}{2} = \frac{42}{5}$$

$$\frac{1}{2}n = 4$$

$$n = 8$$

2. $a = \log_2 y^3$

$d = \log_2 y^5 - \log_2 y^3$
$= 5\log_2 y - 3\log_2 y$
$= 2\log_2 y$

$$T_n = \log_2 y^{21}$$
$$\log_2 y^3 + (n - 1)(2\log_2 y) = \log_2 y^{21}$$
$$3\log_2 y + 2n\log_2 y - 2\log_2 y = 21\log_2 y$$
$$\log_2 y + 2n\log_2 y = 21\log_2 y$$
$$2n\log_2 y = 20\log_2 y$$
$$2n = 20$$
$$n = 10$$

F

1. $12 - (2x^2 + 4x) = 4x - 12$
$$24 = 4x + 2x^2 + 4x$$
$$2x^2 + 8x - 24 = 0$$
$$x^2 + 4x - 12 = 0$$
$$(x + 6)(x - 2) = 0$$
$$x = -6 \text{ atau } x = 2$$

2. $\log_2 (x + 6) - \log_2 x = \log_2 (10x + 12) - \log_2 (x + 6)$

$$\log_2 \frac{x + 6}{x} = \log_2 \frac{10x + 12}{x + 6}$$

$$\frac{x + 6}{x} = \frac{10x + 12}{x + 6}$$

$$(x + 6)(x + 6) = x(10x + 12)$$
$$x^2 + 12x + 36 = 10x^2 + 12x$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

Apabila $x = -2$, $\log_2 (-2)$ tidak tertakrif.
Maka, $x = 2$.

G

1. $a = 2$

$d = \frac{8}{3} - 2 = \frac{2}{3}$

$$S_{10} = \frac{10}{2}\left[2(2) + (10 - 1)\left(\frac{2}{3}\right)\right]$$
$$= 5(4 + 6)$$
$$= 50$$

2. $a = 2h + 8k$

$d = 3h + 2k - (2h + 8k) = h - 6k$

$$S_{12} = \frac{12}{2}[2(2h + 8k) + (12 - 1)(h - 6k)]$$
$$= 6(4h + 16k + 11h - 66k)$$
$$= 6(15h - 50k)$$
$$= 90h - 300k$$

H

1. (a) $T_7 = S_7 - S_6$

$$= \frac{7}{2}[3 + 3(7)] - \frac{6}{2}[3 + 3(6)]$$
$$= 21$$

(b) Hasil tambah dari T_2 hingga T_8
$$= S_8 - S_1$$
$$= \frac{8}{2}[3 + 3(8)] - \frac{1}{2}[3 + 3(1)] = 105$$

2. (a) $a = T_1 = S_1$
$$= (1^2) + 4(1)$$
$$= 5$$

$T_2 = S_2 - S_1$ $d = T_2 - T_1$
$= [(2^2) + 4(2)] - 5$ $= 7 - 5$
$= 7$ $= 2$

(b) $T_9 = S_9 - S_8$
$ = [(9^2) + 4(9)] - [(8^2) + 4(8)]$
$ = 21$

3. $S_{n-1} = 2(n-1)^2 - 3(n-1)$
$\phantom{3. S_{n-1}} = 2(n^2 - 2n + 1) - 3n + 3$
$\phantom{3. S_{n-1}} = 2n^2 - 4n + 2 - 3n + 3$
$\phantom{3. S_{n-1}} = 2n^2 - 7n + 5$

(a) $T_n = S_n - S_{n-1}$
$ = (2n^2 - 3n) - (2n^2 - 7n + 5)$
$ = 2n^2 - 3n - 2n^2 + 7n - 5$
$ = 4n - 5$

(b) $T_n = 4n - 5$

$T_{n-1} = 4(n-1) - 5$
$\phantom{T_{n-1}} = 4n - 4 - 5$
$\phantom{T_{n-1}} = 4n - 9$

$T_n - T_{n-1} = 4n - 5 - (4n - 9)$
$\phantom{T_n - T_{n-1}} = 4n - 5 - 4n + 9$
$\phantom{T_n - T_{n-1}} = 4 \leftarrow$ Pemalar

Oleh sebab $T_n - T_{n-1}$ ialah satu pemalar, maka jujukan itu ialah janjang aritmetik.

I

1. $a = 7.3$

$d = 6.5 - 7.3 = -0.8$

$S_n = 37$

$\dfrac{n}{2}[2(7.3) + (n-1)(-0.8)] = 37$

$\dfrac{n}{2}(15.4 - 0.8n) = 37$

$7.7n - 0.4n^2 = 37$

$0.4n^2 - 7.7n + 37 = 0$

$4n^2 - 77n + 370 = 0$

$(4n - 37)(n - 10) = 0$

Oleh sebab n mesti integer positif, maka $n = 10$.

2. $a = \dfrac{11}{12}$ $\qquad d = \dfrac{5}{4} - \dfrac{11}{12} = \dfrac{1}{3}$

$S_n = 16\dfrac{2}{3}$

$\dfrac{n}{2}\left[2\left(\dfrac{11}{12}\right) + (n-1)\left(\dfrac{1}{3}\right)\right] = 16\dfrac{2}{3}$

$\dfrac{n}{2}\left(\dfrac{3}{2} + \dfrac{1}{3}n\right) = \dfrac{50}{3}$

$\dfrac{3}{4}n + \dfrac{1}{6}n^2 = \dfrac{50}{3}$

$9n + 2n^2 = 200$

$2n^2 + 9n - 200 = 0$

$(2n + 25)(n - 8) = 0$

Oleh sebab n mesti integer positif, maka $n = 8$.

J

1. $T_6 - T_2 = 12$
$(a + 5d) - (a + d) = 12$
$ 4d = 12$
$ d = 3$

$T_{10} = 39$
$a + 9d = 39$
$a + 9(3) = 39$
$ a = 12$

$T_n = 132$
$12 + (n-1)(3) = 132$
$12 + 3n - 3 = 132$
$ n = 41$

Maka, sebutan ke-41 bernilai 132.

2. (a) $\qquad T_7 = 82$
$ a + 6d = 82 \qquad \ldots\ldots \text{①}$
$ T_{15} = -46$
$ a + 14d = -46 \qquad \ldots\ldots \text{②}$
$ \text{②} - \text{①}: \ 8d = -128$
$ d = -16$

Dari ①, $a + 6(-16) = 82$
$ a = 178$

(b) $\qquad T_n < 0$
$178 + (n-1)(-16) < 0$
$178 - 16n + 16 < 0$
$ -16n < -194$
$ n > 12.125$

Maka, $n = 13$.

$T_{13} = 178 + 12(-16)$
$\phantom{T_{13}} = -14$

3. J.A.: $x, 5, y, \ldots$
$5 - x = y - 5$
$ y = 10 - x \quad \ldots\ldots \text{①}$
$x^2 - y^2 = 60 \qquad \ldots\ldots \text{②}$
Gantikan ① ke dalam ②.
$ x^2 - (10 - x)^2 = 60$
$x^2 - 100 + 20x - x^2 - 60 = 0$
$ 20x - 160 = 0$
$ 20x = 160$
$ x = 8$
Dari ①, $y = 10 - 8$
$ = 2$

4. J.A.: $h, k, m, n, \ldots$
$k - h = n - m$
$k + m = h + n$
Diberi persamaan kuadratik $3x^2 - 5x - 2 = 0$.
Hasil tambah punca $= \dfrac{5}{3}$

$h + n = \dfrac{5}{3}$

Maka, $k + m = \dfrac{5}{3}$.

5. (a)
$$S_7 = 105$$
$$\frac{7}{2}[2a + 6d] = 105$$
$$2a + 6d = 30 \qquad \ldots\ldots \text{①}$$
$$S_{12} = 300$$
$$\frac{12}{2}[2a + 11d] = 300$$
$$2a + 11d = 50 \qquad \ldots\ldots \text{②}$$
$$\text{② − ①:} \quad 5d = 20$$
$$d = 4$$

Dari ①, $2a + 6(4) = 30$
$$a = 3$$

(b) Hasil tambah dari T_3 hingga T_9
$$= S_9 - S_2$$
$$= \frac{9}{2}[2(3) + 8(4)] - \frac{2}{2}[2(3) + 1(4)]$$
$$= \frac{9}{2}(38) - 10$$
$$= 161$$

6. (a) $d = -6 - (-10) = 4$
$$T_n = 66$$
$$-10 + (n - 1)(4) = 66$$
$$-10 + 4n - 4 = 66$$
$$4n = 80$$
$$n = 20$$
$$S_{20} = \frac{20}{2}[2(-10) + 19(4)]$$
$$= 560$$

(b)
$$S_n = 560$$
$$\frac{n}{2}[2(-10) + (n - 1)(4)] = 560$$
$$\frac{n}{2}(-20 + 4n - 4) = 560$$
$$2n^2 - 12n = 560$$
$$n^2 - 6n - 280 = 0$$
$$(n - 20)(n + 14) = 0$$
$$n = 20 \quad \text{atau} \quad n = -14 \text{ (diabaikan)}$$

7. $a = 1$
$$d = 5 - 3$$
$$= 2$$

Hasil tambah dari T_n hingga T_{2n}
$$= S_{2n} - S_{n-1}$$
$$= \frac{2n}{2}[2(1) + (2n - 1)(2)] -$$
$$\quad \frac{n - 1}{2}\{2(1) + [(n - 1) - 1](2)\}$$
$$= n(2 + 4n - 2) - \frac{n - 1}{2}(2 + 2n - 4)$$
$$= 4n^2 - \frac{n - 1}{2}(2n - 2)$$
$$= 4n^2 - (n - 1)^2$$
$$= 4n^2 - (n^2 - 2n + 1)$$
$$= 4n^2 - n^2 + 2n - 1$$
$$= 3n^2 + 2n - 1$$

K

1. (a) Bilangan bola mengikut baris: 2, 3, 4, …
J.A.: $a = 2$, $d = 1$
$$S_n = 170$$
$$\frac{n}{2}[2(2) + (n - 1)(1)] = 170$$
$$n(3 + n) = 340$$
$$n^2 + 3n - 340 = 0$$
$$(n + 20)(n - 17) = 0$$
$$n = -20 \text{ atau } n = 17$$

Maka, bilangan baris dalam susunan itu ialah 17.

(b) Baris di tengah = Baris ke-9
$$T_9 = 2 + 8(1) = 10$$

Bilangan bola plastik di baris tengah ialah 10 biji.

2. (a) Jumlah panjang sisi rangka kubus:
$12(x + 2)$, $12(x + 4)$, …, $12(x + 22)$
$12x + 24$, $12x + 48$, …, $12x + 264$

J.A.: $a = 12x + 24$, $d = 24$
$$T_8 = 228$$
$$12x + 24 + 7(24) = 228$$
$$12x + 192 = 228$$
$$12x = 36$$
$$x = 3$$

(b) Apabila $x = 3$, jumlah panjang sisi rangka kubus: 60, 84, …, 300

J.A.: $a = 60$, $d = 24$
$$T_n = 300$$
$$60 + (n - 1)(24) = 300$$
$$24n = 264$$
$$n = 11$$
$$S_{11} = \frac{11}{2}(60 + 300)$$
$$= 1\,980 \text{ cm}$$

$$L = 1\,980 \text{ cm} + 20 \text{ cm}$$
$$= 2\,000 \text{ cm}$$
$$= 20 \text{ m}$$

1.2

A

1. $r_1 = \dfrac{T_2}{T_1}$ $\qquad\qquad$ $r_2 = \dfrac{T_3}{T_2}$

$\qquad = \dfrac{-\dfrac{1}{3}}{1}$ $\qquad\qquad = \dfrac{\dfrac{1}{9}}{-\dfrac{1}{3}}$

$\qquad = -\dfrac{1}{3}$ $\qquad\qquad\quad = -\dfrac{1}{3}$

Jujukan ini ialah J.G. sebab nisbah antara sebarang dua sebutan berturutan adalah sama, iaitu $-\dfrac{1}{3}$.

2. $r_1 = \dfrac{T_2}{T_1}$

$= \dfrac{n}{\dfrac{n}{n-1}}$

$= n - 1$

$r_2 = \dfrac{T_3}{T_2}$

$= \dfrac{n^2 - n}{n}$

$= n - 1$

Jujukan ini ialah J.G. sebab nisbah antara sebarang dua sebutan berturutan adalah sama, iaitu $n - 1$.

3. $r_1 = \dfrac{T_2}{T_1}$

$= \dfrac{5^{2h-k}}{5^{h+k}}$

$= 5^{h-2k}$

$r_2 = \dfrac{T_3}{T_2}$

$= \dfrac{5^{3h+3k}}{5^{2h-k}}$

$= 5^{h+4k}$

Jujukan ini bukan J.G. sebab nisbah antara sebarang dua sebutan berturutan adalah tidak sama.

4. $r_1 = \dfrac{T_2}{T_1}$ $\qquad r_2 = \dfrac{T_3}{T_2}$

$= \dfrac{\log_6 y^n}{\log_6 y}$ $\qquad = \dfrac{\log_6 y^{2n}}{\log_6 y^n}$

$= n$ $\qquad\qquad = 2$

Jujukan ini bukan J.G. sebab nisbah antara sebarang dua sebutan berturutan adalah tidak sama.

B

1. $a = 10\dfrac{4}{5}$

$r = \dfrac{7\dfrac{1}{5}}{10\dfrac{4}{5}}$

$= \dfrac{2}{3}$

$T_8 = \left(\dfrac{54}{5}\right)\left(\dfrac{2}{3}\right)^7$

$= \dfrac{256}{405}$

2. $a = \log_{10} 81$ $\qquad\qquad T_6 = \log_{10} 81\left(\dfrac{1}{2}\right)^5$

$r = \dfrac{\log_{10} 9}{\log_{10} 81}$ $\qquad\qquad = \dfrac{1}{32}\log_{10} 3^4$

$= \dfrac{2\log_{10} 3}{4\log_{10} 3}$ $\qquad\qquad = \dfrac{1}{8}\log_{10} 3$

$= \dfrac{1}{2}$

C

1. $a = \dfrac{1}{4}$ $\qquad r = \dfrac{\dfrac{3}{8}}{\dfrac{1}{4}} = \dfrac{3}{2}$

$T_n = \dfrac{243}{128}$

$\left(\dfrac{1}{4}\right)\left(\dfrac{3}{2}\right)^{n-1} = \dfrac{243}{128}$

$\left(\dfrac{3}{2}\right)^{n-1} = \left(\dfrac{3}{2}\right)^5$

$n = 6$

2. $a = 3.2k$

$r = \dfrac{6.4k}{3.2k}$

$= 2$

$T_n = 819.2k$

$(3.2k)(2)^{n-1} = 819.2k$

$2^{n-1} = 256$

$2^{n-1} = 2^8$

$n = 9$

D

1. $\dfrac{3x-5}{20} = \dfrac{5}{3x-5}$

$(3x-5)^2 = 100$

$3x - 5 = \pm\sqrt{100}$

$3x - 5 = 10$ $\qquad$ atau $\qquad 3x - 5 = -10$

$x = 5$ $\qquad\qquad\qquad\qquad\qquad x = -\dfrac{5}{3}$

2. $\dfrac{x}{\log_{10} 2} = \dfrac{\log_{10} 16}{x}$

$x^2 = \log_{10} 2 \times \log_{10} 16$

$= \log_{10} 2 \times 4\log_{10} 2$

$= 4 \times (\log_{10} 2)^2$

$x = \sqrt{4 \times (\log_{10} 2)^2}$

$= 2\log_{10} 2$

$= \log_{10} 4$

E

1. $a = 2$ $\quad r = \dfrac{\frac{4}{3}}{2} = \dfrac{2}{3}$

$$S_6 = \dfrac{2\left[1 - \left(\frac{2}{3}\right)^6\right]}{1 - \frac{2}{3}}$$

$$= 5\frac{115}{243}$$

2. $a = 3$

$r = \dfrac{3.6}{3}$

$\quad = 1.2$

$$S_5 = \dfrac{3[(1.2)^5 - 1]}{1.2 - 1}$$

$$= 22.32$$

F

1. $T_1 + T_3 = \dfrac{13}{16}$

$\quad a + ar^2 = \dfrac{13}{16}$ ...... ①

$\quad T_2 + T_4 = \dfrac{39}{32}$

$\quad ar + ar^3 = \dfrac{39}{32}$ ...... ②

② ÷ ①:

$$\dfrac{ar + ar^3}{a + ar^2} = \dfrac{3}{2}$$

$$\dfrac{ar(1 + r^2)}{a(1 + r^2)} = \dfrac{3}{2}$$

$$r = \dfrac{3}{2}$$

Dari ①:

$$a + a\left(\dfrac{3}{2}\right)^2 = \dfrac{13}{16}$$

$$\dfrac{13}{4}a = \dfrac{13}{16}$$

$$a = \dfrac{1}{4}$$

2. $T_2 - T_1 = 4$

$\quad ar - a = 4$ ...... ①

$\quad T_3 - T_2 = 10$

$\quad ar^2 - ar = 10$ ...... ②

② ÷ ①: $\dfrac{ar^2 - ar}{ar - a} = \dfrac{10}{4}$

$$\dfrac{ar(r - 1)}{a(r - 1)} = \dfrac{10}{4}$$

$$r = \dfrac{5}{2}$$

Dari ①:

$$a\left(\dfrac{5}{2}\right) - a = 4$$

$$\dfrac{5}{2}a - a = 4$$

$$a = 2\dfrac{2}{3}$$

$$S_9 = \dfrac{\frac{8}{3}\left[\left(\frac{5}{2}\right)^9 - 1\right]}{\frac{5}{2} - 1}$$

$$= 6\,779.9$$

3. J.A.: 8, h, k, ...

$\quad h - 8 = k - h$

$\quad\quad k = 2h - 8$ ...... ①

J.G.: h, k, 36, ...

$\quad \dfrac{k}{h} = \dfrac{36}{k}$

$\quad k^2 = 36h$ ...... ②

Gantikan ① ke dalam ②.

$$(2h - 8)^2 = 36h$$
$$4h^2 - 32h + 64 = 36h$$
$$4h^2 - 68h + 64 = 0$$
$$h^2 - 17h + 16 = 0$$
$$(h - 1)(h - 16) = 0$$
$$h = 1 \text{ atau } h = 16$$

Apabila $h = 1$, dari ①,
$k = 2(1) - 8$
$\quad = -6$

Apabila $h = 16$, dari ①,
$k = 2(16) - 8$
$\quad = 24$

4. J.G.: $2 + 4 + 8 + \ldots$
$a = 2$
$r = 2$

$$T_n < 1\,000$$
$$2(2)^{n-1} < 1\,000$$
$$2^n < 1\,000$$
$$\log_{10} 2^n < \log_{10} 1\,000$$
$$n \log_{10} 2 < 3$$
$$n < \dfrac{3}{\log_{10} 2}$$
$$n < 9.965$$

Maka, $n = 9$.

$S_9 = \dfrac{2(2^9 - 1)}{2 - 1}$
$\quad = 2(2^9 - 1)$

Diberi $S_9 = \dfrac{m}{3}(2^n - 1)$.

Maka, $\dfrac{m}{3} = 2$ $\quad$ dan $\quad n = 9$
$\quad\quad m = 6$

G

1. $a = 0.6$ $r = \dfrac{-0.15}{0.6} = -\dfrac{1}{4}$

$$S_\infty = \dfrac{0.6}{1 - \left(-\dfrac{1}{4}\right)}$$

$$= \dfrac{12}{25}$$

2. $a = \dfrac{5}{2}$ $r = \dfrac{\dfrac{5}{3}}{\dfrac{5}{2}} = \dfrac{2}{3}$

$$S_\infty = \dfrac{\dfrac{5}{2}}{1 - \dfrac{2}{3}} = 7\dfrac{1}{2}$$

H

1. $a = 8$

$$T_3 = \dfrac{1}{2}$$

$$(8)r^2 = \dfrac{1}{2}$$

$$r = \dfrac{1}{4}$$

$$S_\infty = \dfrac{8}{1 - \dfrac{1}{4}}$$

$$= 10\dfrac{2}{3}$$

2. $$T_2 + T_3 = \dfrac{10}{9}a$$

$$ar + ar^2 = \dfrac{10}{9}a$$

$$r + r^2 = \dfrac{10}{9}$$

$$9r + 9r^2 - 10 = 0$$

$$(3r - 2)(3r + 5) = 0$$

$$r = \dfrac{2}{3} \text{ atau } r = -\dfrac{5}{3} \text{ (diabaikan)}$$

$$S_\infty = 168$$

$$\dfrac{a}{1 - \dfrac{2}{3}} = 168$$

$$a = 56$$

I

1. $0.060606 \ldots = 0.06 + 0.0006 + 0.000006 + \ldots$

$a = 0.06$ $r = \dfrac{0.0006}{0.06} = 0.01$

$$S_\infty = \dfrac{0.06}{1 - 0.01}$$

$$= \dfrac{2}{33}$$

2. $2.57777 \ldots = 2.5 + 0.07777 \ldots$
                 $= 2.5 + 0.07 + 0.007 + 0.0007 + \ldots$

$a = 0.07$ $r = \dfrac{0.007}{0.07} = 0.1$

$$S_\infty = \dfrac{0.07}{1 - 0.1}$$

$$= \dfrac{7}{90}$$

Maka, $2.57777 \ldots = 2.5 + \dfrac{7}{90} = 2\dfrac{26}{45}$

J

(a) $2.141414 \ldots$
    $= 2 + 0.141414 \ldots$
    $= 2 + 0.14 + 0.0014 + 0.000014 + \ldots$
    Maka, $h = 2$ dan $k = 0.000014$.

(b) $2.141414 \ldots$
    $= 2 + 0.141414 \ldots$
    $= 2 + 0.14 + 0.0014 + 0.000014 + \ldots$

$a = 0.14$ $r = \dfrac{0.0014}{0.14} = 0.01$

$$S_\infty = \dfrac{0.14}{1 - 0.01}$$

$$= \dfrac{14}{99}$$

$2.141414 \ldots = 2 + \dfrac{14}{99} = 2\dfrac{14}{99}$

Maka, $m = 2$ dan $n = 14$.

K

1. (a) Katakan diameter bagi semibulatan terkecil
      $= d$ cm.
      Maka, $ST = d$ cm, $RT = 2d$ cm,
      $QT = 4d$ cm dan $PT = 8d$ cm.
      Luas semibulatan berdiameter ST

$$= \dfrac{1}{2} \times \pi \left(\dfrac{d}{2}\right)^2 = \dfrac{1}{8}\pi d^2$$

Luas semibulatan berdiameter $RT = \dfrac{1}{2}\pi d^2$

Luas semibulatan berdiameter QT

$$= \dfrac{1}{2} \times \pi(2d)^2 = 2\pi d^2$$

Luas semibulatan berdiameter PT

$$= \dfrac{1}{2} \times \pi(4d)^2 = 8\pi d^2$$

Luas semibulatan:

$$\dfrac{1}{8}\pi d^2, \dfrac{1}{2}\pi d^2, 2\pi d^2, 8\pi d^2 \ldots$$

Luas bagi empat semibulatan itu membentuk

suatu janjang geometri dengan $a = \dfrac{1}{8}\pi d^2$

dan $r = \dfrac{\dfrac{1}{2}\pi d^2}{\dfrac{1}{8}\pi d^2} = 4$.

(b)
$$S_4 = 170\pi$$
$$\frac{\frac{1}{8}\pi d^2(4^4 - 1)}{4 - 1} = 170\pi$$
$$\frac{1}{8}d^2(85) = 170$$
$$d^2 = 16$$
$$d = 4$$

Diameter bagi semibulatan terbesar
= 8d
= 8 × 4
= 32 cm

2. 100, 100(0.8), 100(0.8)^2, 100(0.8)^3 …

$a = 100$, $r = 0.8$

Jumlah jarak lompatan katak = S_∞
$$= \frac{100}{1 - 0.8}$$
$$= 500 \text{ cm}$$
$$= 5 \text{ m}$$

Jarak asal di antara katak dan serangga = 5.43 m.
Oleh sebab 5 m < 5.43 m, maka katak itu gagal menangkap serangga itu.

Praktis Formatif: Kertas 1

1. 120 + (n – 1)(–8) = 90 + (n – 1)(–5)
120 – 8n + 8 = 90 – 5n + 5
3n = 33
n = 11

2. $S_n = \dfrac{n}{2}(17 - 5n)$

$S_{n-1} = \dfrac{n-1}{2}[17 - 5(n-1)]$

$\qquad = \dfrac{n-1}{2}(22 - 5n)$

$T_n = S_n - S_{n-1}$

$\qquad = \dfrac{n}{2}(17 - 5n) - \dfrac{n-1}{2}(22 - 5n)$

$\qquad = \dfrac{17n}{2} - \dfrac{5n^2}{2} - 11(n-1) + \dfrac{5n}{2}(n-1)$

$\qquad = 11 - 5n$

3. Syarikat Setia:
a = RM40 000, r = 1.06, n = 10

$S_{10} = \dfrac{\text{RM40 000}(1.06^{10} - 1)}{1.06 - 1}$

$\qquad = \text{RM527 232}$

Syarikat Cekap:
a = RM37 000, r = 1.08, n = 10

$S_{10} = \dfrac{\text{RM37 000}(1.08^{10} - 1)}{1.08 - 1}$

$\qquad = \text{RM536 003}$

Jefri patut memilih Syarikat Cekap.

Jumlah tabungan = RM536 003 × 25%
$\qquad\qquad\qquad = \text{RM134 001}$

4. (a) $T_1 = S_1 = \dfrac{7}{3}(4 - 1)$
$\qquad\qquad\quad = 7$

(b) $S_2 = \dfrac{7}{3}(4^2 - 1)$
$\qquad\quad = 35$

$T_2 = S_2 - S_1 \qquad\qquad r = \dfrac{28}{7}$
$\quad = 35 - 7 \qquad\qquad\quad = 4$
$\quad = 28$

5. (a) Nisbah sepunya $= \dfrac{3x + 1}{4x + 4} = \dfrac{2}{3}$
$\qquad\qquad 3(3x + 1) = 2(4x + 4)$
$\qquad\qquad\quad 9x + 3 = 8x + 8$
$\qquad\qquad\qquad\quad x = 5$

(b) $\quad T_8 = 4(5) + 4$
$\qquad ar^7 = 24$
$\quad a\left(\dfrac{2}{3}\right)^7 = 24$
$\qquad\quad a = 24\left(\dfrac{3}{2}\right)^7$
$\qquad\qquad = 410\dfrac{1}{16}$

6. $r = \dfrac{12}{4} = 3$
$a = 12 \times 3$
$b = 12 \times 3^2$
$c = 12 \times 3^3$
$\quad = 324$

7. $a = 6$ minit, $r = 1 + 0.1 = 1.1$

Masa yang diambil $= \dfrac{6(1.1^{10} - 1)}{1.1 - 1}$
$\qquad\qquad\qquad = 95.62 \text{ minit}$
$\qquad\qquad\qquad = 1 \text{ jam } 35.62 \text{ minit}$

Ya, Subramaniam layak untuk menerima pingat.

8. (a) $r \neq 0$, maka $k = 0$.

(b) $T_1 = \dfrac{4r^0}{3}$
$\qquad = \dfrac{4}{3}$

Praktis Formatif: Kertas 2

1. $\qquad T_n = a + (n - 1)d$
$\quad 1\,000 = 5\,000 + (n - 1)(-400)$
$400(n - 1) = 4\,000$
$\qquad n - 1 = 10$
$\qquad\qquad n = 11$

Bilangan hari yang diperlukan = 11

$S_{11} = \dfrac{11}{2}(5\,000 + 1\,000)$
$\qquad = 33\,000$

Jumlah kos = 33 000 × RM0.80
$\qquad\qquad = \text{RM26 400}$

2. (a) $a = 18$, $d = 5$, $n = 60$

    (i) $T_{60} = 18 + 59(5)$

          $= 313$ cm

    (ii) $S_{60} = \dfrac{60}{2}(18 + 313)$

             $= 9\,930$ cm

  (b) Luas segi empat tepat $= 40\,200$ cm^2

     Tinggi dinding $= 1.5$ m $= 150$ cm

     Panjang sisi segi empat tepat $= \dfrac{40\,200}{150}$

                               $= 268$ cm

$$18 + (n - 1)(5) = 268$$
$$5(n - 1) = 250$$
$$n - 1 = 50$$
$$n = 51$$

B: 1, 4, 7, 10, … $3n - 2$
M: 2, 5, 8, 11, … $3n - 1$
H: 3, 6, 9, 12, … $3n$

$$3n = 51$$
$$n = 17 \text{ (integer)}$$

Maka, segi empat tepat berwarna ke-51 mempunyai luas $40\,200$ cm^2 dan berwarna hijau.

3. (a) $\dfrac{x}{351} = \dfrac{3\,159}{x}$

      $x^2 = 351 \times 3\,159$

       $x = \sqrt{351 \times 3\,159}$

         $= 1\,053$

    Nisbah sepunya $= \dfrac{x}{351}$

                   $= \dfrac{1\,053}{351}$

                   $= 3$

  (b) $S_6 = \dfrac{a(3^6 - 1)}{3 - 1} = 4\,732$

           $a\left(\dfrac{728}{2}\right) = 4\,732$

                  $a = 13$

  (c) $T_n > 50\,000$

      $13(3)^{n-1} > 50\,000$

         $3^{n-1} > \dfrac{50\,000}{13}$

   $(n - 1)\log_{10} 3 > \log_{10}\left(\dfrac{50\,000}{13}\right)$

        $n - 1 > \dfrac{\log_{10}\left(\dfrac{50\,000}{13}\right)}{\log_{10} 3}$

        $n - 1 > 7.514$

            $n > 8.514$

Nilai n yang paling kecil ialah 9.

4. (a) $T_6 = 9T_4$

     $ar^5 = 9ar^3$

      $r^2 = 9$

       $r = 3 \; (r > 0)$

    Nisbah sepunya $= 3$

  (b) (i) $a = 4$, $r = 3$, $S_r = 13\,120$

       $\dfrac{4(3^n - 1)}{3 - 1} = 13\,120$

          $3^n - 1 = 6\,560$

              $3^n = 6\,561$

                $= 3^8$

             $n = 8$

    (ii) $a = 4$, $r = 3$, $n = 8$

       $T_8 = 4(3)^7$

           $= 8\,748$ cm

FOKUS KBAT

$$T_8 - T_5 = 105$$
$$a + (8 - 1)d - [a + (5 - 1)d] = 105$$
$$3d = 105$$
$$d = 35$$

$$T_{12} > 2\,500$$
$$a + (12 - 1)(35) > 2\,500$$
$$a > 2\,115$$

dan

$$T_{11} < 2\,500$$
$$a + (11 - 1)(35) < 2\,500$$
$$a < 2\,150$$

Julat yang mungkin bagi bilangan kereta yang dihasilkan dalam bulan Disember 2018 ialah $2\,115 < a < 2\,150$.
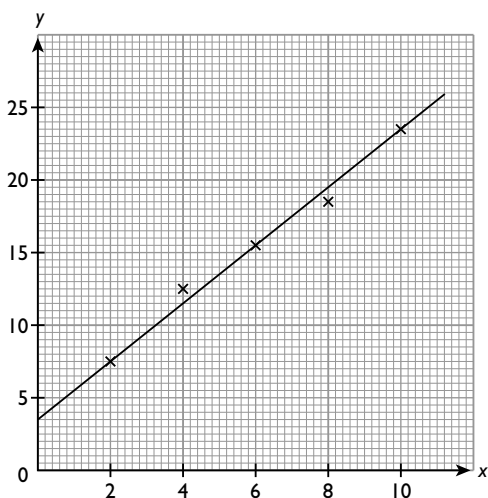
BAB 2: HUKUM LINEAR

2.1

A

1.
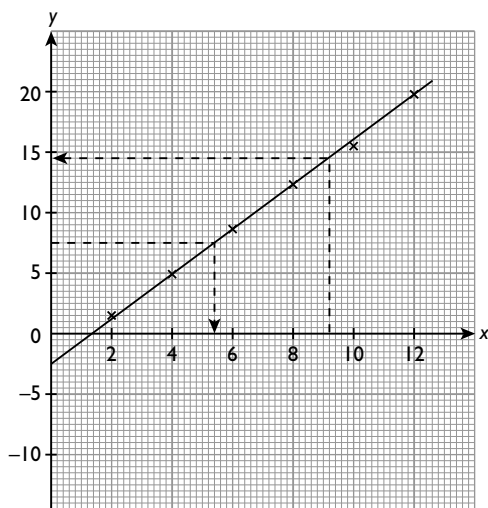

Daripada graf,

pintasan-y = 3.5

$$\text{kecerunan} = \frac{23.5 - 3.5}{10 - 0}$$
$$= 2$$

B

1. (a)

(b) Daripada graf, pintasan-y = –2.5

∴ q = –2.5

$$\text{kecerunan} = \frac{19.7 - (-2.5)}{12 - 0}$$
$$= 1.85$$

∴ p = 1.85

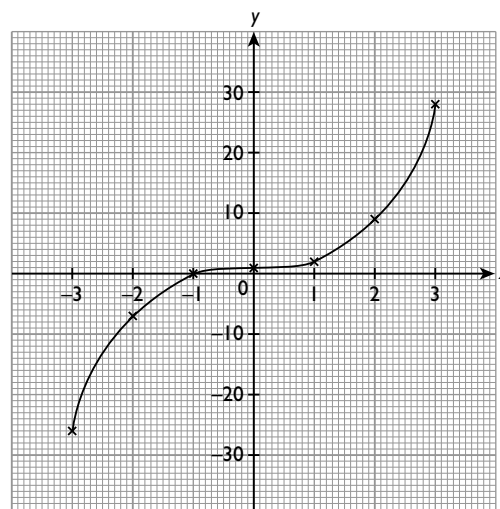Persamaan garis lurus penyuaian terbaik ialah $y = 1.85x - 2.5$.

(c) Daripada graf,
   (i) apabila x = 9.2, y = 14.5.
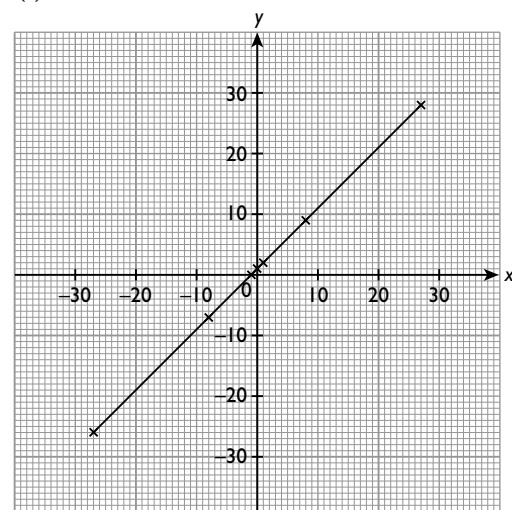   (ii) apabila y = 7.5, x = 5.4.

2.2

A

1. (a)
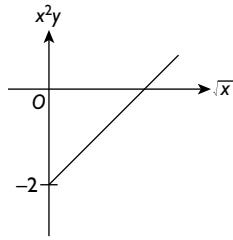

Graf berbentuk kubik.

(b) (i)

Graf berbentuk garis lurus.
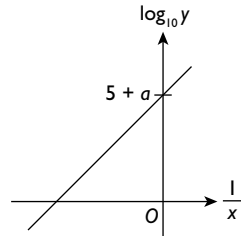
(ii) Kecerunan $= \dfrac{28 - (-26)}{27 - (-27)}$
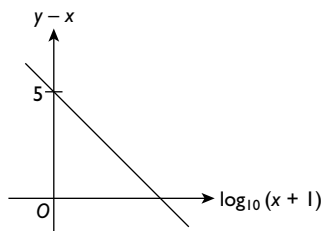$$= 1$$

(iii) Pintasan-y = 1

B

1. $Y = x^2 y$, $X = \sqrt{x}$, $m = 1$, $c = -2$

2. $Y = \log_{10} y$, $X = \dfrac{1}{x}$, $m = 3$, $c = 5 + a$

3. $Y = y - x$, $X = \log_{10}(x + 1)$, $m = -2$, $c = 5$

C

1. $y = -2bx + \dfrac{a}{x}$ ← $\times x$

$xy = -2bx^2 + a$

$Y = xy$, $X = x^2$, $m = -2b$, $c = a$

2. $y + bx = x^2 + a$

$y - x^2 = -bx + a$

$Y = y - x^2$, $X = x$, $m = -b$, $c = a$

3. $ay + 1 = yx^3 + b^2xy$ ← $\div y$

$a + \dfrac{1}{y} = x^3 + b^2x$

$\dfrac{1}{y} - x^3 = b^2x - a$

$Y = \dfrac{1}{y} - x^3$, $X = x$, $m = b^2$, $c = -a$

4. $2y = \dfrac{kx^2}{x - h}$

$\dfrac{1}{2y} = \dfrac{x - h}{kx^2}$

$\dfrac{x^2}{y} = \dfrac{2x}{k} - \dfrac{2h}{k}$

$Y = \dfrac{x^2}{y}$, $X = x$, $m = \dfrac{2}{k}$, $c = -\dfrac{2h}{k}$

5. $-\dfrac{1}{y} = \dfrac{h}{\sqrt{x}} + 3k\sqrt{x}$ ← $\times \sqrt{x}$

$-\dfrac{\sqrt{x}}{y} = h + 3kx$

$\dfrac{\sqrt{x}}{y} = -h - 3kx$

$Y = \dfrac{\sqrt{x}}{y}$, $X = x$, $m = -3k$, $c = -h$

6. $ay = ax\sqrt{x} + \dfrac{b}{\sqrt{x}}$ ← $\times \sqrt{x}$

$\sqrt{x}\,ay = ax^2 + b$ ← $\div a$

$\sqrt{x}\,y = x^2 + \dfrac{b}{a}$

$Y = \sqrt{x}\,y$, $X = x^2$, $m = 1$, $c = \dfrac{b}{a}$

7. $y = \sqrt{ax^2 + 2 - b}$ ← Kuasa duakan kedua-dua belah persamaan.

$y^2 = ax^2 + 2 - b$

$Y = y^2$, $X = x^2$, $m = a$, $c = 2 - b$

8. $x = \left(\dfrac{h - y}{2k}\right)^2$ ← Punca kuasa duakan kedua-dua belah persamaan.

$\sqrt{x} = \dfrac{h - y}{2k}$

$h - y = 2k\sqrt{x}$

$y = -2k\sqrt{x} + h$

$Y = y$, $X = \sqrt{x}$, $m = -2k$, $c = h$

9. $xy = h(2k + x - y)$

$xy = 2hk + h(x - y)$

$Y = xy$, $X = (x - y)$, $m = h$, $c = 2hk$

10. $y(y - h) = x - 5k$

$y^2 - hy = x - 5k$

$y^2 - x = hy - 5k$

$Y = y^2 - x$, $X = y$, $m = h$, $c = -5k$

11. $y = bx^{-a}$

$\log_{10} y = \log_{10} bx^{-a}$

$\log_{10} y = \log_{10} b + \log_{10} x^{-a}$

$\log_{10} y = -a \log_{10} x + \log_{10} b$

$Y = \log_{10} y$, $X = \log_{10} x$, $m = -a$, $c = \log_{10} b$

12. $\dfrac{h}{y} = k^x$

$\log_{10}\left(\dfrac{h}{y}\right) = \log_{10} k^x$

$\log_{10} h - \log_{10} y = x \log_{10} k$

$\log_{10} y = -(\log_{10} k)x + \log_{10} h$

$Y = \log_{10} y$, $X = x$, $m = -\log_{10} k$, $c = \log_{10} h$

D

1. $hy^2 + \dfrac{ky}{h} = x$

$$y + \dfrac{k}{h^2} = \dfrac{x}{hy}$$

$$y = \dfrac{x}{hy} - \dfrac{k}{h^2}$$

$\therefore Y = y,\ X = \dfrac{x}{y},\ m = \dfrac{1}{h},\ c = -\dfrac{k}{h^2}$

Daripada graf, kecerunan, $\dfrac{1}{h} = \dfrac{7+1}{6-2} = 2$

$$h = \dfrac{1}{2}$$

Persamaan garis lurus ialah $Y = 2X + c$.

Gantikan $(6, 7)$ ke dalam $Y = 2X + c$.

$7 = 2(6) + c$

$c = -5$

$\therefore -\dfrac{k}{h^2} = -5$

$-\dfrac{k}{\dfrac{1}{4}} = -5$

$k = \dfrac{5}{4}$

2. $\dfrac{3h}{y} = \dfrac{k}{x^2} + \dfrac{6}{x}$

$$\dfrac{x}{y} = \dfrac{k}{3hx} + \dfrac{2}{h}$$

$\therefore\ Y = \dfrac{x}{y},\ X = \dfrac{1}{x},\ m = \dfrac{k}{3h},\ c = \dfrac{2}{h}$

Daripada graf, $c = \dfrac{2}{h} = 6$

$$h = \dfrac{1}{3}$$

Kecerunan, $\dfrac{k}{3h} = \dfrac{6-0}{0+3} = 2$

$$k = 6h$$

$$= 2$$

E

$\dfrac{y-12}{x} + 2x = 0$

$y - 12 + 2x^2 = 0$

$$y = -2x^2 + 12$$

Persamaan garis lurus ialah $Y = -2X + 12$.

Gantikan $A(4, 2k - 4)$ ke dalam $Y = -2X + 12$.

$2k - 4 = -2(4) + 12$

$k = 4$

Gantikan $B(h + k, 2)$ ke dalam $Y = -2X + 12$.

$2 = -2(h + k) + 12$

$2(h + k) = 10$

$h = 1$

F

1. Daripada graf, $c = 3$.

Persamaan garis lurus ialah $Y = mX + 3$.

Apabila $x = 1$, $y = 3$.

$9 = m(3 - 1) + 3$

$9 = 3m - m + 3$

$m = 3$

Kecerunan, $m = \dfrac{3 - 0}{0 - k}$

$3 = -\dfrac{3}{k}$

$k = -1$

2. Daripada graf, kecerunan, $m = \dfrac{7 - 3}{-6 + 4}$

$$= -2$$

Persamaan garis lurus ialah $Y = -2X + c$.

Gantikan $(-4, 3)$ ke dalam $Y = -2X + c$.

$3 = -2(-4) + c$

$c = -5$

$\therefore y^2 = -2x^2y - 5$

Diberi $x^2y + 7 = 0$

$$x^2y = -7$$

$y^2 = -2(-7) - 5$

$\quad = 9$

$y = \pm 3$

3. $h(y + x - 3k) - kx^3 = 0$

$h(y + x - 3k) = kx^3$

$$y + x = \dfrac{k}{h}x^3 + 3k$$

Diberi $m = \dfrac{3}{4}$.

$\therefore \dfrac{k}{h} = \dfrac{3}{4}\ \ldots\ldots$ ①

Persamaan garis lurus ialah $Y = \dfrac{3}{4}X + 3k$.

Gantikan $(8, 3)$ ke dalam $Y = \dfrac{3}{4}X + c$.

$3 = \dfrac{3}{4}(8) + c$

$c = -3$

$\therefore 3k = -3$

$k = -1$

Gantikan $k = -1$ ke dalam ①.

$\dfrac{-1}{h} = \dfrac{3}{4}$

$h = -\dfrac{4}{3}$

4.
$$y = 2px^q + 10$$
$$y - 10 = 2px^q$$
$$\log_{10}(y - 10) = \log_{10} 2p + q \log_{10} x$$

Persamaan garis lurus ialah $Y = qX + \log_{10} 2p$.

Gantikan $(2.0, 2.5)$ dan $(2.4, 3.0)$ ke dalam $Y = qX + \log_{10} 2p$.

$$2.5 = 2q + \log_{10} 2p \quad \ldots\ldots \text{①}$$
$$3.0 = 2.4q + \log_{10} 2p \quad \ldots\ldots \text{②}$$

② – ①: $0.4q = 0.5$
$$q = 1.25$$

Gantikan $q = 1.25$ ke dalam ①.
$$2.5 = 2(1.25) + \log_{10} 2p$$
$$\log_{10} 2p = 0$$
$$2p = 10^0$$
$$p = \frac{1}{2}$$

5. (a) $\sqrt{xy} = 8x - 3\sqrt{x}$
$$\sqrt{y} = 8\sqrt{x} - 3$$
$$\therefore Y = \sqrt{y}, X = \sqrt{x}, m = 8, c = -3$$

(b) (i) $\sqrt{y} = 8\sqrt{x} - 3$

Kecerunan, $m = 8$

(ii) Pada titik M, $\sqrt{y} = 0$.
$$0 = 8\sqrt{x} - 3$$
$$\sqrt{x} = \frac{3}{8}$$
$$\therefore M\left(\frac{3}{8}, 0\right)$$

Pada titik N, $\sqrt{x} = 0$.
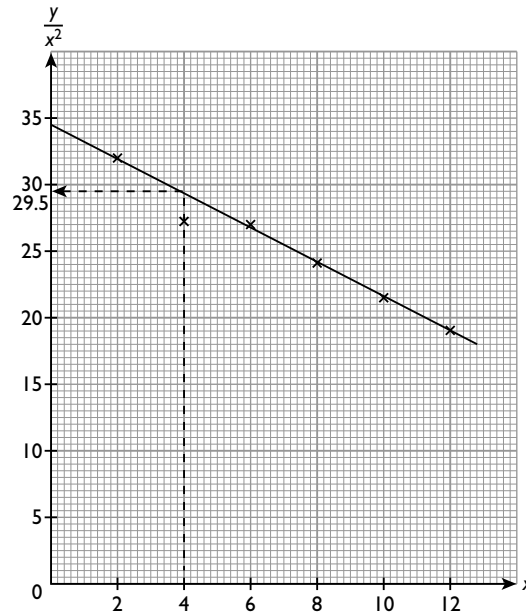$$\sqrt{y} = 8(0) - 3$$
$$\sqrt{y} = -3$$
$$\therefore N(0, -3)$$

G

1. (a)

x	2	4	6	8	10	12
$\dfrac{y}{x^2}$	32.00	27.25	27.00	24.14	21.50	19.00

(b) Daripada graf, terdapat satu nilai yang tidak betul bagi $\dfrac{y}{x^2}$ apabila $x = 4$. Nilai yang betul bagi $\dfrac{y}{x^2}$ ialah 29.5.

Maka, $\dfrac{y}{x^2} = 29.5$
$$\frac{y}{16} = 29.5$$
$$y = 472$$

(c) $y = (h + 1)x^3 + 2kx^2$
$$\frac{y}{x^2} = (h + 1)x + 2k$$

Pintasan-$\dfrac{y}{x^2} = 34.5$
$$2k = 34.5$$
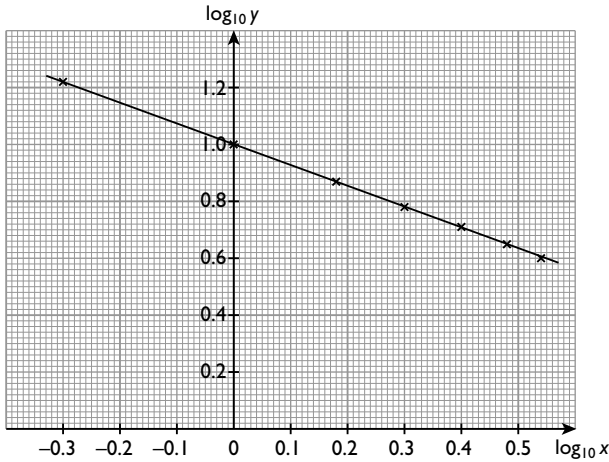$$k = 17.25$$

Kecerunan graf $= \dfrac{32 - 19}{2 - 12}$
$$= -1.3$$
$$h + 1 = -1.3$$
$$h = -2.3$$

2. (a)

$\log_{10} x$	−0.30	0.00	0.18	0.30	0.40	0.48	0.54
$\log_{10} y$	1.22	1.00	0.87	0.78	0.71	0.65	0.60

(b)
$$y^p = \frac{1\,000}{x^q}$$

$$\log_{10} y^p = \log_{10} \frac{1\,000}{x^q}$$

$$\log_{10} y^p = \log_{10} 1\,000 - \log_{10} x^q$$

$$p \log_{10} y = 3 - q \log_{10} x$$

$$p \log_{10} y = -q \log_{10} x + 3$$

$$\log_{10} y = -\frac{q}{p} \log_{10} x + \frac{3}{p}$$

Pintasan-$\log_{10} y = 1$

$$\frac{3}{p} = 1$$

$$p = 3$$

Kecerunan graf $= \dfrac{0.60 - 1.22}{0.54 + 0.3}$

$$= -0.74$$

$$-\frac{q}{p} = -0.74$$

$$-\frac{q}{3} = -0.74$$

$$q = 2.22$$

Praktis Formatif: Kertas 1

1. (a) $\dfrac{a}{y} = 1 + \dfrac{b}{x^3}$

$$\frac{1}{y} = \frac{1}{a} + \frac{b}{ax^3}$$

$$\frac{1}{a} = \text{Pintasan-}\frac{1}{y} = 3$$

$$a = \frac{1}{3}$$

(b) Kecerunan garis lurus, $\dfrac{b}{a} = \dfrac{8 - 3}{4 - 0}$

$$3b = \frac{5}{4}$$

$$b = \frac{5}{12}$$

2. (a)
$$y = 10\,000a^x$$

$$\log_{10} y = \log_{10} 10\,000a^x$$

$$= \log_{10} 10\,000 + \log_{10} a^x$$

$$\log_{10} y = x \log_{10} a + 4$$

(b) Pintasan-$\log_{10} y$, $b = 4$

$$2 = 1 \log_{10} a + 4$$

$$\log_{10} a = -2$$

$$a = 10^{-2}$$

$$= \frac{1}{100}$$

3. (a) $xy = 5x - 3x^4$

$$y = 5 - 3x^3$$

(b) (i) kecerunan $= -3$

(ii) koordinat M ialah $(0, 5)$.

4. Kecerunan garis lurus $= -\dfrac{12}{(-4)}$

$$= 3$$

Pintasan-$(y + x) = 12$.

Persamaan garis lurus ialah
$$y + x = 3x^2 + 12$$
$$y = 3x^2 - x + 12$$

5. $y = 3x - \dfrac{2k}{x^2}$

$$x^2y = 3x^3 - 2k$$

Kecerunan garis lurus $= \dfrac{19 - 4}{m - 0} = 3$

$$\frac{15}{m} = 3$$

$$m = 5$$

Pintasan-x^2y, $-2k = 4$

$$k = -2$$

6. $y = 2x + \dfrac{m}{x^3}$

$$y - 2x = \frac{m}{x^3}$$

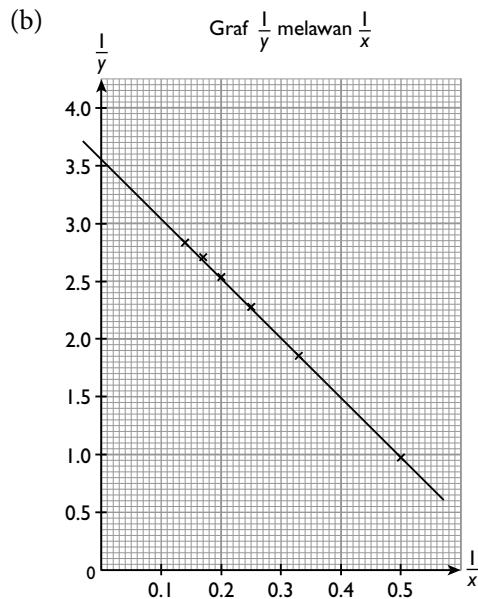Gantikan $\left(\dfrac{h}{3}, 4k\right)$ ke dalam $Y = mX$.

$$4k = m\left(\frac{h}{3}\right)$$

$$h = \frac{12k}{m}$$

1. (a)

$\dfrac{1}{x}$	0.50	0.33	0.25	0.20	0.17	0.14
$\dfrac{1}{y}$	0.98	1.85	2.28	2.54	2.71	2.83

(b)

Graf $\dfrac{1}{y}$ melawan $\dfrac{1}{x}$

(c) $px = qy + xy$

$px = (q + x)y$

$\dfrac{1}{y} = \dfrac{q + x}{px}$

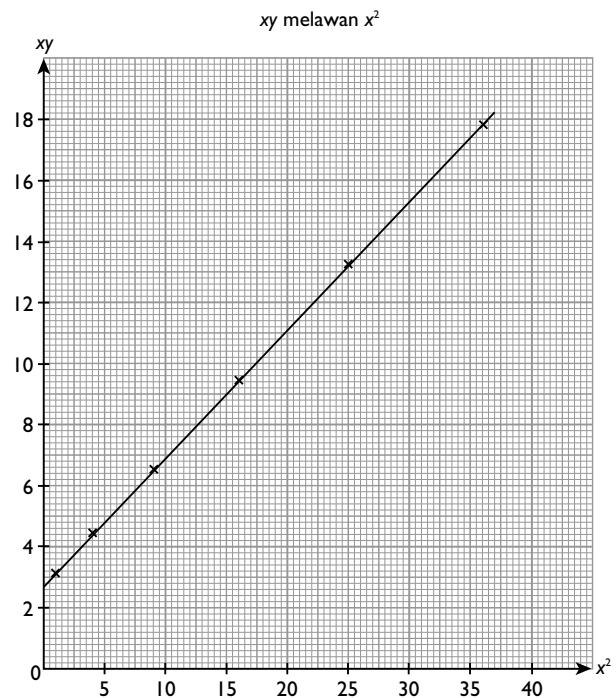$\dfrac{1}{y} = \dfrac{q}{p}\left(\dfrac{1}{x}\right) + \dfrac{1}{p}$

(i) Pintasan-$\dfrac{1}{y}$, $\dfrac{1}{p} = 3.55$

$p = \dfrac{1}{3.55}$

$= 0.28$

(ii) Kecerunan, $\dfrac{q}{p} = \dfrac{0.98 - 3.05}{0.5 - 0.1}$

$\dfrac{q}{0.28} = -5.175$

$q = -1.45$

2. (a)

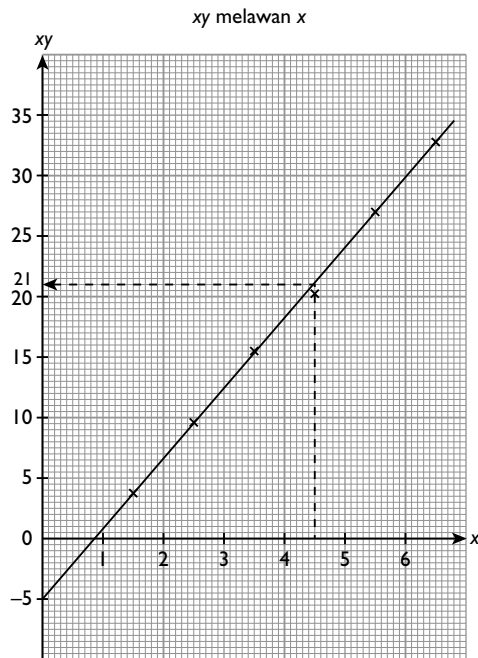| x^2	1	4	9	16	25	36
xy	3.15	4.42	6.51	9.44	13.25	17.88

(b)

xy melawan x^2

(c) $y = \dfrac{m}{2}x + \dfrac{3k}{x}$

$xy = \dfrac{m}{2}x^2 + 3k$

(i) Kecerunan, $\dfrac{m}{2} = \dfrac{17.5 - 4.8}{35 - 5}$

$= 0.423$

$m = 0.846$

(ii) Pintasan-xy, $3k = 2.70$

$k = 0.90$

3. (a)

x	1.5	2.5	3.5	4.5	5.5	6.5
xy	3.9	9.7	15.5	20.3	27.1	32.9

xy melawan x

(b) $y - \sqrt{a} = \dfrac{a}{bx}$

$xy - \sqrt{a}\,x = \dfrac{a}{b}$

$xy = \sqrt{a}\,x + \dfrac{a}{b}$

(i) Kecerunan, $\sqrt{a} = \dfrac{31 - 2}{6.2 - 1.2} = 5.8$

$a = 33.64$

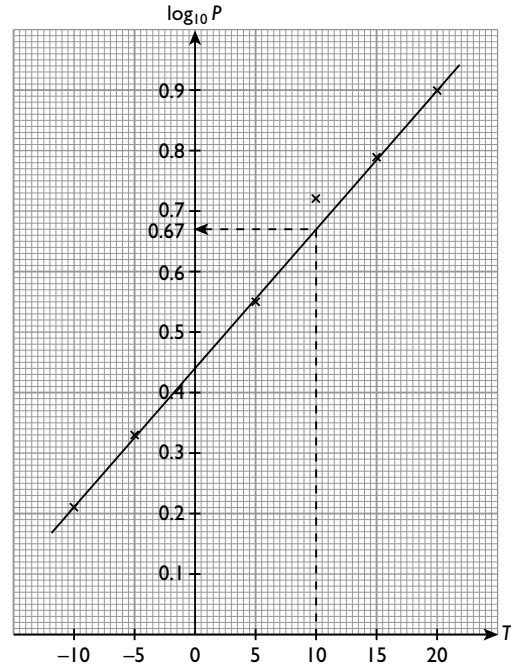Pintasan-xy, $\dfrac{a}{b} = -5$

$b = \dfrac{33.64}{-5}$

$= -6.728$

(ii) Daripada graf, terdapat satu nilai yang tidak betul bagi xy apabila $x = 4.5$.
Nilai yang betul bagi xy ialah 21.
Maka, $xy = 21$
$4.5y = 21$
$y = 4.67$

(a)

T	-10	-5	5	10	15	20
$\log_{10} P$	0.21	0.33	0.55	0.72	0.79	0.90

$\log_{10} P$

(b) Daripada graf, terdapat satu nilai yang tidak betul bagi $\log_{10} P$ apabila $T = 10$.
Nilai yang betul bagi $\log_{10} P$ ialah 0.67.
Maka, $P = 4.677$.

(c) $P^2 = nk^T$

$\log_{10} P^2 = \log_{10} nk^T$

$2 \log_{10} P = \log_{10} n + T \log_{10} k$

$\log_{10} P = \left(\dfrac{1}{2} \log_{10} k\right)T + \dfrac{1}{2} \log_{10} n$

Kecerunan graf $= \dfrac{1}{2} \log_{10} k$

$\dfrac{1}{2} \log_{10} k = 0.023$

$k = 1.112$

Pintasan-$\log_{10} P = 0.44$

$\dfrac{1}{2} \log_{10} n = 0.44$

$n = 7.586$

BAB 3: PENGAMIRAN

3.1

A

1. $y = (2x + 5)^4$

$$\frac{dy}{dx} = 4(2x + 5)^3(2) = 8(2x + 5)^3$$

$$\int(2x + 5)^3 \, dx = \frac{1}{8}\int 8(2x + 5)^3 \, dx$$

$$= \frac{1}{8}(2x + 5)^4 + c$$

2. $y = \dfrac{x}{3x + 2}$

$$\frac{dy}{dx} = \frac{(3x + 2)(1) - x(3)}{(3x + 2)^2} = \frac{2}{(3x + 2)^2}$$

$$\int\frac{1}{(3x + 2)^2} \, dx = \frac{1}{2}\int\frac{2}{(3x + 2)^2} \, dx$$

$$= \frac{1}{2}\left(\frac{x}{3x + 2}\right) + c$$

3. $y = \left(x + \dfrac{1}{x}\right)^2$

$$y = x^2 + \frac{1}{x^2} + 2$$

$$\frac{dy}{dx} = 2x + (-2)x^{-3} + 0 = 2x - \frac{2}{x^3}$$

$$\int\left(x - \frac{1}{x^3}\right) \, dx = \frac{1}{2}\int\left(2x - \frac{2}{x^3}\right) \, dx$$

$$= \frac{1}{2}\left(x + \frac{1}{x}\right)^2 + c$$

B

1. $\int x^5 \, dx = \dfrac{x^6}{6} + c$

2. $\int 8x^2 \, dx = \dfrac{8x^3}{3} + c$

3. $\int 8 \, dx = 8x + c$

4. $\int\dfrac{10}{3}x^4 \, dx = \dfrac{10}{3}\left(\dfrac{x^5}{5}\right) + c$

$$= \frac{2}{3}x^5 + c$$

5. $\int 5x^{-4} \, dx = \dfrac{5x^{-3}}{-3} + c$

$$= -\frac{5}{3x^3} + c$$

6. $\int\dfrac{10}{x^6} \, dx = \int 10x^{-6} \, dx$

$$= \frac{10x^{-5}}{-5} + c$$

$$= -\frac{2}{x^5} + c$$

C

1. $\int x(4x - 3) \, dx = \int(4x^2 - 3x) \, dx$

$$= \frac{4x^3}{3} - \frac{3x^2}{2} + c$$

2. $\int(x - 3)(x + 3) \, dx = \int(x^2 - 9) \, dx$

$$= \frac{x^3}{3} - 9x + c$$

3. $\int(x - 1)(x + 4) \, dx = \int(x^2 + 3x - 4) \, dx$

$$= \frac{x^3}{3} + \frac{3x^2}{2} - 4x + c$$

4. $\int(2x - 1)^2 \, dx = \int(4x^2 - 4x + 1) \, dx$

$$= \frac{4x^3}{3} - 2x^2 + x + c$$

5. $\int\dfrac{3x^4 - 5}{x^2} \, dx = \int\left(3x^2 - \dfrac{5}{x^2}\right) \, dx$

$$= \frac{3x^3}{3} - \frac{5x^{-1}}{(-1)} + c$$

$$= x^3 + \frac{5}{x} + c$$

D

1. $\dfrac{dy}{dx} = 3 - 2x$

$$y = \int(3 - 2x) \, dx$$
$$y = 3x - x^2 + c \qquad \text{...... ①}$$

Gantikan $x = 4$ dan $y = 1$ ke dalam ①.
$$1 = 3(4) - 4^2 + c$$
$$c = 5$$

Persamaan lengkung ialah $y = 3x - x^2 + 5$.

2. $\dfrac{dy}{dx} = 2x - \dfrac{1}{x^2}$

$$y = \int\left(2x - \frac{1}{x^2}\right) \, dx$$

$$= x^2 - \frac{x^{-1}}{(-1)} + c$$

$$y = x^2 + \frac{1}{x} + c \qquad \text{...... ①}$$

Gantikan $x = 3$ dan $y = -2$ ke dalam ①.

$$-2 = 3^2 + \frac{1}{3} + c$$

$$c = -11\frac{1}{3}$$

Persamaan lengkung ialah $y = x^2 + \dfrac{1}{x} - 11\dfrac{1}{3}$.

3. $f'(x) = \left(2x + \dfrac{1}{x}\right)^2$

$\quad y = \int \left(2x + \dfrac{1}{x}\right)^2 dx$

$\qquad = \int \left(4x^2 + \dfrac{1}{x^2} + 4\right) dx$

$\quad y = \dfrac{4x^3}{3} - \dfrac{1}{x} + 4x + c \qquad \ldots\ldots ①$

Gantikan $x = 1$ dan $y = 2$ ke dalam ①.

$2 = \dfrac{4}{3} - 1 + 4 + c$

$c = -2\dfrac{1}{3}$

Persamaan lengkung ialah

$y = \dfrac{4x^3}{3} - \dfrac{1}{x} + 4x - 2\dfrac{1}{3}.$

E

1. $\int (2x + 3)^5 \, dx = \dfrac{(2x + 3)^6}{2(6)} + c$

$\qquad\qquad\qquad = \dfrac{1}{12}(2x + 3)^6 + c$

2. $\int 14(3x - 4)^6 \, dx = \dfrac{14(3x - 4)^7}{3(7)} + c$

$\qquad\qquad\qquad\ = \dfrac{2}{3}(3x - 4)^7 + c$

3. $\int 6(5 - 2x)^7 \, dx = \dfrac{6(5 - 2x)^8}{(-2)(8)} + c$

$\qquad\qquad\qquad = -\dfrac{3}{8}(5 - 2x)^8 + c$

4. $\int 3(4 - x)^5 \, dx = \dfrac{3(4 - x)^6}{(-1)(6)} + c$

$\qquad\qquad\qquad = -\dfrac{1}{2}(4 - x)^6 + c$

5. $\int 8(1 + 2x)^{-3} \, dx = \dfrac{8(1 + 2x)^{-2}}{(2)(-2)} + c$

$\qquad\qquad\qquad\ = -2(1 + 2x)^{-2} + c$

$\qquad\qquad\qquad\ = \dfrac{-2}{(1 + 2x)^2} + c$

6. $\int 20(2 - 5x)^{-4} \, dx = \dfrac{20(2 - 5x)^{-3}}{(-5)(-3)} + c$

$\qquad\qquad\qquad\ = \dfrac{4}{3}(2 - 5x)^{-3} + c$

$\qquad\qquad\qquad\ = \dfrac{4}{3(2 - 5x)^3} + c$

7. $\int \dfrac{1}{(3x - 2)^2} \, dx = \int (3x - 2)^{-2} \, dx$

$\qquad\qquad\qquad = \dfrac{(3x - 2)^{-1}}{(3)(-1)} + c$

$\qquad\qquad\qquad = \dfrac{-1}{3(3x - 2)} + c$

8. $\int \dfrac{24}{(4x + 1)^3} \, dx = \int 24(4x + 1)^{-3} \, dx$

$\qquad\qquad\qquad = \dfrac{24(4x + 1)^{-2}}{4(-2)} + c$

$\qquad\qquad\qquad = -\dfrac{3}{(4x + 1)^2} + c$

9. $\int \dfrac{12}{(1 - 3x)^5} \, dx = \int 12(1 - 3x)^{-5} \, dx$

$\qquad\qquad\qquad = \dfrac{12(1 - 3x)^{-4}}{(-3)(-4)} + c$

$\qquad\qquad\qquad = \dfrac{1}{(1 - 3x)^4} + c$

10. $\int \dfrac{18}{(3 - 2x)^7} \, dx = \int 18(3 - 2x)^{-7} \, dx$

$\qquad\qquad\qquad = \dfrac{18(3 - 2x)^{-6}}{(-2)(-6)} + c$

$\qquad\qquad\qquad = \dfrac{3}{2(3 - 2x)^6} + c$

3.2

A

1. $\int_1^3 (4 - x) \, dx = \left[4x - \dfrac{x^2}{2}\right]_1^3$

$\qquad\qquad\qquad = \left(12 - \dfrac{9}{2}\right) - \left(4 - \dfrac{1}{2}\right)$

$\qquad\qquad\qquad = 4$

2. $\int_1^2 x(2x - 3) \, dx = \int_1^2 (2x^2 - 3x) \, dx$

$\qquad\qquad\qquad = \left[\dfrac{2x^3}{3} - \dfrac{3x^2}{2}\right]_1^2$

$\qquad\qquad\qquad = \left(\dfrac{16}{3} - 6\right) - \left(\dfrac{2}{3} - \dfrac{3}{2}\right)$

$\qquad\qquad\qquad = \dfrac{1}{6}$

3. $\int_2^3 \left(3 + \dfrac{4}{x^2}\right) dx = \left[3x - \dfrac{4}{x}\right]_2^3$

$\qquad\qquad\qquad = \left(9 - \dfrac{4}{3}\right) - (6 - 2)$

$\qquad\qquad\qquad = 3\dfrac{2}{3}$

4. $\int_1^2 (3x - 2)^3 \, dx = \left[\dfrac{(3x - 2)^4}{(3)(4)}\right]_1^2$

$\qquad\qquad\qquad = \dfrac{1}{12}\left(4^4 - 1\right)$

$\qquad\qquad\qquad = 21\dfrac{1}{4}$

5. $\int_1^3 (1 - 2x)^{-2} \, dx = \left[\dfrac{(1 - 2x)^{-1}}{(-2)(-1)}\right]_1^3$

$\qquad\qquad\qquad = \dfrac{1}{2}\left[\dfrac{1}{1 - 2x}\right]_1^3$

$\qquad\qquad\qquad = \dfrac{1}{2}\left[-\dfrac{1}{5} - (-1)\right]$

$\qquad\qquad\qquad = \dfrac{2}{5}$

6. $\displaystyle\int_{-2}^{-1} \left(x + \frac{1}{x}\right)^2 dx = \int_{-2}^{-1} \left(x^2 + 2 + \frac{1}{x^2}\right) dx$

$\qquad\qquad = \left[\dfrac{x^3}{3} + 2x - \dfrac{1}{x}\right]_{-2}^{-1}$

$\qquad\qquad = \left(-\dfrac{1}{3} - 2 + 1\right) - \left(-\dfrac{8}{3} - 4 + \dfrac{1}{2}\right)$

$\qquad\qquad = 4\dfrac{5}{6}$

7. $\displaystyle\int_{-1}^{0} \frac{1}{(2x-3)^3} dx = \left[\frac{(2x-3)^{-2}}{(2)(-2)}\right]_{-1}^{0}$

$\qquad\qquad = -\dfrac{1}{4}\left[\dfrac{1}{(2x-3)^2}\right]_{-1}^{0}$

$\qquad\qquad = -\dfrac{1}{4}\left[\dfrac{1}{9} - \left(\dfrac{1}{25}\right)\right]$

$\qquad\qquad = -\dfrac{4}{225}$

8. $\displaystyle\int_{-3}^{-1} \frac{x^3 - 4}{x^2} dx = \int_{-3}^{-1} \left(x - \frac{4}{x^2}\right) dx$

$\qquad\qquad = \left[\dfrac{x^2}{2} + \dfrac{4}{x}\right]_{-3}^{-1}$

$\qquad\qquad = \left(\dfrac{1}{2} - 4\right) - \left(\dfrac{9}{2} - \dfrac{4}{3}\right)$

$\qquad\qquad = -6\dfrac{2}{3}$

B

1. $\displaystyle\int_{1}^{3} 5f(x)\, dx = 5\int_{1}^{3} f(x)\, dx$

$\qquad\qquad = 5(8)$
$\qquad\qquad = 40$

2. $\displaystyle\int_{4}^{3} 6f(x)\, dx = -6\int_{3}^{4} f(x)\, dx$

$\qquad\qquad = -6(15)$
$\qquad\qquad = -90$

3. $3\displaystyle\int_{1}^{4} f(x)\, dx = 3\left[\int_{1}^{3} f(x)\, dx + \int_{3}^{4} f(x)\, dx\right]$

$\qquad\qquad = 3(8 + 15)$
$\qquad\qquad = 69$

4. $\displaystyle\int_{1}^{3} [2f(x) + 4]\, dx = \int_{1}^{3} 2f(x)\, dx + \int_{1}^{3} 4\, dx$

$\qquad\qquad = 2\displaystyle\int_{1}^{3} f(x)\, dx + \left[4x\right]_{1}^{3}$

$\qquad\qquad = 2(8) + 4(3) - 4$
$\qquad\qquad = 24$

5. $\displaystyle\int_{1}^{3} [3f(x) - 4x^2]\, dx = 3\int_{1}^{3} f(x)\, dx - \int_{1}^{3} 4x^2\, dx$

$\qquad\qquad = 3(8) - \left[\dfrac{4x^3}{3}\right]_{1}^{3}$

$\qquad\qquad = 24 - \dfrac{4}{3}(3^3 - 1)$

$\qquad\qquad = 24 - \dfrac{4}{3}(26) = -10\dfrac{2}{3}$

6. $\displaystyle\int_{3}^{4} [4x - 3f(x)]\, dx = \int_{3}^{4} 4x\, dx - \int_{3}^{4} 3f(x)\, dx$

$\qquad\qquad = \left[2x^2\right]_{3}^{4} - 3(15)$

$\qquad\qquad = 2(4^2 - 3^2) - 45$
$\qquad\qquad = -31$

7. $\displaystyle\int_{3}^{4} [4f(x) - kx]\, dx = 30$

$\displaystyle\int_{3}^{4} 4f(x)\, dx - \int_{3}^{4} kx\, dx = 30$

$\qquad 4(15) - \left[\dfrac{kx^2}{2}\right]_{3}^{4} = 30$

$\qquad -\dfrac{k}{2}(4^2 - 3^2) = 30 - 60$

$\qquad\qquad -\dfrac{7}{2}k = -30$

$\qquad\qquad k = 8\dfrac{4}{7}$

8. $\displaystyle\int_{1}^{3} [x^2 - kf(x)]\, dx = 20$

$\displaystyle\int_{1}^{3} x^2\, dx - \int_{1}^{3} kf(x)\, dx = 20$

$\qquad \left[\dfrac{x^3}{3}\right]_{1}^{3} - k(8) = 20$

$\qquad \dfrac{1}{3}(3^3 - 1) - 8k = 20$

$\qquad\qquad 8k = \dfrac{26}{3} - 20$

$\qquad\qquad k = -1\dfrac{5}{12}$

C

1. Luas $= \displaystyle\int_{1}^{2} (x^2 + 4)\, dx$

$\qquad = \left[\dfrac{x^3}{3} + 4x\right]_{1}^{2}$

$\qquad = \left(\dfrac{8}{3} + 8\right) - \left(\dfrac{1}{3} + 4\right)$

$\qquad = 6\dfrac{1}{3} \text{ unit}^2$

2. Luas $= \displaystyle\int_{1}^{3} \frac{4}{x^2}\, dx$

$\qquad = \left[-\dfrac{4}{x}\right]_{1}^{3}$

$\qquad = \left(-\dfrac{4}{3}\right) - (-4)$

$\qquad = 2\dfrac{2}{3} \text{ unit}^2$

3. Luas $= \displaystyle\int_{1}^{4} x(4 - x)\, dx$

$\qquad = \displaystyle\int_{1}^{4} (4x - x^2)\, dx$

$\qquad = \left[2x^2 - \dfrac{x^3}{3}\right]_{1}^{4}$

$\qquad = \left(32 - \dfrac{64}{3}\right) - \left(2 - \dfrac{1}{3}\right)$

$\qquad = 9 \text{ unit}^2$

4. Luas $= \displaystyle\int_{-1}^{3} (x - 2)^2\, dx$

$\qquad = \left[\dfrac{(x-2)^3}{3}\right]_{-1}^{3}$

$\qquad = \dfrac{1}{3} - \left(\dfrac{-27}{3}\right)$

$\qquad = 9\dfrac{1}{3} \text{ unit}^2$

5. Luas $= -\int_0^3 x(x-3)\,dx$

$\qquad = -\int_0^3 (x^2 - 3x)\,dx$

$\qquad = -\left[\dfrac{x^3}{3} - \dfrac{3x^2}{2}\right]_0^3$

$\qquad = -\left[\left(9 - \dfrac{27}{2}\right) - 0\right]$

$\qquad = 4\dfrac{1}{2}$ unit^2

6. Luas

$= \int_{-2}^0 x(x+2)(x-2)\,dx - \int_0^1 x(x+2)(x-2)\,dx$

$= \int_{-2}^0 (x^3 - 4x)\,dx - \int_0^1 (x^3 - 4x)\,dx$

$= \left[\dfrac{x^4}{4} - 2x^2\right]_{-2}^0 - \left[\dfrac{x^4}{4} - 2x^2\right]_0^1$

$= 0 - (4-8) - \left(\dfrac{1}{4} - 2\right)$

$= 5\dfrac{3}{4}$ unit^2

7. Luas $= -\int_1^2 (x^3 - 8)\,dx$

$\qquad = -\left[\dfrac{x^4}{4} - 8x\right]_1^2$

$\qquad = -\left[(4-16) - \left(\dfrac{1}{4} - 8\right)\right]$

$\qquad = 4\dfrac{1}{4}$ unit^2

8. Luas

$= -\int_1^2 (x^2 - 3x + 2)\,dx + \int_2^3 (x^2 - 3x + 2)\,dx$

$= -\left[\dfrac{x^3}{3} - \dfrac{3x^2}{2} + 2x\right]_1^2 + \left[\dfrac{x^3}{3} - \dfrac{3x^2}{2} + 2x\right]_2^3$

$= -\left[\left(\dfrac{8}{3} - 6 + 4\right) - \left(\dfrac{1}{3} - \dfrac{3}{2} + 2\right)\right] + \left[\left(9 - \dfrac{27}{2} + 6\right) - \left(\dfrac{8}{3} - 6 + 4\right)\right]$

$= 1$ unit^2

D

1. Luas $= \int_0^3 y(3-y)\,dy$

$\qquad = \int_0^3 (3y - y^2)\,dy$

$\qquad = \left[\dfrac{3y^2}{2} - \dfrac{y^3}{3}\right]_0^3$

$\qquad = \dfrac{27}{2} - 9 - 0$

$\qquad = 4\dfrac{1}{2}$ unit^2

2. Luas $= \int_1^3 \dfrac{4}{y^2}\,dy$

$\qquad = \left[-\dfrac{4}{y}\right]_1^3$

$\qquad = -\dfrac{4}{3} + 4$

$\qquad = 2\dfrac{2}{3}$ unit^2

3. Luas $= \int_{-1}^3 (y^2 + 2)\,dy$

$\qquad = \left[\dfrac{y^3}{3} + 2y\right]_{-1}^3$

$\qquad = (9 + 6) - \left(-\dfrac{1}{3} - 2\right)$

$\qquad = 17\dfrac{1}{3}$ unit^2

4. Luas $= -\int_{-1}^2 (y^2 - 4)\,dy$

$\qquad = -\left[\dfrac{y^3}{3} - 4y\right]_{-1}^2$

$\qquad = -\left[\left(\dfrac{8}{3} - 8\right) - \left(-\dfrac{1}{3} + 4\right)\right]$

$\qquad = 9$ unit^2

5. Luas

$= -\int_1^2 y(y-2)\,dy + \int_2^3 y(y-2)\,dy$

$= -\int_1^2 (y^2 - 2y)\,dy + \int_2^3 (y^2 - 2y)\,dy$

$= -\left[\dfrac{y^3}{3} - y^2\right]_1^2 + \left[\dfrac{y^3}{3} - y^2\right]_2^3$

$= -\left[\left(\dfrac{8}{3} - 4\right) - \left(\dfrac{1}{3} - 1\right)\right] + \left[(9-9) - \left(\dfrac{8}{3} - 4\right)\right]$

$= 2$ unit^2

6. Luas

$= \int_{-1}^0 y(y+1)(y-2)\,dy - \int_0^2 y(y+1)(y-2)\,dy$

$= \int_{-1}^0 (y^3 - y^2 - 2y)\,dy - \int_0^2 (y^3 - y^2 - 2y)\,dy$

$= \left[\dfrac{y^4}{4} - \dfrac{y^3}{3} - y^2\right]_{-1}^0 - \left[\dfrac{y^4}{4} - \dfrac{y^3}{3} - y^2\right]_0^2$

$= 0 - \left(\dfrac{1}{4} + \dfrac{1}{3} - 1\right) - \left(4 - \dfrac{8}{3} - 4\right)$

$= 3\dfrac{1}{12}$ unit^2

E

1. Luas $= \int_0^3 [x(4-x) - x]\,dx$

$\qquad = \int_0^3 (3x - x^2)\,dx$

$\qquad = \left[\dfrac{3x^2}{2} - \dfrac{x^3}{3}\right]_0^3$

$\qquad = \dfrac{3}{2}(9) - \dfrac{27}{3} - 0$

$\qquad = 4\dfrac{1}{2}$ unit^2

2. Luas $= \int_1^2 \left[7 - 3x - \dfrac{4}{x^2}\right]\,dx$

$\qquad = \left[7x - \dfrac{3x^2}{2} + \dfrac{4}{x}\right]_1^2$

$\qquad = (14 - 6 + 2) - \left(7 - \dfrac{3}{2} + 4\right)$

$\qquad = \dfrac{1}{2}$ unit^2

3. Luas $= \int_0^1 2x^2 \, dx + \int_1^3 (3 - x) \, dx$

$$= \left[\frac{2x^3}{3}\right]_0^1 + \left[3x - \frac{x^2}{2}\right]_1^3$$

$$= \left(\frac{2}{3} - 0\right) + \left[\left(9 - \frac{9}{2}\right) - \left(3 - \frac{1}{2}\right)\right]$$

$$= 2\frac{2}{3} \text{ unit}^2$$

4. Luas $= \int_0^1 x(6 - x) \, dx + \int_1^6 (6 - x) \, dx$

$$= \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[6x - \frac{x^2}{2}\right]_1^6$$

$$= \left[\left(3 - \frac{1}{3}\right) - 0\right] + \left[(36 - 18) - \left(6 - \frac{1}{2}\right)\right]$$

$$= 15\frac{1}{6} \text{ unit}^2$$

F

1. Isi padu janaan $= \pi\int_1^4 \left(\frac{2}{x}\right)^2 dx$

$$= \pi\int_1^4 \frac{4}{x^2} \, dx$$

$$= \pi\left[-\frac{4}{x}\right]_1^4$$

$$= \pi(-1 + 4)$$

$$= 3\pi \text{ unit}^3$$

2. Isi padu janaan

$$= \pi\int_1^3 [x(3 - x)]^2 \, dx$$

$$= \pi\int_1^3 (9x^2 - 6x^3 + x^4) \, dx$$

$$= \pi\left[3x^3 - \frac{3x^4}{2} + \frac{x^5}{5}\right]_1^3$$

$$= \pi\left[\left(81 - \frac{243}{2} + \frac{243}{5}\right) - \left(3 - \frac{3}{2} + \frac{1}{5}\right)\right]$$

$$= 6\frac{2}{5}\pi \text{ unit}^3$$

3. Isi padu janaan $= \pi\int_{-2}^3 (x + 2) \, dx$

$$= \pi\left[\frac{x^2}{2} + 2x\right]_{-2}^3$$

$$= \pi\left[\left(\frac{9}{2} + 6\right) - (2 - 4)\right]$$

$$= 12\frac{1}{2}\pi \text{ unit}^3$$

4. Isi padu janaan $= \pi\int_1^3 (2x - 3)^4 \, dx$

$$= \pi\left[\frac{(2x - 3)^5}{2(5)}\right]_1^3$$

$$= \frac{\pi}{10}[3^5 - (-1)]$$

$$= 24\frac{2}{5}\pi \text{ unit}^3$$

G

1. Isi padu janaan $= \pi\int_1^2 \left(-\frac{2}{y}\right)^2 dy$

$$= \pi\int_1^2 \left(\frac{4}{y^2}\right) dy$$

$$= \pi\left[-\frac{4}{y}\right]_1^2$$

$$= \pi(-2 + 4)$$

$$= 2\pi \text{ unit}^3$$

2. Isi padu janaan $= \pi\int_3^5 (y - 2) \, dy$

$$= \pi\left[\frac{y^2}{2} - 2y\right]_3^5$$

$$= \pi\left[\left(\frac{25}{2} - 10\right) - \left(\frac{9}{2} - 6\right)\right]$$

$$= 4\pi \text{ unit}^3$$

3. Isi padu janaan $= \pi\int_0^2 [y(y - 2)]^2 \, dy$

$$= \pi\int_0^2 (y^4 - 4y^3 + 4y^2) \, dy$$

$$= \pi\left[\frac{y^5}{5} - y^4 + \frac{4y^3}{3}\right]_0^2$$

$$= \pi\left(\frac{32}{5} - 16 + \frac{32}{3}\right)$$

$$= 1\frac{1}{15}\pi \text{ unit}^3$$

4. Isi padu janaan $= \pi\int_{-1}^1 (2y + 1)^4 \, dy$

$$= \pi\left[\frac{(2y + 1)^5}{2(5)}\right]_{-1}^1$$

$$= \frac{\pi}{10}[3^5 - (-1)]$$

$$= 24\frac{2}{5}\pi \text{ unit}^3$$

H

1. Isi padu janaan $= \pi\int_0^3 (x + 1) \, dx - \pi\int_0^3 \left(\frac{2}{3}x\right)^2 dx$

$$= \pi\int_0^3 \left(x + 1 - \frac{4}{9}x^2\right) dx$$

$$= \pi\left[\frac{x^2}{2} + x - \frac{4}{9}\left(\frac{x^3}{3}\right)\right]_0^3$$

$$= \pi\left[\left(\frac{9}{2} + 3 - 4\right) - 0\right]$$

$$= 3\frac{1}{2}\pi \text{ unit}^3$$

2. Isi padu janaan $= \pi\int_0^2 (2x)^2 \, dx + \pi\int_2^3 \left(\frac{8}{x}\right)^2 dx$

$$= \pi\left[\frac{4x^3}{3}\right]_0^2 + \pi\left[-\frac{64}{x}\right]_2^3$$

$$= \pi\left(\frac{32}{3} - 0\right) + \pi\left(-\frac{64}{3} + 32\right)$$

$$= 21\frac{1}{3}\pi \text{ unit}^3$$

I

1. Isi padu janaan

$$= \pi\int_0^1 (1 - y)\, dy - \pi\int_0^1 (y - 1)^2\, dy$$

$$= \pi\left[y - \frac{y^2}{2} - \frac{(y - 1)^3}{3}\right]_0^1$$

$$= \pi\left\{\left[1 - \frac{1}{2} - 0\right] - \left[0 - 0 - \left(-\frac{1}{3}\right)\right]\right\}$$

$$= \frac{1}{6}\pi \text{ unit}^3$$

2. Isi padu janaan

$$= \pi\int_0^1 (y^2 + 1)^2\, dy + \pi\int_1^3 (3 - y)^2\, dy$$

$$= \pi\int_0^1 (y^4 + 2y^2 + 1)\, dy + \pi\int_1^3 (3 - y)^2\, dy$$

$$= \pi\left[\frac{y^5}{5} + \frac{2y^3}{3} + y\right]_0^1 + \pi\left[\frac{(3 - y)^3}{-3}\right]_1^3$$

$$= \pi\left[\left(\frac{1}{5} + \frac{2}{3} + 1\right) - 0\right] + \pi\left(0 + \frac{8}{3}\right)$$

$$= 4\frac{8}{15}\pi \text{ unit}^3$$

Praktis Formatif: Kertas 1

1. (a) $\dfrac{dy}{dx} = kx - 10$ Pada $(2, 7)$, $\dfrac{dy}{dx} = 0$.

Apabila $x = 2$, $k(2) - 10 = 0$

$$k = 5$$

(b) $y = \int(5x - 10)\, dx$

$$y = \frac{5x^2}{2} - 10x + c \quad \text{...... } ①$$

Gantikan $x = 2$ dan $y = 7$ ke dalam $①$.

$$7 = \frac{5}{2}(2^2) - 10(2) + c$$

$$c = 17$$

Persamaan lengkung ialah

$$y = \frac{5x^2}{2} - 10x + 17.$$

2. $\displaystyle\int \frac{8}{(3x + 4)^n}\, dx = \int 8(3x + 4)^{-n}\, dx$

$$= \frac{8(3x + 4)^{-n + 1}}{3(-n + 1)} + c$$

$$-n + 1 = -6$$

$$n = 7$$

$$m = \frac{8}{3(1 - n)}$$

$$= \frac{8}{3(1 - 7)}$$

$$= -\frac{4}{9}$$

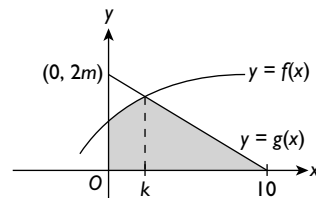**3.** Luas $= \displaystyle\int_1^k \frac{4}{x^2}\, dx = \frac{16}{5}$

$$\left[-\frac{4}{x}\right]_1^k = \frac{16}{5}$$

$$-\frac{4}{k} + 4 = \frac{16}{5}$$

$$\frac{4}{k} = 4 - \frac{16}{5}$$

$$= \frac{4}{5}$$

$$k = 5$$

4. $\dfrac{dy}{dx} = g(x) \Rightarrow y = \int g(x)\, dx$

$$\int_0^5 3g(x)\, dx = 3\int_0^5 g(x)\, dx$$

$$= 3\left[y\right]_0^5$$

$$= 3\left[\frac{4x}{x^2 + 2}\right]_0^5$$

$$= 3\left[\frac{4(5)}{5^2 + 2} - 0\right]$$

$$= 2\frac{2}{9}$$

5. (a) $\displaystyle\int_4^2 3f(x)\, dx = -3\int_2^4 f(x)\, dx$

$$= -3(5)$$

$$= -15$$

(b) $\displaystyle\int_2^4 [3 + f(x)]\, dx = \int_2^4 3\, dx + \int_2^4 f(x)\, dx$

$$= \left[3x\right]_2^4 + 5$$

$$= 3(4 - 2) + 5$$

$$= 11$$

6. (a)

(b) Luas $= \dfrac{1}{2}(10)(2m) - 15$

$$= (10m - 15) \text{ unit}^2$$

7. $\displaystyle\int_1^k (5 - 4x)\, dx = -15$

$$\left[5x - 2x^2\right]_1^k = -15$$

$$5k - 2k^2 - (5 - 2) = -15$$

$$-2k^2 + 5k + 12 = 0$$

$$2k^2 - 5k - 12 = 0$$

$$(k - 4)(2k + 3) = 0$$

$k - 4 = 0$ atau $2k + 3 = 0$

$$k = 4 \qquad\qquad k = -\frac{3}{2}$$

8. (a) $a = 3$, $b = 8$

(b) Luas kawasan yang dibatasi oleh $y = f(x)$ dan paksi-x dari $x = -2$ ke $x = 8$

$$= \left| \int_{-2}^{3} f(x)\ dx \right| + \int_{3}^{8} f(x)\ dx$$

$$13 = \left| \int_{-2}^{3} f(x)\ dx \right| + 5$$

$$\left| \int_{-2}^{3} f(x)\ dx \right| = 8$$

$$\int_{-2}^{3} f(x)\ dx = -8 \text{ (Luas di bawah paksi-}x.\text{)}$$

Praktis Formatif: Kertas 2

1. (a) $y = x + 5$ $\qquad$ ...... ①

$y = \dfrac{1}{4}x^2 + 2$ $\qquad$ ...... ②

Gantikan ① ke dalam ②.

$$x + 5 = \frac{1}{4}x^2 + 2$$

$$\frac{1}{4}x^2 - x - 3 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x + 2)(x - 6) = 0$$

$$x = -2 \text{ atau } x = 6$$

Apabila $x = -2$, $y = -2 + 5$
$\qquad\qquad\qquad = 3$

Koordinat X ialah $(-2, 3)$.

(b) (i) $y = x + 5$
Pada paksi-x, $y = 0$.
$$x + 5 = 0$$
$$x = -5$$

Luas rantau berlorek A

$$= \frac{1}{2}(3)(3) + \int_{-2}^{0} \left(\frac{1}{4}x^2 + 2 \right) dx$$

$$= \frac{9}{2} + \left[\frac{x^3}{12} + 2x \right]_{-2}^{0}$$

$$= \frac{9}{2} + \left[0 - \left(-\frac{8}{12} \right) - 2(-2) \right]$$

$$= 9\frac{1}{6} \text{ unit}^2$$

(ii) $y = \dfrac{1}{4}x^2 + 2$

$$\frac{1}{4}x^2 = y - 2$$

$$x^2 = 4(y - 2)$$

Isi padu janaan

$$= \pi \int_{2}^{3} 4(y - 2)\ dy$$

$$= 4\pi \left[\frac{y^2}{2} - 2y \right]_{2}^{3}$$

$$= 4\pi \left[\left(\frac{9}{2} - 6 \right) - \left(\frac{4}{2} - 4 \right) \right]$$

$$= 2\pi \text{ unit}^3$$

2. (a) $y = \dfrac{4}{x^2} + 2$

$$\frac{dy}{dx} = 4(-2)x^{-3}$$

$$= -\frac{8}{x^3}$$

Apabila $x = 2$, $\dfrac{dy}{dx} = -\dfrac{8}{(2)^3} = -1$

$m = -1$

Gantikan $(2, 3)$ ke dalam $y = mx + c$.
$3 = (-1)(2) + c$
$c = 5$

(b) Persamaan tangen ialah $y = -x + 5$.
Pada paksi-x, $y = 0$.
Maka, $x = 5$.

Luas kawasan berlorek

$$= \int_{2}^{6} \left(\frac{4}{x^2} + 2 \right) dx - \frac{1}{2}(5 - 2)(3)$$

$$= \left[-\frac{4}{x} + 2x \right]_{2}^{6} - \frac{9}{2}$$

$$= \left(-\frac{2}{3} + 12 \right) - (-2 + 4) - \frac{9}{2}$$

$$= 4\frac{5}{6} \text{ unit}^2$$

(c) Isi padu janaan $= \pi \int_{2}^{4} \left(\dfrac{4}{x^2} + 2 \right)^2 dx$

$$= \pi \int_{2}^{4} \left(\frac{16}{x^4} + \frac{16}{x^2} + 4 \right) dx$$

$$= \pi \left[-\frac{16}{3x^3} - \frac{16}{x} + 4x \right]_{2}^{4}$$

$$= \pi \left[\left(\frac{143}{12} \right) - \left(-\frac{2}{3} \right) \right]$$

$$= 12\frac{7}{12}\pi \text{ unit}^3$$

3. (a) $y = 3x^2 - 27$

$$\frac{dy}{dx} = 6x$$

Kecerunan tangen pada titik $P = 6$.
$6x = 6$
$x = 1$
Apabila $x = 1$,
$y = 3 - 27$
$\quad = -24$

Maka, koordinat titik P ialah $(1, -24)$.

(b) Pada paksi-x, $y = 0$.
$3x^2 - 27 = 0$
$x^2 = 9$
$x = \pm 3$

Luas rantau berlorek

$$= \frac{1}{2}(5 - 1)(24) - \left| \int_{1}^{3} (3x^2 - 27)\ dx \right|$$

$$= 48 - \left| \left[x^3 - 27x \right]_{1}^{3} \right|$$

$$= 48 - \left| (27 - 81) - (1 - 27) \right|$$

$$= 48 - 28$$

$$= 20 \text{ unit}^2$$

(c) $y = 3x^2 - 27$

$x^2 = \dfrac{y}{3} + 9$

Isi padu kisaran $= 48\pi$

$\displaystyle\int_k^0 \pi\left(\dfrac{y}{3} + 9\right) dy = 48\pi$

$\left[\dfrac{y^2}{6} + 9y\right]_k^0 = 48$

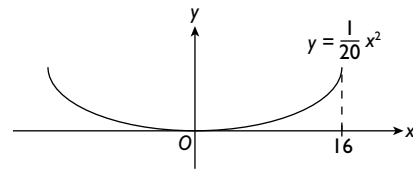$0 - \left(\dfrac{k^2}{6} + 9k\right) = 48$

$-k^2 - 54k = 288$

$k^2 + 54k + 288 = 0$

$(k + 6)(k + 48) = 0$

$k = -6$ atau $k = -48$

Oleh sebab $k > -27$, maka $k = -6$.

FOKUS KBAT

Apabila $x = 16$, $y = \dfrac{1}{20}(16)^2$

$= 12.8$

Isi padu sup yang perlu ditambahkan,

$I = \pi \displaystyle\int_5^{12.8} (20y)\, dy$

$= \pi\left[10y^2\right]_5^{12.8}$

$= \pi\{[10(12.8)^2] - [10(5)^2]\}$

$= 1\,388.4\pi$ cm^3

JAWAPAN

BAB 4: VEKTOR

4.1

A

1. $\overrightarrow{CD} = \underset{\sim}{b}$

2. $\overrightarrow{EF} = \underset{\sim}{x}$

3. $\overrightarrow{HG} = -\underset{\sim}{y}$

4. $\overrightarrow{LK} = -\underset{\sim}{b}$

5. $\overrightarrow{RS} = -\underset{\sim}{x}$

6. $\overrightarrow{MN} = -\underset{\sim}{y}$

B

1. $\overrightarrow{EF} = \underset{\sim}{x}$

2. $\overrightarrow{GH} = \underset{\sim}{b}$

3. $\overrightarrow{PQ} = \underset{\sim}{a}$

4. $\overrightarrow{MN} = \underset{\sim}{c}$

C

1. $\overrightarrow{KL} = \dfrac{1}{2}\underset{\sim}{a}$

$\overrightarrow{MN} = \dfrac{3}{2}\underset{\sim}{a}$

$\overrightarrow{PQ} = -\underset{\sim}{a}$

2. $\overrightarrow{PQ} = \dfrac{1}{2}\underset{\sim}{a}$

$\overrightarrow{RS} = \dfrac{3}{2}\underset{\sim}{a}$

$\overrightarrow{TX} = -\dfrac{1}{2}\underset{\sim}{a}$

D

1. $\overrightarrow{CD}$ dan $\underline{\quad\overrightarrow{RS}\quad}$ ialah vektor selari.

$\overrightarrow{CD} = \dfrac{2}{3}\overrightarrow{RS}$

2. $\overrightarrow{EF}$ dan $\underline{\quad\overrightarrow{GH}\quad}$ ialah vektor selari.

$\overrightarrow{EF} = -2\overrightarrow{GH}$

3. $\overrightarrow{PQ}$ dan $\underline{\quad\overrightarrow{XY}\quad}$ ialah vektor selari.

$\overrightarrow{PQ} = -2\overrightarrow{XY}$

E

1.
$$2m + n - 1 = 0 \quad \text{......} \,①$$
$$3m - 2n + 16 = 0 \quad \text{......} \,②$$
$$① \times 2: \quad 4m + 2n - 2 = 0 \quad \text{......} \,③$$

$$② + ③: \quad 7m + 14 = 0$$
$$7m = -14$$
$$m = -2$$

Gantikan $m = -2$ ke dalam ①.
$$2(-2) + n - 1 = 0$$
$$n = 5$$

2.
$$m + 3n + 2 = 0 \quad \text{......} \,①$$
$$3m + n - 10 = 0 \quad \text{......} \,②$$
$$① \times 3: \quad 3m + 9n + 6 = 0 \quad \text{......} \,③$$

$$② - ③: \quad -8n - 16 = 0$$
$$n = -2$$

Gantikan $n = -2$ ke dalam ①.
$$m + 3(-2) + 2 = 0$$
$$m = 4$$

F

1. $\overrightarrow{AB} = 6\underset{\sim}{x}$

$\quad\quad = \dfrac{2}{7}(21\underset{\sim}{x})$

$\overrightarrow{AB} = \dfrac{2}{7}\overrightarrow{BC}$

AB dan BC adalah selari dan B ialah titik sepunya. Maka, A, B dan C adalah segaris.

2. $\dfrac{XY}{YZ} = \dfrac{8}{12}$

$\quad\quad = \dfrac{2}{3}$

$XY = \dfrac{2}{3}YZ$

Diberi X, Y dan Z adalah segaris. Maka, XY adalah selari dengan YZ dan $\overrightarrow{XY} = \dfrac{2}{3}\overrightarrow{YZ}$.

3. $\dfrac{\left|\overrightarrow{AB}\right|}{\left|\overrightarrow{KL}\right|} = \dfrac{14}{8}$

$\quad\quad = \dfrac{7}{4}$

$\left|\overrightarrow{AB}\right| = \dfrac{7}{4}\left|\overrightarrow{KL}\right|$

Diberi $\overrightarrow{AB}$ dan $\overrightarrow{KL}$ adalah selari.

Maka, $\overrightarrow{AB} = \dfrac{7}{4}\overrightarrow{KL}$.

4.2

A

1. $\underset{\sim}{a} + \dfrac{1}{2}\underset{\sim}{a} + \dfrac{1}{3}\underset{\sim}{a} = \left(1 + \dfrac{1}{2} + \dfrac{1}{3}\right)\underset{\sim}{a}$

$\quad\quad\quad\quad\quad\quad = 1\dfrac{5}{6}\underset{\sim}{a}$

2. $4\underset{\sim}{x} + \dfrac{1}{3}\underset{\sim}{x} + \dfrac{1}{4}\underset{\sim}{x} = \left(4 + \dfrac{1}{3} + \dfrac{1}{4}\right)\underset{\sim}{x}$

$\quad\quad\quad\quad\quad\quad = 4\dfrac{7}{12}\underset{\sim}{x}$

3. $(\underset{\sim}{a} + 2\underset{\sim}{b}) + \left(2\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}\right) = (1 + 2)\underset{\sim}{a} + \left(2 + \frac{3}{4}\right)\underset{\sim}{b}$

$$= 3\underset{\sim}{a} + 2\frac{3}{4}\underset{\sim}{b}$$

4. $(3\underset{\sim}{x} + 4\underset{\sim}{y}) + \left(\frac{1}{2}\underset{\sim}{x} + \frac{2}{3}\underset{\sim}{y}\right) = \left(3 + \frac{1}{2}\right)\underset{\sim}{x} + \left(4 + \frac{2}{3}\right)\underset{\sim}{y}$

$$= 3\frac{1}{2}\underset{\sim}{x} + 4\frac{2}{3}\underset{\sim}{y}$$

B

1. (a) $\underset{\sim}{a} + \underset{\sim}{b} = \overrightarrow{AC}$

(b) $\underset{\sim}{b} + \underset{\sim}{c} = \overrightarrow{BD}$

(c) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(d) $\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$

2. (a) $\underset{\sim}{x} + \underset{\sim}{y} = \overrightarrow{KM}$

(b) $\underset{\sim}{v} + \underset{\sim}{x} = \overrightarrow{OL}$

(c) $\overrightarrow{KL} + \overrightarrow{LM} = \overrightarrow{KM}$

(d) $\overrightarrow{OL} + \overrightarrow{LM} = \overrightarrow{OM}$

C

1. (a) $\overrightarrow{KL} + \overrightarrow{KN} = \overrightarrow{KM}$

(b) $\overrightarrow{LK} + \overrightarrow{LM} = \overrightarrow{LN}$

(c) $\overrightarrow{ML} + \overrightarrow{MN} = \overrightarrow{MK}$

(d) $\overrightarrow{NK} + \overrightarrow{NM} = \overrightarrow{NL}$

2. (a) $\overrightarrow{AB} + \overrightarrow{AF} = \overrightarrow{AO}$

(b) $\overrightarrow{BA} + \overrightarrow{BC} = \overrightarrow{BO}$

(c) $\overrightarrow{OC} + \overrightarrow{OE} = \overrightarrow{OD}$

(d) $\overrightarrow{OA} + \overrightarrow{OE} = \overrightarrow{OF}$

D

1. (a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$

(b) $\overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$

(c) $\overrightarrow{BD} + \overrightarrow{DC} + \overrightarrow{CA} = \overrightarrow{BA}$

2. (a) $\overrightarrow{KL} + \overrightarrow{LN} + \overrightarrow{NO} = \overrightarrow{KO}$

(b) $\overrightarrow{PK} + \overrightarrow{KL} + \overrightarrow{LO} + \overrightarrow{OM} = \overrightarrow{PM}$

(c) $\overrightarrow{MN} + \overrightarrow{NP} + \overrightarrow{PL} + \overrightarrow{LO} = \overrightarrow{MO}$

E

1. $10\underset{\sim}{a} + 6\underset{\sim}{a} - 7\underset{\sim}{a} = (10 + 6 - 7)\underset{\sim}{a}$

$$= 9\underset{\sim}{a}$$

2. $4\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{b} = \left(4 - \frac{1}{2} - \frac{3}{4}\right)\underset{\sim}{b}$

$$= 2\frac{3}{4}\underset{\sim}{b}$$

3. $4\underset{\sim}{x} - 5\underset{\sim}{y} - (2\underset{\sim}{x} + 6\underset{\sim}{y}) = 4\underset{\sim}{x} - 5\underset{\sim}{y} - 2\underset{\sim}{x} - 6\underset{\sim}{y}$

$$= 2\underset{\sim}{x} - 11\underset{\sim}{y}$$

4. $3\underset{\sim}{x} - 2\underset{\sim}{y} - \left(\frac{3}{2}\underset{\sim}{x} - \frac{1}{3}\underset{\sim}{y}\right)$

$$= 3\underset{\sim}{x} - 2\underset{\sim}{y} - \frac{3}{2}\underset{\sim}{x} + \frac{1}{3}\underset{\sim}{y}$$

$$= \frac{3}{2}\underset{\sim}{x} - \frac{5}{3}\underset{\sim}{y}$$

F

1. (a) $\overrightarrow{AB} - \overrightarrow{CB} = \overrightarrow{AB} + \overrightarrow{BC}$

$$= \overrightarrow{AC}$$

(b) $\overrightarrow{AD} - \overrightarrow{CD} = \overrightarrow{AD} + \overrightarrow{DC}$

$$= \overrightarrow{AC}$$

(c) $\overrightarrow{AC} - \overrightarrow{BC} - \overrightarrow{EB} = \overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BE}$

$$= \overrightarrow{AE}$$

2. (a) $\overrightarrow{KL} - \overrightarrow{NL} = \overrightarrow{KL} + \overrightarrow{LN}$

$$= \overrightarrow{KN}$$

(b) $\overrightarrow{KL} - \overrightarrow{OL} - \overrightarrow{PO} = \overrightarrow{KL} + \overrightarrow{LO} + \overrightarrow{OP}$

$$= \overrightarrow{KP}$$

(c) $\overrightarrow{PM} - \overrightarrow{LM} - \overrightarrow{KL} = \overrightarrow{PM} + \overrightarrow{ML} + \overrightarrow{LK}$

$$= \overrightarrow{PK}$$

G

1. (a) $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$

$$= \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AD}\right)$$

$$= \frac{1}{2}(4\underset{\sim}{x} + 6\underset{\sim}{y})$$

$$= 2\underset{\sim}{x} + 3\underset{\sim}{y}$$

(b) $\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

$$= \overrightarrow{AB} + \frac{2}{3}\overrightarrow{BC}$$

$$= \overrightarrow{AB} + \frac{2}{3}\overrightarrow{AD}$$

$$= 4\underset{\sim}{x} + \frac{2}{3}(6\underset{\sim}{y})$$

$$= 4\underset{\sim}{x} + 4\underset{\sim}{y}$$

(c) $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF}$

$$= -2\underset{\sim}{x} - 3\underset{\sim}{y} + 4\underset{\sim}{x} + 4\underset{\sim}{y}$$

$$= 2\underset{\sim}{x} + \underset{\sim}{y}$$

2. (a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$= \overrightarrow{AB} + \frac{1}{3}\overrightarrow{AD}$$

$$= 4\underset{\sim}{x} + \frac{1}{3}(9\underset{\sim}{y})$$

$$= 4\underset{\sim}{x} + 3\underset{\sim}{y}$$

(b) $\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CE}$

$\qquad = \overrightarrow{AC} + \dfrac{1}{3}\overrightarrow{CD}$

$\qquad = 4\underset{\sim}{x} + 3\underset{\sim}{y} + \dfrac{1}{3}(\overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AD})$

$\qquad = 4\underset{\sim}{x} + 3\underset{\sim}{y} + \dfrac{1}{3}(-3\underset{\sim}{y} - 4\underset{\sim}{x} + 9\underset{\sim}{y})$

$\qquad = 4\underset{\sim}{x} + 3\underset{\sim}{y} - \dfrac{4}{3}\underset{\sim}{x} + 2\underset{\sim}{y}$

$\qquad = \dfrac{8}{3}\underset{\sim}{x} + 5\underset{\sim}{y}$

H

1. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\qquad = (\mu\underset{\sim}{a} + 5\underset{\sim}{b}) - (3\underset{\sim}{a} - 2\underset{\sim}{b})$

$\qquad = (\mu - 3)\underset{\sim}{a} + 7\underset{\sim}{b}$

$\quad \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$\qquad = \underset{\sim}{a} + 4\underset{\sim}{b} - (3\underset{\sim}{a} - 2\underset{\sim}{b})$

$\qquad = -2\underset{\sim}{a} + 6\underset{\sim}{b}$

(b) A, B dan C adalah segaris.

$\qquad \overrightarrow{AB} = \lambda\overrightarrow{AC}$

$(\mu - 3)\underset{\sim}{a} + 7\underset{\sim}{b} = \lambda(-2\underset{\sim}{a} + 6\underset{\sim}{b})$

$\qquad\qquad 7 = 6\lambda$

$\qquad\qquad \lambda = \dfrac{7}{6} \qquad \cdots\cdots ①$

$\qquad \mu - 3 = -2\lambda \qquad \cdots\cdots ②$

Gantikan ① ke dalam ②. $\mu - 3 = -2\left(\dfrac{7}{6}\right)$

$\qquad\qquad\qquad\qquad\qquad \mu = 3 - \dfrac{7}{3}$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{2}{3}$

2. (a) $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$

$\qquad = -24\underset{\sim}{x} + 20\underset{\sim}{y}$

$\quad \overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

$\qquad = \overrightarrow{AB} + \dfrac{3}{4}\overrightarrow{BD}$

$\qquad = 24\underset{\sim}{x} + \dfrac{3}{4}(-24\underset{\sim}{x} + 20\underset{\sim}{y})$

$\qquad = 24\underset{\sim}{x} - 18\underset{\sim}{x} + 15\underset{\sim}{y}$

$\qquad = 6\underset{\sim}{x} + 15\underset{\sim}{y}$

(b) $\overrightarrow{DC} = \dfrac{4}{3}\overrightarrow{AB}$

$\qquad = \dfrac{4}{3}(24\underset{\sim}{x})$

$\qquad = 32\underset{\sim}{x}$

$\quad \overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$

$\qquad = -24\underset{\sim}{x} + 20\underset{\sim}{y} + 32\underset{\sim}{x}$

$\qquad = 8\underset{\sim}{x} + 20\underset{\sim}{y}$

$\qquad = \dfrac{4}{3}(6\underset{\sim}{x} + 15\underset{\sim}{y})$

$\quad \overrightarrow{BC} = \dfrac{4}{3}\overrightarrow{AE}$

Maka, AE adalah selari dengan BC.

4.3

A

1. $\underset{\sim}{a} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} = 3\underset{\sim}{j}$

$\quad |\underset{\sim}{a}| = 3$ unit

2. $\underset{\sim}{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} = -4\underset{\sim}{i}$

$\quad |\underset{\sim}{b}| = 4$ unit

3. $\underset{\sim}{c} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$

$\quad |\underset{\sim}{c}| = \sqrt{(-2)^2 + 2^2}$

$\qquad = \sqrt{8} = 2\sqrt{2}$ unit

4. $\underset{\sim}{d} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = -2\underset{\sim}{i} - 3\underset{\sim}{j}$

$\quad |\underset{\sim}{d}| = \sqrt{(-2)^2 + (-3)^2}$

$\qquad = \sqrt{13}$ unit

5. $\underset{\sim}{e} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3\underset{\sim}{i} + 3\underset{\sim}{j}$

$\quad |\underset{\sim}{e}| = \sqrt{3^2 + 3^2}$

$\qquad = \sqrt{18} = 3\sqrt{2}$ unit

6. $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$

$\quad |\overrightarrow{AB}| = \sqrt{3^2 + (-4)^2}$

$\qquad = 5$ unit

7. $\overrightarrow{CD} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} = -4\underset{\sim}{i} + \underset{\sim}{j}$

$\quad |\overrightarrow{CD}| = \sqrt{(-4)^2 + 1^2}$

$\qquad = \sqrt{17}$ unit

8. $\overrightarrow{EF} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} = -3\underset{\sim}{i} + \underset{\sim}{j}$

$\quad |\overrightarrow{EF}| = \sqrt{(-3)^2 + 1^2}$

$\qquad = \sqrt{10}$ unit

B

1. $|\underset{\sim}{b}| = \sqrt{5^2 + (-12)^2}$

$\qquad = 13$ unit

$\quad \hat{\underset{\sim}{b}} = \dfrac{1}{13}(5\underset{\sim}{i} - 12\underset{\sim}{j})$

$\qquad = \dfrac{5}{13}\underset{\sim}{i} - \dfrac{12}{13}\underset{\sim}{j}$

2. $|\underset{\sim}{u}| = \sqrt{(-1)^2 + 3^2}$

$\qquad = \sqrt{10}$ unit

$\quad \hat{\underset{\sim}{u}} = \dfrac{1}{\sqrt{10}}(-\underset{\sim}{i} + 3\underset{\sim}{j})$

$\qquad = -\dfrac{1}{\sqrt{10}}\underset{\sim}{i} + \dfrac{3}{\sqrt{10}}\underset{\sim}{j}$

3. $|\underset{\sim}{v}| = \sqrt{(-4)^2 + (-2)^2}$

$\quad\quad = \sqrt{20}$

$\quad\quad = 2\sqrt{5}\text{ unit}$

$\quad\hat{\underset{\sim}{v}} = \dfrac{1}{2\sqrt{5}}(-4\underset{\sim}{i} - 2\underset{\sim}{j})$

$\quad\quad = -\dfrac{2}{\sqrt{5}}\underset{\sim}{i} - \dfrac{1}{\sqrt{5}}\underset{\sim}{j}$

C

1. $\underset{\sim}{u} + 2\underset{\sim}{v} - \underset{\sim}{w} = 3\underset{\sim}{i} + 5\underset{\sim}{j} + 2(2\underset{\sim}{i} - \underset{\sim}{j}) - (-4\underset{\sim}{i} + 2\underset{\sim}{j})$

$\quad\quad = 3\underset{\sim}{i} + 5\underset{\sim}{j} + 4\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{i} - 2\underset{\sim}{j}$

$\quad\quad = 11\underset{\sim}{i} + \underset{\sim}{j}$

2. $2\underset{\sim}{u} - \underset{\sim}{v} + 4\underset{\sim}{w}$

$\quad = 2(3\underset{\sim}{i} + 5\underset{\sim}{j}) - (2\underset{\sim}{i} - \underset{\sim}{j}) + 4(-4\underset{\sim}{i} + 2\underset{\sim}{j})$

$\quad = 6\underset{\sim}{i} + 10\underset{\sim}{j} - 2\underset{\sim}{i} + \underset{\sim}{j} - 16\underset{\sim}{i} + 8\underset{\sim}{j}$

$\quad = -12\underset{\sim}{i} + 19\underset{\sim}{j}$

3. $2\underset{\sim}{u} + 3\underset{\sim}{v} - \dfrac{1}{2}\underset{\sim}{w}$

$\quad = 2(3\underset{\sim}{i} + 5\underset{\sim}{j}) + 3(2\underset{\sim}{i} - \underset{\sim}{j}) - \dfrac{1}{2}(-4\underset{\sim}{i} + 2\underset{\sim}{j})$

$\quad = 6\underset{\sim}{i} + 10\underset{\sim}{j} + 6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{i} - \underset{\sim}{j}$

$\quad = 14\underset{\sim}{i} + 6\underset{\sim}{j}$

D

1. $\underset{\sim}{p} + 2\underset{\sim}{q} + 3\underset{\sim}{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\quad\quad = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 8 \end{pmatrix} + \begin{pmatrix} 15 \\ -3 \end{pmatrix}$

$\quad\quad = \begin{pmatrix} 16 \\ 7 \end{pmatrix}$

2. $2\underset{\sim}{p} + 4\underset{\sim}{q} - \underset{\sim}{r} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} + 4\begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\quad\quad = \begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 16 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\quad\quad = \begin{pmatrix} -3 \\ 21 \end{pmatrix}$

3. $3\underset{\sim}{p} - 2\underset{\sim}{q} - \underset{\sim}{r} = 3\begin{pmatrix} 3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\quad\quad = \begin{pmatrix} 9 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\quad\quad = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

E

1. (a) $\overrightarrow{OA} + 3\overrightarrow{OB} + 2\overrightarrow{OC} = \underset{\sim}{0}$

$\quad\quad 2\overrightarrow{OC} = -\overrightarrow{OA} - 3\overrightarrow{OB}$

$\quad\quad\quad = -(4\underset{\sim}{i} - \underset{\sim}{j}) - 3(-2\underset{\sim}{i} + 3\underset{\sim}{j})$

$\quad\quad\quad = -4\underset{\sim}{i} + \underset{\sim}{j} + 6\underset{\sim}{i} - 9\underset{\sim}{j}$

$\quad\quad\quad = 2\underset{\sim}{i} - 8\underset{\sim}{j}$

$\quad\quad \overrightarrow{OC} = \underset{\sim}{i} - 4\underset{\sim}{j}$

(b) $|\overrightarrow{OC}| = \sqrt{1^2 + (-4)^2} = \sqrt{17}$ unit

(c) Vektor unit dalam arah $\overrightarrow{OC}$

$\quad = \dfrac{1}{\sqrt{17}}(\underset{\sim}{i} - 4\underset{\sim}{j})$

$\quad = \dfrac{1}{\sqrt{17}}\underset{\sim}{i} - \dfrac{4}{\sqrt{17}}\underset{\sim}{j}$

2. (a) $\overrightarrow{OA} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$, $\overrightarrow{OB} = 5\underset{\sim}{i} - 6\underset{\sim}{j}$

$\quad\quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\quad\quad\quad = (5\underset{\sim}{i} - 6\underset{\sim}{j}) - (2\underset{\sim}{i} + 3\underset{\sim}{j})$

$\quad\quad\quad = 3\underset{\sim}{i} - 9\underset{\sim}{j}$

(b) $AP : PB = 2 : 1$

$\quad\quad \overrightarrow{AP} = \dfrac{2}{3}\overrightarrow{AB}$

$\quad\quad \overrightarrow{OP} - \overrightarrow{OA} = \dfrac{2}{3}\overrightarrow{AB}$

$\quad\quad\quad \overrightarrow{OP} = \dfrac{2}{3}(3\underset{\sim}{i} - 9\underset{\sim}{j}) + \overrightarrow{OA}$

$\quad\quad\quad\quad = 2\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{i} + 3\underset{\sim}{j}$

$\quad\quad\quad\quad = 4\underset{\sim}{i} - 3\underset{\sim}{j}$

(c) $|\overrightarrow{OP}| = \sqrt{4^2 + (-3)^2}$

$\quad\quad = 5$ unit

Vektor unit dalam arah $\overrightarrow{OP}$

$\quad = \dfrac{1}{5}(4\underset{\sim}{i} - 3\underset{\sim}{j})$

$\quad = \dfrac{4}{5}\underset{\sim}{i} - \dfrac{3}{5}\underset{\sim}{j}$

Praktis Formatif: Kertas 1

1. $\quad \underset{\sim}{a} = k\underset{\sim}{b}$

$\quad \begin{pmatrix} 5 \\ 6 \end{pmatrix} = k\begin{pmatrix} 10 \\ m - 2 \end{pmatrix}$

$\quad\quad 5 = 10k$

$\quad\quad k = \dfrac{1}{2}$

$\quad\quad 6 = k(m - 2)$

$\quad\quad 6 = \dfrac{1}{2}(m - 2)$

$\quad 12 = m - 2$

$\quad m = 14$

2. (a) $|-\overrightarrow{QP}| = \sqrt{4^2 + 6^2}$

$\quad\quad\quad = \sqrt{52}$

$\quad\quad\quad = 2\sqrt{13}$ unit

(b) (i) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$\quad\quad\quad = -\overrightarrow{PQ} + \overrightarrow{PR}$

$\quad\quad\quad = -\underset{\sim}{p} + \underset{\sim}{q}$

(ii) $2\overrightarrow{PR} = \overrightarrow{PS} + \overrightarrow{PQ}$

$\quad\quad \overrightarrow{PS} = 2\overrightarrow{PR} - \overrightarrow{PQ}$

$\quad\quad\quad = 2\underset{\sim}{q} - \underset{\sim}{p}$

3. (a) $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$
$= 2\underset{\sim}{a} - 8\underset{\sim}{b}$

(b) $\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$

$= \frac{1}{4}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

$= \frac{1}{4}(8\underset{\sim}{b}) + \frac{1}{2}(2\underset{\sim}{a} - 8\underset{\sim}{b})$

$= \underset{\sim}{a} - 2\underset{\sim}{b}$

4. (a) $\overrightarrow{OA} = 7\underset{\sim}{i} + 13\underset{\sim}{j}$

(b) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 7 \\ 13 \end{pmatrix}$

$= \begin{pmatrix} -11 \\ -10 \end{pmatrix}$

5. A, B dan C terletak pada satu garis lurus.
$$\overrightarrow{AB} = m\overrightarrow{BC}$$
$$\overrightarrow{OB} - \overrightarrow{OA} = m\left(\overrightarrow{OC} - \overrightarrow{OB}\right)$$
$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} - \begin{pmatrix} h \\ 5 \end{pmatrix} = m\left[\begin{pmatrix} k \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}\right]$$
$$\begin{pmatrix} 1 - h \\ -1 \end{pmatrix} = m\begin{pmatrix} k - 1 \\ -7 \end{pmatrix}$$
$$\frac{1 - h}{-1} = \frac{k - 1}{-7}$$
$$7 - 7h = k - 1$$
$$7h = 8 - k$$
$$h = \frac{8 - k}{7}$$

6. (a) $|\underset{\sim}{a}| = \sqrt{(-5)^2 + 12^2}$
$= \sqrt{25 + 144}$
$= 13$ unit

(b) $\underset{\sim}{a} + \underset{\sim}{b} = \begin{pmatrix} -5 \\ 12 \end{pmatrix} + \begin{pmatrix} 3 \\ k \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 12 + k \end{pmatrix}$

$12 + k = 0$
$k = -12$

7. (a) $\overrightarrow{PR} + \overrightarrow{RT} + \overrightarrow{RQ} = \overrightarrow{PT} + \overrightarrow{RQ}$
$= \overrightarrow{QS} + \overrightarrow{RQ}$
$= \overrightarrow{RQ} + \overrightarrow{QS}$
$= \overrightarrow{RS}$

(b) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \underset{\sim}{q} - \underset{\sim}{p}$

$|\overrightarrow{PQ}| = 5$ unit

Vektor unit dalam arah $\overrightarrow{PQ} = \frac{1}{5}(\underset{\sim}{q} - \underset{\sim}{p})$

Praktis Formatif: Kertas 2

1. (a) (i) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$
$= -\overrightarrow{AB} + \overrightarrow{AC}$
$= -12\underset{\sim}{y} + 8\underset{\sim}{x}$

(ii) $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$
$= \overrightarrow{AB} + \frac{1}{4}\overrightarrow{BC}$
$= 12\underset{\sim}{y} + \frac{1}{4}(-12\underset{\sim}{y} + 8\underset{\sim}{x})$
$= 2\underset{\sim}{x} + 9\underset{\sim}{y}$

(b) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = h\overrightarrow{AD}$
$12\underset{\sim}{y} + k(4\underset{\sim}{x} - 9\underset{\sim}{y}) = h(2\underset{\sim}{x} + 9\underset{\sim}{y})$
$4k\underset{\sim}{x} + (12 - 9k)\underset{\sim}{y} = 2h\underset{\sim}{x} + 9h\underset{\sim}{y}$
$4k = 2h$
$h = 2k$ ...... ①
$12 - 9k = 9h$ ...... ②
Gantikan ① ke dalam ②.
$12 - 9k = 9(2k)$
$27k = 12$
$k = \frac{4}{9}$

Gantikan $k = \frac{4}{9}$ ke dalam ①.
$h = 2\left(\frac{4}{9}\right)$
$= \frac{8}{9}$

(c) Biarkan $\overrightarrow{AF} = n\overrightarrow{AD}$
$m\underset{\sim}{x} + 12\underset{\sim}{y} = n(2\underset{\sim}{x} + 9\underset{\sim}{y})$
$12 = 9n$
$n = \frac{4}{3}$
$m = 2n$
$= 2\left(\frac{4}{3}\right)$
$= \frac{8}{3}$

2. (a) (i) $\overrightarrow{CP} = \overrightarrow{CA} + \overrightarrow{AP}$
$= -6\underset{\sim}{y} + 4\underset{\sim}{x}$

(ii) $\overrightarrow{CR} = \frac{1}{4}\overrightarrow{CB}$
$= \frac{1}{4}\left(\overrightarrow{AB} - \overrightarrow{AC}\right)$
$= \frac{1}{4}[4(4\underset{\sim}{x}) - 6\underset{\sim}{y}]$
$= 4\underset{\sim}{x} - \frac{3}{2}\underset{\sim}{y}$

(b) $\overrightarrow{CR} = 4(3\underset{\sim}{i} - \underset{\sim}{j}) - \frac{3}{2}(4\underset{\sim}{i})$
$= 6\underset{\sim}{i} - 4\underset{\sim}{j}$
$|\overrightarrow{CR}| = \sqrt{6^2 + (-4)^2}$
$= \sqrt{52}$
$= 2\sqrt{13}$ unit

(c) $\quad\overrightarrow{CQ} = \lambda\overrightarrow{CP},\ \overrightarrow{QR} = \mu\overrightarrow{AR}$

$$\overrightarrow{CR} = \overrightarrow{CQ} + \overrightarrow{QR}$$
$$= \lambda\overrightarrow{CP} + \mu\overrightarrow{AR}$$
$$4\underset{\sim}{x} - \frac{3}{2}\underset{\sim}{y} = \lambda(4\underset{\sim}{x} - 6\underset{\sim}{y}) + \mu(\overrightarrow{AC} + \overrightarrow{CR})$$
$$= 4\lambda\underset{\sim}{x} - 6\lambda\underset{\sim}{y} + \mu\left(6\underset{\sim}{y} + 4\underset{\sim}{x} - \frac{3}{2}\underset{\sim}{y}\right)$$
$$= (4\lambda + 4\mu)\underset{\sim}{x} + \left(\frac{9}{2}\mu - 6\lambda\right)\underset{\sim}{y}$$

Bandingkan vektor-vektor.
$$4 = 4\lambda + 4\mu$$
$$\lambda + \mu = 1$$
$$\lambda = 1 - \mu \qquad \text{...... } ①$$
$$-\frac{3}{2} = \frac{9}{2}\mu - 6\lambda \qquad \text{...... } ②$$

Gantikan ① ke dalam ②.
$$-\frac{3}{2} = \frac{9}{2}\mu - 6(1 - \mu)$$
$$\frac{21}{2}\mu = \frac{9}{2}$$
$$\mu = \frac{3}{7}$$

Dari ①, $\lambda = 1 - \dfrac{3}{7}$
$$= \frac{4}{7}$$

3. (a) Halaju paduan kano $P = (3\underset{\sim}{i} - \underset{\sim}{j}) + \left(\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j}\right)$
$$= 4\underset{\sim}{i} - \frac{4}{3}\underset{\sim}{j}$$

Halaju paduan kano Q
$$= (9\underset{\sim}{i} - 3\underset{\sim}{j}) + \left(\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j}\right)$$
$$= 10\underset{\sim}{i} - \frac{10}{3}\underset{\sim}{j}$$
$$= \frac{10}{4}\left(4\underset{\sim}{i} - \frac{4}{3}\underset{\sim}{j}\right)$$
$$= \frac{5}{2} \times \text{Halaju paduan kano } P$$

(b) (i) Halaju paduan kano R
$$= 2\underset{\sim}{i} - \frac{8}{3}\underset{\sim}{j} + \left(\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j}\right)$$
$$= 3\underset{\sim}{i} - 3\underset{\sim}{j}$$

(ii) $\quad\underset{\sim}{r} = 2\underset{\sim}{i} - \frac{8}{3}\underset{\sim}{j}$
$$|\underset{\sim}{r}| = \sqrt{2^2 + \left(-\frac{8}{3}\right)^2}$$
$$= \sqrt{\frac{100}{9}}$$
$$= \frac{10}{3}$$

Vektor unit dalam arah kano R
$$= \frac{2\underset{\sim}{i} - \frac{8}{3}\underset{\sim}{j}}{\frac{10}{3}}$$
$$= \frac{3}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$$

FOKUS KBAT

(a) (i) $\quad\overrightarrow{LS} = \overrightarrow{LB} + \overrightarrow{BS}$
$$= -12\underset{\sim}{x} + (16\underset{\sim}{x} + 10\underset{\sim}{y})$$
$$= 4\underset{\sim}{x} + 10\underset{\sim}{y}$$

(ii) $BL : LP = 3 : 1$ dan $PC : CS = 1 : 1$
$$\overrightarrow{BC} = \overrightarrow{BP} + \overrightarrow{PC}$$
$$= 16\underset{\sim}{x} + \frac{1}{2}\overrightarrow{PS}$$
$$= 16\underset{\sim}{x} + \frac{1}{2}(\overrightarrow{PB} + \overrightarrow{BS})$$
$$= 16\underset{\sim}{x} + \frac{1}{2}[-16\underset{\sim}{x} + (16\underset{\sim}{x} + 10\underset{\sim}{y})]$$
$$= 16\underset{\sim}{x} + 5\underset{\sim}{y}$$

(b) Biarkan $\overrightarrow{BT} = \lambda\overrightarrow{BC}$ dan $\overrightarrow{LT} = k\overrightarrow{LS}$.
$$\overrightarrow{BT} = \overrightarrow{BL} + \overrightarrow{LT}$$
$$\lambda\overrightarrow{BC} = 12\underset{\sim}{x} + k\overrightarrow{LS}$$
$$\lambda(16\underset{\sim}{x} + 5\underset{\sim}{y}) = 12\underset{\sim}{x} + k(4\underset{\sim}{x} + 10\underset{\sim}{y})$$
$$16\lambda\underset{\sim}{x} + 5\lambda\underset{\sim}{y} = (12 + 4k)\underset{\sim}{x} + 10k\underset{\sim}{y}$$

Banding kedua-dua belah persamaan.
$$16\lambda = 12 + 4k \quad \text{...... } ①$$
$$5\lambda = 10k$$
$$k = \frac{1}{2}\lambda \qquad \text{...... } ②$$

Gantikan ② ke dalam ①.
$$16\lambda = 12 + 4\left(\frac{1}{2}\lambda\right)$$
$$\lambda = \frac{6}{7}$$

Maka, $\overrightarrow{BT} = \dfrac{6}{7}\overrightarrow{BC}$.

$BT : BC = 6 : 7$
Dengan itu, $BT : TC = 6 : 1$.

BAB 5: FUNGSI TRIGONOMETRI

5.1

A
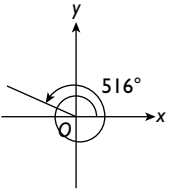
1. Sudut positif = 180° + 47° = 227°
   Sudut negatif = –(180° – 47°) = –133°
   Sukuan = III

2. Sudut positif = 270° + 58° = 328°
   Sudut negatif = –(90° – 58°) = –32°
   Sukuan = IV

3. Sudut positif = $\pi + \dfrac{5}{6}\pi = \dfrac{11}{6}\pi$

   Sudut negatif = $-\left(\pi - \dfrac{5}{6}\pi\right) = -\dfrac{1}{6}\pi$

   Sukuan = IV

4. Sudut positif = $\dfrac{1}{2}\pi + \dfrac{1}{4}\pi = \dfrac{3}{4}\pi$

   Sudut negatif = $-\left(\dfrac{1}{2}\pi + \dfrac{3}{4}\pi\right) = -\dfrac{5}{4}\pi$

   Sukuan = II

B

1. $\alpha = 90° - 56° = 34°$
2. $\alpha = 222° - 180° = 42°$
3. $\alpha = \dfrac{1}{2}\pi - \left(\dfrac{4}{3}\pi - \pi\right) = \dfrac{1}{6}\pi$

C
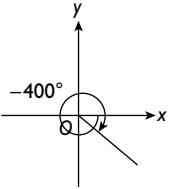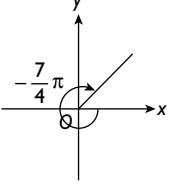
1.
   Sukuan = II
   $\alpha = 540° - 516°$
   $= 24°$

2.
   Sukuan = IV
   $\alpha = 400° - 360°$
   $= 40°$

3.
   Sukuan = I
   $\alpha = 2\pi - \dfrac{7}{4}\pi$
   $= \dfrac{1}{4}\pi$

D

1. Sukuan I
   $\theta = 65°, 360° + 65°$
   $= 65°, 425°$

   Sukuan II
   $\theta = 180° - 65°, 360° + 115°$
   $= 115°, 475°$

   Sukuan III
   $\theta = 180° + 65°, 360° + 245°$
   $= 245°, 605°$

   Sukuan IV
   $\theta = 360° - 65°, 360° + 295°$
   $= 295°, 655°$
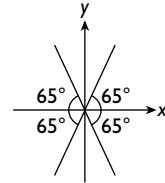
2. Sukuan I
   $\theta = \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6} = \dfrac{\pi}{6}, \dfrac{13\pi}{6}$

   Sukuan II
   $\theta = \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{5\pi}{6} = \dfrac{5\pi}{6}, \dfrac{17\pi}{6}$

   Sukuan III
   $\theta = \pi + \dfrac{\pi}{6}, 2\pi + \dfrac{7\pi}{6} = \dfrac{7\pi}{6}, \dfrac{19\pi}{6}$

   Sukuan IV
   $\theta = 2\pi - \dfrac{\pi}{6}, 2\pi + \dfrac{11\pi}{6} = \dfrac{11\pi}{6}, \dfrac{23\pi}{6}$

5.2

A
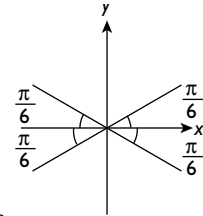
1. (a) $\text{kos } 42° = \dfrac{1}{\text{sek } 42°}$

   $= \dfrac{1}{1.3456}$

   $= 0.7432$

   (b) $\tan 42° = \dfrac{\sin 42°}{\text{kos } 42°}$

   $= \dfrac{0.6691}{0.7432}$

   $= 0.9003$

2. (a) $\sin 200° = \dfrac{1}{\text{kosek } 200°}$

   $= \dfrac{1}{-2.9238}$

   $= -0.3420$

   (b) $\text{kos } 200° = \dfrac{\sin 200°}{\tan 200°}$ ← $\tan 200° = \dfrac{\sin 200°}{\text{kos } 200°}$

   $= \sin 200° \times \text{kot } 200°$
   $= (-0.3420) \times (2.7475)$
   $= -0.9396$

B

1. $\text{kosek } \theta + \text{sek } \theta = \dfrac{1}{\sin \theta} + \dfrac{1}{\text{kos } \theta}$

$\qquad\qquad\qquad\quad = \left(-\dfrac{10}{6}\right) + \left(-\dfrac{10}{8}\right)$

$\qquad\qquad\qquad\quad = -\dfrac{35}{12}$

2. $\dfrac{2}{\tan \theta} - \dfrac{5}{\text{kot } \theta} = \dfrac{2}{\frac{6}{8}} - \dfrac{5}{\frac{8}{6}}$

$\qquad\qquad\qquad = \dfrac{8}{3} - \dfrac{15}{4}$

$\qquad\qquad\qquad = -\dfrac{13}{12}$

3. $\text{sek } \theta \times \dfrac{1}{3 \sin \theta} = \dfrac{1}{\text{kos } \theta} \times \dfrac{1}{3 \sin \theta}$

$\qquad\qquad\qquad\quad = \left(-\dfrac{10}{8}\right) \times \dfrac{1}{3}\left(-\dfrac{10}{6}\right)$

$\qquad\qquad\qquad\quad = \dfrac{25}{36}$

4. $\tan \theta \times \dfrac{5}{\text{kosek } \theta} - \dfrac{7}{3} = \dfrac{6}{8} \times 5\left(-\dfrac{3}{5}\right) - \dfrac{7}{3}$

$\qquad\qquad\qquad\qquad\qquad = -\dfrac{9}{4} - \dfrac{7}{3}$

$\qquad\qquad\qquad\qquad\qquad = -\dfrac{55}{12}$

C

1. $\sin 30° = \text{kos } (90° - 30°)$

$\qquad\quad = \text{kos } 60°$

$\qquad\quad = 0.5$

2. $2 \text{ kot } 45° + 5 \sin 30° = 2 \tan 45° + 5 \text{ kos } 60°$

$\qquad\qquad\qquad\qquad\quad = 2(1) + 5(0.5)$

$\qquad\qquad\qquad\qquad\quad = 4.5$

3. $\text{kosek } 30° + 2 \text{ sek } 60° + \dfrac{3}{\text{sek } 60°}$

$= \text{sek } 60° + 2 \text{ sek } 60° + 3 \text{ kos } 60°$

$= \dfrac{1}{0.5} + 2\left(\dfrac{1}{0.5}\right) + 3(0.5)$

$= 7.5$

D

1. $\sqrt{5} \text{ sek } (90° - \theta) + \text{kosek } (90° - \theta)$

$= \sqrt{5} \text{ kosek } \theta + \text{sek } \theta$

$= \sqrt{5}\left(\dfrac{1}{\sin \theta}\right) + \dfrac{1}{\text{kos } \theta}$

$= \sqrt{5} \times \dfrac{3}{\sqrt{5}} + \dfrac{3}{2}$

$= \dfrac{9}{2}$

2. $\dfrac{\text{kot } (90° - \theta) + \tan (90° - \theta)}{\text{kot } (90° - \theta) - \tan (90° - \theta)}$

$= \dfrac{\tan \theta + \text{kot } \theta}{\tan \theta - \text{kot } \theta}$

$= \dfrac{\frac{\sqrt{5}}{2} + \frac{2}{\sqrt{5}}}{\frac{\sqrt{5}}{2} - \frac{2}{\sqrt{5}}} = \dfrac{\frac{5+4}{2\sqrt{5}}}{\frac{5-4}{2\sqrt{5}}} = 9$

E

1. $\dfrac{\sin 34° \sin 22°}{2 \tan 22° \text{ kos } 22° \text{ kos } 56°}$

$= \dfrac{\text{kos } 56° \sin 22°}{2 \tan 22° \text{ kos } 22° \text{ kos } 56°}$

$= \dfrac{\sin 22°}{2 \tan 22° \text{ kos } 22°}$

$= \dfrac{\tan 22°}{2 \tan 22°}$

$= \dfrac{1}{2}$

2. $\dfrac{8 \sin \frac{\pi}{5}}{\sin \frac{\pi}{5} + \text{kos} \frac{3\pi}{10}}$

$= \dfrac{8 \sin \frac{\pi}{5}}{\sin \frac{\pi}{5} + \sin\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)}$

$= \dfrac{8 \sin \frac{\pi}{5}}{\sin \frac{\pi}{5} + \sin \frac{\pi}{5}}$

$= \dfrac{8 \sin \frac{\pi}{5}}{2 \sin \frac{\pi}{5}} = 4$

F

1. $\text{kos } (-662°) = \text{kos } 662°$

$\qquad\qquad\qquad = \text{kos } 58°$

$\qquad\qquad\qquad = 0.5299$

2. $\tan (-932°) = -\tan 932°$

$\qquad\qquad\qquad = -\tan 212°$

$\qquad\qquad\qquad = -(\tan 32°)$

$\qquad\qquad\qquad = -0.6249$

3. $\text{kosek } (-198°) = \dfrac{1}{\sin (-198°)}$

$\qquad\qquad\qquad\quad = \dfrac{1}{-\sin 198°}$

$\qquad\qquad\qquad\quad = \dfrac{1}{-(-\sin 18°)}$

$\qquad\qquad\qquad\quad = \dfrac{1}{\sin 18°} = 3.2361$

4. $\cot\left(-\dfrac{5}{3}\pi\right) = \cot\left(-\dfrac{5}{3}\pi \times \dfrac{180°}{\pi}\right)$

$= \cot(-300°)$

$= \dfrac{1}{\tan(-300°)}$

$= \dfrac{1}{-\tan 300°}$

$= \dfrac{1}{-(-\tan 60°)}$

$= \dfrac{1}{\tan 60°} = 0.5774$

5. $\sec\left(-\dfrac{1}{3}\pi\right) = \sec\left(-\dfrac{1}{3}\pi \times \dfrac{180°}{\pi}\right)$

$= \sec(-60°)$

$= \dfrac{1}{\kos(-60°)}$

$= \dfrac{1}{\kos 60°} = 2$

6. $\kosek\left(-\dfrac{9}{8}\pi\right) = \kosek\left(-\dfrac{9}{8}\pi \times \dfrac{180°}{\pi}\right)$

$= \kosek(-202.5°)$

$= \dfrac{1}{\sin(-202.5°)}$

$= \dfrac{1}{-\sin 202.5°}$

$= \dfrac{1}{-(-\sin 22.5°)}$

$= \dfrac{1}{\sin 22.5°} = 2.6131$

G

1. $\kos 210° = -\kos(210° - 180°)$

$= -\kos 30°$

$= -\dfrac{\sqrt{3}}{2}$

2. $\sin(-225°) = -\sin 225°$

$= \sin(225° - 180°)$

$= \sin 45°$

$= \dfrac{1}{\sqrt{2}}$

3. $\cot(-405°) = \dfrac{1}{\tan(-405°)}$

$= \dfrac{1}{-\tan 405°}$

$= \dfrac{1}{-\tan(405° - 360°)}$

$= \dfrac{1}{-\tan 45°} = -1$

4. $\kos\left(-\dfrac{5}{6}\pi\right) = \kos\left(-\dfrac{5}{6}\pi \times \dfrac{180°}{\pi}\right)$

$= \kos(-150°)$

$= \kos 150°$

$= -\kos(180° - 150°)$

$= -\kos 30°$

$= -\dfrac{\sqrt{3}}{2}$

H

1. Sudut rujukan $= 36°\,48'$

$x = 180° - 36°\,48'$,

$\qquad 360° - 36°\,48'$

$\quad = 143°\,12',\ 323°\,12'$
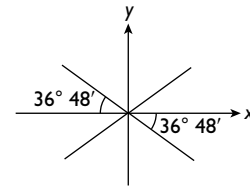
2. $\sin 2x = -0.5$

Sudut rujukan $= 30°$

$0° \leqslant x \leqslant 360°$,

$0° \leqslant 2x \leqslant 720°$
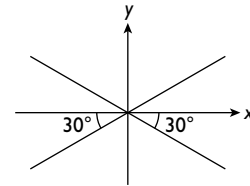
$2x = 180° + 30°,\ 360° - 30°$

$\qquad 540° + 30°,\ 720° - 30°$

$\quad = 210°,\ 330°,\ 570°,\ 690°$

$x = 105°,\ 165°,\ 285°,\ 345°$

I

1. $\tan x + 2 = -0.5$

$\qquad \tan x = -2.5$

Sudut rujukan $= 68°\,12'$
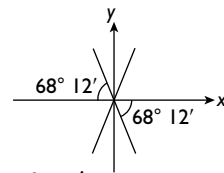
$x = 180° - 68°\,12',\ 360° - 68°\,12'$

$\quad = 111°\,48',\ 291°\,48'$

2. $\kos x = -0.7536$

Sudut rujukan $= 41°\,6'$

$x = 180° - 41°\,6'$,

$\qquad 180° + 41°\,6'$

$\quad = 138°\,54',\ 221°\,6'$
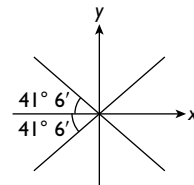
3. $\sin 2x = -\kos 66°$

$\sin 2x = -\sin 24°$
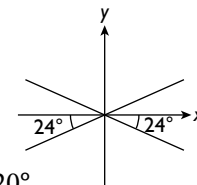
Sudut rujukan $= 24°$

$0° \leqslant x \leqslant 360°,\ 0° \leqslant 2x \leqslant 720°$

$2x = 180° + 24°,\ 360° - 24°$

$\qquad 540° + 24°,\ 720° - 24°$

$\quad = 204°,\ 336°,\ 564°,\ 696°$

$x = 102°,\ 168°,\ 282°,\ 348°$

J

1. $\sec\left(\dfrac{3}{2}x + 75°\right) = -2$

$\kos\left(\dfrac{3}{2}x + 75°\right) = -0.5$

Sudut rujukan $= 60°$
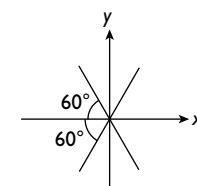
$\dfrac{3}{2}x + 75° = 180° - 60°,\ 180° + 60°$

$\qquad\qquad 540° - 60°,\ 540° + 60°$

$\qquad\quad = 120°,\ 240°,\ 480°,\ 600°$

$\qquad x = 30°,\ 110°,\ 270°,\ 350°$

2. $\text{kos}\left(\dfrac{1}{2}x - 25°\right) = -(-\text{kos } 60°)$

$\qquad\qquad\qquad = \text{kos } 60°$

Sudut rujukan $= 60°$

$\dfrac{1}{2}x - 25° = 60°, 360° - 60°$

$\qquad\qquad = 60°, 300°$

$\qquad\qquad x = 170°$
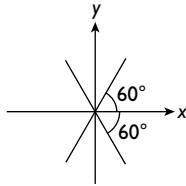
3. $\tan (x - 45°) = 1.12$

Sudut rujukan $= 48° 14'$

$x - 45°$
$= 48° 14', 180° + 48° 14'$
$= 48° 14', 228° 14'$
$x = 93° 14', 273° 14'$

K
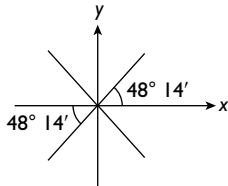
1. $\dfrac{\sin x}{\text{kos } x} = -3$

$\tan x = -3$

Sudut rujukan $= 71° 34'$

$x = 180° - 71° 34',$
$\qquad 360° - 71° 34'$
$\qquad = 108° 26', 288° 26'$

2. $\text{kos } x + 1 = 0$
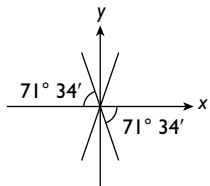
$\qquad \text{kos } x = -1$

$\qquad\qquad x = 180°$

$\tan x + 1 = 0$

$\qquad \tan x = -1$

Sudut rujukan $= 45°$

$x = 180° - 45°, 360° - 45°$
$\quad = 135°, 315°$

Maka, $x = 135°, 180°, 315°$

3. $\quad 3 \text{ kos}^2 x + 8 \text{ kos } x - 3 = 0$

$(3 \text{ kos } x - 1)(\text{kos } x + 3) = 0$

$\qquad\qquad 3 \text{ kos } x - 1 = 0$
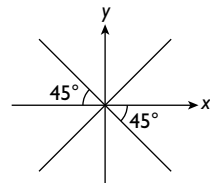
$\qquad\qquad\qquad \text{kos } x = 0.3333$

Sudut rujukan $= 70° 32'$

$x = 70° 32', 360° - 70° 32'$
$\quad = 70° 32', 289° 28'$

$\text{kos } x + 3 = 0$

$\quad \text{kos } x = -3$ (tidak diterima)

5.3

A

1.

2.

3.

4.

5.

6.
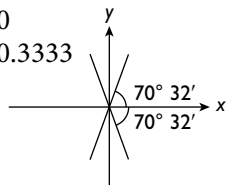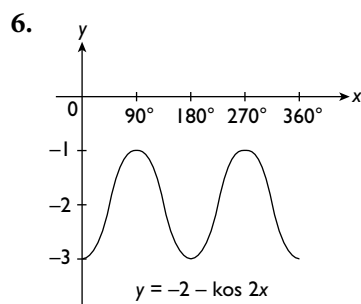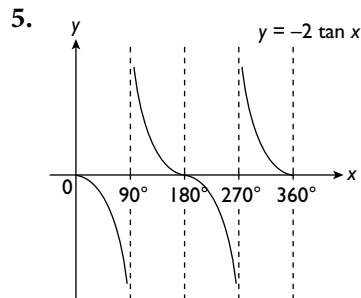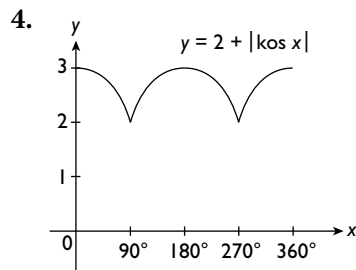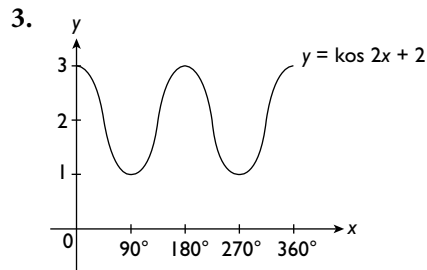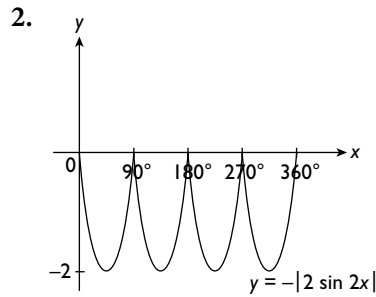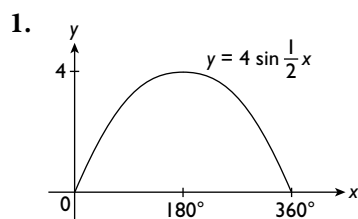
B

1. L①: Lakar graf $y = 3 \sin 2x$.

L②: Daripada persamaan $x - \pi \sin 2x = 0$,

$$\pi \sin 2x = x$$

$$\sin 2x = \frac{x}{\pi}$$

$\times 3$: $\quad 3 \sin 2x = \frac{3x}{\pi}$

Lukis garis lurus $y = \frac{3x}{\pi}$ pada paksi yang sama.

x	0	π
y	0	3
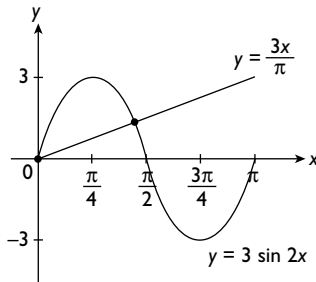
Bilangan penyelesaian = 2

2. L①: Lakar graf $y = 2 \tan x - 1$.

L②: Daripada persamaan $\frac{2x}{\pi} - \left(\tan x - \frac{1}{2}\right) = \frac{3}{2}$,

$$\frac{2x}{\pi} - \frac{3}{2} = \tan x - \frac{1}{2}$$

$\times 2$: $\frac{4x}{\pi} - 3 = 2 \tan x - 1$

Lukis garis lurus $y = \frac{4x}{\pi} - 3$ pada paksi yang sama.

x	0	π
y	−3	1
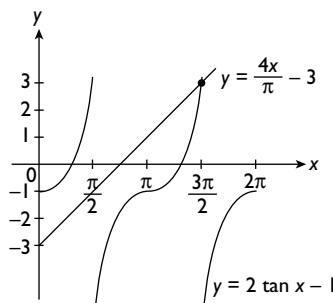
Bilangan penyelesaian = 1

5.4

A

1. $3 - \dfrac{3 \text{ kos}^2 A}{1 + \sin A} = \dfrac{3 + 3 \sin A - 3 \text{ kos}^2 A}{1 + \sin A}$

$$= \frac{3 - 3 \text{ kos}^2 A + 3 \sin A}{1 + \sin A}$$

$$= \frac{3 \sin A(\sin A + 1)}{1 + \sin A}$$

$$= 3 \sin A$$

2. $\dfrac{1 + \sin A}{1 - \sin A} - \dfrac{1 - \sin A}{1 + \sin A}$

$$= \frac{(1 + \sin A)^2 - (1 - \sin A)^2}{(1 - \sin A)(1 + \sin A)}$$

$$= \frac{(1 + 2 \sin A + \sin^2 A) - (1 - 2 \sin A + \sin^2 A)}{1 - \sin^2 A}$$

$$= \frac{4 \sin A}{\text{kos}^2 A}$$

$$= \frac{4 \sin A}{\text{kos } A} \times \frac{1}{\text{kos } A}$$

$$= 4 \tan A \text{ sek } A$$

3. $\dfrac{1}{\text{kot } A - 1} - \dfrac{1}{\text{kot } A + 1}$

$$= \frac{(\text{kot } A + 1) - (\text{kot } A - 1)}{(\text{kot } A - 1)(\text{kot } A + 1)}$$

$$= \frac{\text{kot } A + 1 - \text{kot } A + 1}{(\text{kot } A - 1)(\text{kot } A + 1)}$$

$$= \frac{2}{\text{kot}^2 A - 1}$$

$$= \frac{2}{(\text{kosek}^2 A - 1) - 1}$$

$$= \frac{2}{\text{kosek}^2 A - 2}$$

B

$7 \text{ kosek } A \text{ kos } A + 8 \text{ sek } A \sin A$

$$= \frac{7 \text{ kos } A}{\sin A} + \frac{8 \sin A}{\text{kos } A}$$

$$= \frac{7 \text{ kos}^2 A + 8 \sin^2 A}{\text{kos } A \sin A}$$

$$= \frac{7 \text{ kos}^2 A + 7 \sin^2 A + \sin^2 A}{\text{kos } A \sin A}$$

$$= \frac{7(\text{kos}^2 A + \sin^2 A) + \sin^2 A}{\text{kos } A \sin A}$$

$$= \frac{7 + \sin^2 A}{\text{kos } A \sin A}$$

$$= \frac{\sin^2 A + 7}{\sin A \text{ kos } A}$$

Banding dengan $\dfrac{n \sin^2 A + m}{\sin A \text{ kos } A}$.

Maka, $m = 7$ dan $n = 1$.

C

1.
$$6 \text{ kos}^2 x + 8 \sin^2 x = 13 \sin x$$
$$6(1 - \sin^2 x) + 8 \sin^2 x - 13 \sin x = 0$$
$$6 - 6 \sin^2 x + 8 \sin^2 x - 13 \sin x = 0$$
$$2 \sin^2 x - 13 \sin x + 6 = 0$$
$$(2 \sin x - 1)(\sin x - 6) = 0$$
$2 \sin x - 1 = 0 \quad$ atau $\quad \sin x - 6 = 0$

$\sin x = \dfrac{1}{2}$ $\qquad\qquad\qquad \sin x = 6$

$\qquad\qquad\qquad\qquad\qquad$ (tidak diterima)

$x = 30°, 150°$

2.
$$\text{kos } x \times \frac{\text{kos } x}{\sin x} = 4 \sin x - 4$$
$$\text{kos}^2 x = 4 \sin^2 x - 4 \sin x$$
$$1 - \sin^2 x = 4 \sin^2 x - 4 \sin x$$
$$1 - \sin^2 x - 4 \sin^2 x + 4 \sin x = 0$$
$$5 \sin^2 x - 4 \sin x - 1 = 0$$
$$(5 \sin x + 1)(\sin x - 1) = 0$$

$5 \sin x + 1 = 0$	atau	$\sin x - 1 = 0$

$$\sin x = -\frac{1}{5} \qquad \qquad \sin x = 1$$
$$x = 90°$$
$$x = 191° \ 32', \ 348° \ 28'$$
Maka, $x = 90°, \ 191° \ 32', \ 348° \ 28'$

3.
$$3 \text{ sek}^2 x + 2 \text{ sek } x + \tan^2 x = 1$$
$$3 \text{ sek}^2 x + 2 \text{ sek } x + \text{sek}^2 x - 1 = 1$$
$$4 \text{ sek}^2 x + 2 \text{ sek } x - 2 = 0$$
$$2 \text{ sek}^2 x + \text{sek } x - 1 = 0$$
$$(2 \text{ sek } x - 1)(\text{sek } x + 1) = 0$$

$2 \text{ sek } x - 1 = 0$	atau	$\text{sek } x + 1 = 0$

$$\text{sek } x = \frac{1}{2} \qquad \qquad \text{kos } x = -1$$
$$\text{kos } x = 2 \qquad \qquad x = 180°$$
(tidak diterima)

D

$$\frac{3}{\sin^2 x} - \frac{4}{\tan x} = 7$$
$$3 \text{ kosek}^2 x - 4 \text{ kot } x = 7$$
$$3(1 + \text{kot}^2 x) - 4 \text{ kot } x = 7$$
$$3 + 3 \text{ kot}^2 x - 4 \text{ kot } x - 7 = 0$$
$$3 \text{ kot}^2 x - 4 \text{ kot } x - 4 = 0$$
$$(3 \text{ kot } x + 2)(\text{kot } x - 2) = 0$$
$$3 \text{ kot } x + 2 = 0$$
$$\text{kot } x = -\frac{2}{3}$$
$$\tan x = -\frac{3}{2}$$
$$= -1.5$$
$$x = 123° \ 41', \ 303° \ 41'$$

atau

$$\text{kot } x - 2 = 0$$
$$\text{kot } x = 2$$
$$\tan x = \frac{1}{2}$$
$$= 0.5$$
$$x = 26° \ 34', \ 206° \ 34'$$
Maka, $x = 26° \ 34', \ 123° \ 41', \ 206° \ 34', \ 303° \ 41'$

5.5

A

1.
$$\sin 15° = \sin (45° - 30°)$$
$$= \sin 45° \text{ kos } 30° - \text{kos } 45° \sin 30°$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

2.
$$\text{kos } 168° \text{ kos } 123° + \sin 168° \sin 123°$$
$$= \text{kos } (168° - 123°)$$
$$= \text{kos } 45°$$
$$= \frac{1}{\sqrt{2}}$$

3.
$$\text{kos } 345° = \text{kos } 15°$$
$$= \text{kos } (45° - 30°)$$
$$= \text{kos } 45° \text{ kos } 30° + \sin 45° \sin 30°$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

4.
$$\frac{1 + \tan 195°}{1 - \tan 195°}$$
$$= \frac{\tan 45° + \tan 195°}{1 - \tan 45° \tan 195°} \quad \leftarrow \boxed{\tan 45° = 1}$$
$$= \tan (45° + 195°)$$
$$= \tan 240°$$
$$= \tan 60°$$
$$= \sqrt{3}$$

B

1.
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$
$$= \frac{\left(-\frac{5}{12}\right) - \left(\frac{4}{3}\right)}{1 + \left(-\frac{5}{12}\right)\left(\frac{4}{3}\right)}$$
$$= \frac{-\frac{63}{36}}{\frac{16}{36}} = -\frac{63}{16}$$

2.
$$\text{kos } (A - B) = \text{kos } A \text{ kos } B + \sin A \sin B$$
$$= \left(-\frac{12}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$
$$= -\frac{36}{65} + \frac{20}{65}$$
$$= -\frac{16}{65}$$

3.
$$\text{kosek } (60° - A)$$
$$= \frac{1}{\sin (60° - A)}$$
$$= \frac{1}{\sin 60° \text{ kos } A - \text{kos } 60° \sin A}$$
$$= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{12}{13}\right) - \left(\frac{1}{2}\right)\left(\frac{5}{13}\right)}$$
$$= \frac{1}{-\frac{12\sqrt{3}}{26} - \frac{5}{26}}$$
$$= -\frac{26}{12\sqrt{3} + 5}$$

C

1. $2 \text{ kos } 67.5° \sin 67.5° = 2 \sin 67.5° \text{ kos } 67.5°$
$$= \sin (2 \times 67.5°)$$
$$= \sin 135°$$
$$= \sin 45°$$
$$= \frac{1}{\sqrt{2}}$$

2. $\text{kos}^2 22.5° = \dfrac{\text{kos } (2 \times 22.5°) + 1}{2}$
$$= \frac{\text{kos } 45° + 1}{2}$$
$$= \frac{\frac{1}{\sqrt{2}} + 1}{2}$$
$$= \frac{1 + \sqrt{2}}{2\sqrt{2}}$$

3. $\dfrac{2 \tan 210°}{\tan 45° - \tan^2 210°} = \dfrac{2 \tan 210°}{1 - \tan^2 210°}$
$$= \tan (2 \times 210°)$$
$$= \tan 420°$$
$$= \tan 60°$$
$$= \sqrt{3}$$

D

1. $\text{kos}^2 \dfrac{A}{2} = \dfrac{\text{kos } A + 1}{2}$
$$= \frac{-\frac{5}{13} + 1}{2}$$
$$= \frac{\frac{8}{13}}{2}$$
$$= \frac{4}{13}$$

2. $\text{kosek } 2B = \dfrac{1}{\sin 2B}$
$$= \frac{1}{2 \sin B \text{ kos } B}$$
$$= \frac{1}{2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)}$$
$$= \frac{25}{24}$$

3. $\dfrac{1}{1 + \text{kot}^2 \frac{A}{2}} = \dfrac{1}{\text{kosek}^2 \frac{A}{2}}$
$$= \sin^2 \frac{A}{2}$$
$$= \frac{1 - \text{kos } A}{2}$$
$$= \frac{1 - \left(-\frac{5}{13}\right)}{2}$$
$$= \frac{\frac{18}{13}}{2} = \frac{9}{13}$$

E

1. $\dfrac{\sin 2A + \text{kos } 2A - 1}{\text{kos } A - \sin A}$
$$= \frac{2 \sin A \text{ kos } A + 1 - 2 \sin^2 A - 1}{\text{kos } A - \sin A}$$
$$= \frac{2 \sin A \text{ kos } A - 2 \sin^2 A}{\text{kos } A - \sin A}$$
$$= \frac{2 \sin A(\text{kos } A - \sin A)}{\text{kos } A - \sin A}$$
$$= 2 \sin A$$

2. $2 \text{ kos}^2 A + \text{kot}^2 A - \text{kosek}^2 A$
$$= 2 \text{ kos}^2 A + \text{kot}^2 A - (1 + \text{kot}^2 A)$$
$$= 2 \text{ kos}^2 A + \text{kot}^2 A - 1 - \text{kot}^2 A$$
$$= 2 \text{ kos}^2 A - 1$$
$$= \text{kos } 2A$$

3. $\dfrac{1 + \text{kos } 2A}{\sin 2A} = \dfrac{1 + 2 \text{ kos}^2 A - 1}{2 \sin A \text{ kos } A}$
$$= \frac{2 \text{ kos}^2 A}{2 \sin A \text{ kos } A}$$
$$= \frac{\text{kos } A}{\sin A}$$
$$= \text{kot } A$$

4. $\dfrac{\tan A}{1 + \tan^2 A} = \dfrac{\frac{\sin A}{\text{kos } A}}{1 + \frac{\sin^2 A}{\text{kos}^2 A}}$
$$= \frac{\frac{\sin A}{\text{kos } A}}{\frac{\text{kos}^2 A + \sin^2 A}{\text{kos}^2 A}}$$
$$= \frac{\frac{\sin A}{\text{kos } A}}{\frac{1}{\text{kos}^2 A}}$$
$$= \frac{\sin A}{\text{kos } A} \times \frac{\text{kos}^2 A}{1}$$
$$= \sin A \text{ kos } A$$
$$= \frac{\sin 2A}{2}$$

5. $\dfrac{\sin 4A + \text{kos } 4A + 1}{\sin 4A - \text{kos } 4A + 1}$
$$= \frac{2 \sin 2A \text{ kos } 2A + 2 \text{ kos}^2 2A - 1 + 1}{2 \sin 2A \text{ kos } 2A - (1 - 2 \sin^2 2A) + 1}$$
$$= \frac{2 \sin 2A \text{ kos } 2A + 2 \text{ kos}^2 2A}{2 \sin 2A \text{ kos } 2A + 2 \sin^2 2A}$$
$$= \frac{2 \text{ kos } 2A (\sin 2A + \text{kos } 2A)}{2 \sin 2A (\text{kos } 2A + \sin 2A)}$$
$$= \frac{\text{kos } 2A}{\sin 2A}$$
$$= \text{kot } 2A$$

F

1.
$$2 \text{ kos } 2x - 4 \text{ kos } x - 1 = 0$$
$$2(2 \text{ kos}^2 x - 1) - 4 \text{ kos } x - 1 = 0$$
$$4 \text{ kos}^2 x - 4 \text{ kos } x - 3 = 0$$
$$(2 \text{ kos } x + 1)(2 \text{ kos } x - 3) = 0$$
$$2 \text{ kos } x + 1 = 0$$
$$\text{kos } x = -\frac{1}{2}$$
$$x = 120°, 240° \qquad \text{atau}$$
$$2 \text{ kos } x - 3 = 0$$
$$\text{kos } x = 1.5$$
(tiada penyelesaian)

2.
$$\tan x = \frac{3}{2} \times \frac{\tan 45° - \tan x}{1 + \tan 45° \tan x}$$
$$2 \tan x = 3 \times \frac{1 - \tan x}{1 + \tan x}$$
$$2 \tan x + 2 \tan^2 x = 3 - 3 \tan x$$
$$2 \tan^2 x + 5 \tan x - 3 = 0$$
$$(2 \tan x - 1)(\tan x + 3) = 0$$
$$2 \tan x - 1 = 0$$
$$\tan x = \frac{1}{2}$$
$$x = 26° 34', 206° 34' \qquad \text{atau}$$
$$\tan x + 3 = 0$$
$$\tan x = -3$$
$$x = 108° 26', 288° 26'$$
Maka, $x = 26° 34', 108° 26', 206° 34', 288° 26'$

3.
$$\frac{1}{\text{kot } 2x} + \tan x = 0$$
$$\tan 2x + \tan x = 0$$
$$\frac{2 \tan x}{1 - \tan^2 x} + \tan x = 0$$
$$\tan x \left(\frac{2}{1 - \tan^2 x} + 1 \right) = 0$$
$$\tan x = 0$$
$$x = 0°, 180°, 360° \qquad \text{atau}$$
$$\frac{2}{1 - \tan^2 x} + 1 = 0$$
$$\tan^2 x = 3$$
$$\tan x = \pm\sqrt{3}$$
$$x = 60°, 120°, 240°, 300°$$
Maka, $x = 0°, 60°, 120°, 180°, 240°, 300°, 360°$

4.
$$\sin (x - \pi) \text{ kos } (x - \pi) = \frac{1}{4}$$
$$2 \sin (x - \pi) \text{ kos } (x - \pi) = \frac{1}{2}$$
$$\sin 2(x - \pi) = \frac{1}{2}$$
$$2(x - 180°) = 30°, 150°$$
$$x - 180° = 15°, 75°$$
$$x = 195°, 255°$$

5.
$$\sin x = \text{kos } (x - 30°)$$
$$\sin x = \text{kos } x \text{ kos } 30° + \sin x \sin 30°$$
$$\sin x = \text{kos } x \,(0.8660) + \sin x \,(0.5)$$
$$0.5 \sin x = 0.8660 \text{ kos } x$$
$$\frac{\sin x}{\text{kos } x} = \frac{0.8660}{0.5}$$
$$\tan x = 1.732$$
$$x = 60°, 240°$$

Praktis Formatif: Kertas 1

1.
$$5 \text{ kot } x = \tan x + 4$$
$$5 \frac{1}{\tan x} = \tan x + 4$$
$$5 = \tan^2 x + 4 \tan x$$
$$\tan^2 x + 4 \tan x - 5 = 0$$
$$(\tan x - 1)(\tan x + 5) = 0$$
$$\tan x - 1 = 0 \qquad \text{atau} \qquad \tan x + 5 = 0$$
$$\tan x = 1 \qquad\qquad\qquad \tan x = -5$$
$$x = 45° \qquad\qquad\qquad x = 101.30°$$
Maka, $x = 45°$ dan $101.30°$

2. (a) Persamaan lengkung ialah $y = 3 \sin 4x + 1$.
  (i) $p = 3$ (ii) $q = 4$

  (b) Garis lurus $y = -1$ bersilang dengan lengkung pada 4 titik untuk $0 \le x \le \pi$.
  Maka, 4 penyelesaian.

3. (a) kosek $\theta = \dfrac{1}{\sin \theta} = \dfrac{1}{c}$

  (b) $\sin 2\theta = 2 \sin \theta \text{ kos } \theta$
$$= 2 \sin \theta \sqrt{1 - \sin^2 \theta}$$
$$= 2c\sqrt{1 - c^2}$$

4.
$$\sin 2x + \sin x = 0$$
$$2 \sin x \text{ kos } x + \sin x = 0$$
$$\sin x(2 \text{ kos } x + 1) = 0$$
$$\sin x = 0$$
$$x = 0°, 180°, 360°$$
atau
$$2 \text{ kos } x + 1 = 0$$
$$\text{kos } x = -\frac{1}{2}$$
$$x = 120°, 240°$$
Maka, $x = 0°, 120°, 180°, 240°, 360°$

5. $\tan (\theta - 45°)$
$$= \frac{\tan \theta - \tan 45°}{1 + \tan \theta \tan 45°}$$
$$= \frac{-\dfrac{12}{5} - 1}{1 + \left(-\dfrac{12}{5}\right)(1)}$$
$$= \frac{-\dfrac{17}{5}}{-\dfrac{7}{5}} = \frac{17}{7}$$

6. (a) kos $(90° + x) = \text{kos } 90° \text{ kos } x - \sin 90° \sin x$
$$= 0 - \sin x$$
$$= -k$$

(b) $\text{sek } 2x = \dfrac{1}{\text{kos } 2x}$
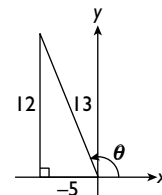
$= \dfrac{1}{1 - 2\sin^2 x}$

$= \dfrac{1}{1 - 2k^2}$

Praktis Formatif: Kertas 2
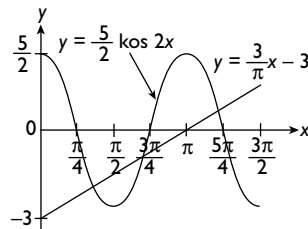
1. (a), (b)
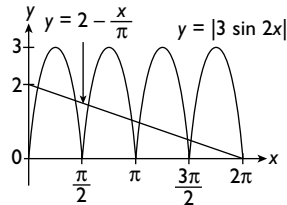
$2 - |3\sin 2x| = \dfrac{x}{\pi}$

$|3\sin 2x| = 2 - \dfrac{x}{\pi}$

Lukis garis lurus $y = 2 - \dfrac{x}{\pi}$.

Bilangan penyelesaian = 8

2. (a), (b)

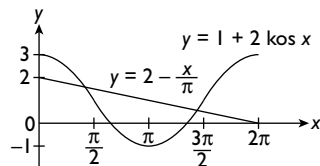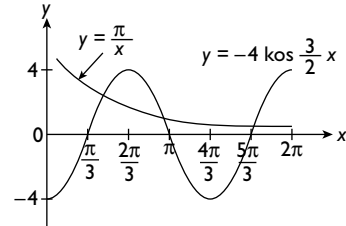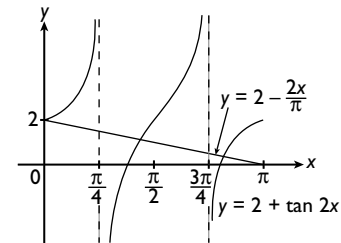$\dfrac{6}{5\pi}x - \text{kos } 2x = \dfrac{6}{5}$

$\dfrac{3}{\pi}x - \dfrac{5}{2}\text{kos } 2x = 3$

$\dfrac{5}{2}\text{kos } 2x = \dfrac{3}{\pi}x - 3$

Lukis garis lurus $y = \dfrac{3}{\pi}x - 3$.

Bilangan penyelesaian = 3

3. (a), (b)

$2\pi \text{ kos } x = \pi - x$

$2 \text{ kos } x = 1 - \dfrac{x}{\pi}$

$1 + 2 \text{ kos } x = 2 - \dfrac{x}{\pi}$

Lukis garis lurus $y = 2 - \dfrac{x}{\pi}$.

Bilangan penyelesaian = 2

4. (a), (b)

$\dfrac{\pi}{x} + 4 \text{ kos } \dfrac{3}{2}x = 0$

$\dfrac{\pi}{x} = -4 \text{ kos } \dfrac{3}{2}x$

Lukis lengkung $y = \dfrac{\pi}{x}$.

Bilangan penyelesaian = 3

5. (a), (b)

$2x + \pi \tan 2x = 0$

$-\dfrac{2x}{\pi} = \tan 2x$

$2 - \dfrac{2x}{\pi} = 2 + \tan 2x$

Lukis garis lurus $y = 2 - \dfrac{2x}{\pi}$.

Bilangan penyelesaian = 3

6. (a) $\dfrac{2\sin x \text{ kos } x}{\text{sek}^2 x - 2\sin^2 x - \tan^2 x}$

$= \dfrac{\sin 2x}{(\text{sek}^2 x - \tan^2 x) - 2\sin^2 x}$

$= \dfrac{\sin 2x}{1 - 2\sin^2 x}$

$= \dfrac{\sin 2x}{\text{kos } 2x}$

$= \tan 2x$

(b), (c)
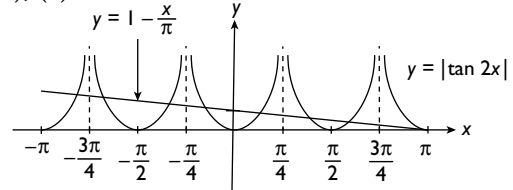
$\left| \dfrac{2\sin x \text{ kos } x}{\text{sek}^2 x - 2\sin^2 x - \tan^2 x} \right| + \dfrac{x}{\pi} = 1$

$|\tan 2x| = 1 - \dfrac{x}{\pi}$

$y = 1 - \dfrac{x}{\pi}$

Lukis garis lurus $y = 1 - \dfrac{x}{\pi}$.

Bilangan penyelesaian = 8

7. (a) (i) $2 \sin (x + 45°) \sin (x – 45°)$

$= 2[(\sin x \text{ kos } 45° + \text{kos } x \sin 45°)$
$(\sin x \text{ kos } 45° – \text{kos } x \sin 45°)]$

$= 2\left[\left(\dfrac{1}{\sqrt{2}} \sin x + \dfrac{1}{\sqrt{2}} \text{ kos } x\right)\right.$

$\left.\left(\dfrac{1}{\sqrt{2}} \sin x – \dfrac{1}{\sqrt{2}} \text{ kos } x\right)\right]$

$= 2\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right)(\sin x + \text{kos } x)(\sin x – \text{kos } x)$

$= \sin^2 x – \text{kos}^2 x$

$= –(\text{kos}^2 x – \sin^2 x)$

$= –\text{kos } 2x$

(ii) $2 \sin (x + 45°) \sin (x – 45°) = \dfrac{\sqrt{3}}{2}$

$–\text{kos } 2x = \dfrac{\sqrt{3}}{2}$

$\text{kos } 2x = –\dfrac{\sqrt{3}}{2}$

$2x = 180° – 30°, 180° + 30°, 540° – 30°,$
$540° + 30°$

$x = 75°, 105°, 255°, 285°$

(b)

$y = –\text{kos } 2x$

8. (a) $2 \text{ kot } \theta \sin^2 \theta = 2\left(\dfrac{\text{kos } \theta}{\sin \theta}\right) \sin^2 \theta$

$= 2 \text{ kos } \theta \sin \theta$

$= \sin 2\theta$
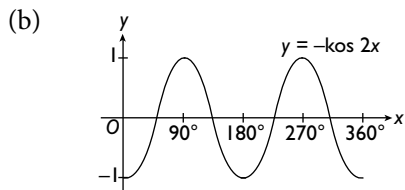
(b) $4 \text{ kot } \theta \sin^2 \theta = \sqrt{3}$

$2 \sin 2\theta = \sqrt{3}$

$\sin 2\theta = \dfrac{\sqrt{3}}{2}$

$2\theta = 60°, 120°, 420°, 480°$

$\theta = 30°, 60°, 210°, 240°$

$= \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{7\pi}{6}, \dfrac{4\pi}{3}$

(c) (i), (ii)

$4\pi \text{ kot } \theta \sin^2 \theta = 2\pi – \theta$

$2\pi \sin 2\theta = 2\pi – \theta$

$\sin 2\theta = 1 – \dfrac{\theta}{2\pi}$
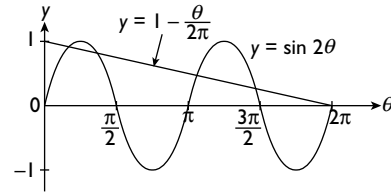
Lukis garis lurus $y = 1 – \dfrac{\theta}{2\pi}$.

Bilangan penyelesaian = 5

FOKUS KBAT

$\text{kot } x = \text{kot } (B – A)$

$= \dfrac{1 + \tan B \tan A}{\tan B – \tan A}$

$= \dfrac{1 + \left(\dfrac{2}{p}\right)\left(\dfrac{2}{p + 1}\right)}{\left(\dfrac{2}{p}\right) – \left(\dfrac{2}{p + 1}\right)}$

$= \dfrac{p(p + 1) + 4}{2(p + 1) – 2p}$

$= \dfrac{p^2 + p + 4}{2}$

BAB 6: PILIH ATUR DAN GABUNGAN

6.1

A

1. $6 \times 8 = 48$

2. $6 \times 6 \times 2 = 72$

B

1. $4! = 24$

2. $7! = 5\ 040$

3. $9! = 362\ 880$

C

1. Bilangan cara = $5!$
   $= 120$

2. Bilangan cara = $7!$
   $= 5\ 040$

D

1. $^5P_2 = 20$

2. $^6P_3 = 120$

3. $^9P_4 = 3\ 024$

E

1. Bilangan nombor 4 digit
   $= {}^7P_4$
   $= 7 \times 6 \times 5 \times 4$
   $= 840$

2. Bilangan cara
   $= {}^8P_5$
   $= 8 \times 7 \times 6 \times 5 \times 4$
   $= 6\ 720$

F

1. Nombor ganjil berakhir dengan digit 9, 7 atau 3.

   Bilangan nombor ganjil
   $= 3 \times 4!$
   $= 3 \times 24$
   $= 72$

2. Bilangan susunan = $2 \times 7!$
   $= 10\ 080$

3. Digit pertama bermula dengan 2, 3 atau 5.
   Bilangan nombor 4 digit
   $= 3 \times 3!$
   $= 18$

G

1. (a) Bilangan pilih atur bagi huruf $O = 4$

   Bilangan kod 4 huruf
   $= 4 \times {}^6P_3$
   $= 4 \times 120$
   $= 480$

   (b) Bilangan huruf konsonan = 5

   Bilangan kod 4 huruf
   $= {}^5P_4$
   $= 120$

2. (a) Nombor genap berakhir dengan digit 2, 4 atau 8.

   Bilangan nombor genap
   $= 3 \times {}^6P_5$
   $= 3 \times 720$
   $= 2\ 160$

   (b) Bilangan pilih atur bagi digit 9 = 6

   Bilangan nombor yang mengandungi digit 9
   $= 6 \times {}^6P_5$
   $= 6 \times 720$
   $= 4\ 320$

6.2

A

1. Bilangan cara
   $= {}^{10}C_7$
   $= \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}$
   $= 120$

2. Bilangan pilihan
   $= {}^8C_5$
   $= \dfrac{8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5}$
   $= 56$

3. Bilangan cara
   $= {}^9C_4$
   $= \dfrac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4}$
   $= 126$

4. Bilangan sisi empat
   $= {}^8C_4$
   $= \dfrac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}$
   $= 70$

5. Bilangan gabungan

$= {}^9C_5$

$= \dfrac{9 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4 \times 5}$

$= 126$

B

1. Bilangan cara

$= {}^9C_5 \times {}^4C_4$

$= 126 \times 1$

$= 126$

2. (a) Bilangan cara

$= {}^{15}C_4 \times {}^{10}C_2$

$= 1\,365 \times 45$

$= 61\,425$

(b) Bilangan cara

$= {}^{15}C_4 \times {}^{10}C_2 + {}^{15}C_5 \times {}^{10}C_1 + {}^{15}C_6 \times {}^{10}C_0$

$= 61\,425 + 30\,030 + 5\,005$

$= 96\,460$

3. (a) Bilangan cara

$= {}^5C_2 \times {}^{11}C_7$

$= 10 \times 330$

$= 3\,300$

(b) Bilangan cara

$= {}^5C_3 \times {}^3C_2 \times {}^8C_4$

$= 10 \times 3 \times 70$

$= 2\,100$

Praktis Formatif: Kertas 1

1. (a) Bilangan cara $= 7! = 5\,040$

(b) Katakan 2 orang guru dianggap sebagai 1 'objek'.

Bilangan pilih atur bagi 6 'objek'. $= 6!$

Bilangan pilih atur bagi 2 orang guru $= 2$

Bilangan cara $= 6! \times 2$

$ = 1\,440$

2. (a) Bilangan kod lima digit yang dapat dibentuk

$= {}^6P_5$

$= 720$

(b) Bilangan pilih atur bagi digit pertama

$= {}^4P_1 = 4$

Bilangan pilih atur bagi digit terakhir

$= {}^2P_1 = 2$

Bilangan pilih atur bagi digit kedua, ketiga dan keempat $= {}^4P_3 = 24$

Bilangan kod lima digit yang dapat dibentuk

$= 4 \times 2 \times 24$

$= 192$

3. (a) Bilangan cara $= 6!$

$ = 720$

(b) Bilangan cara $= 4 \times 3 \times 4!$

$ = 288$

4. (a) Bilangan cara $= {}^9C_3$

$ = 84$

(b) Bilangan cara $= {}^9C_7 + {}^9C_8 + {}^9C_9$

$ = 36 + 9 + 1$

$ = 46$

5. (a) Bilangan cara $= 7!$

$ = 5\,040$

(b) Bilangan cara untuk memilih 3 konsonan daripada 5 konsonan dan 1 vokal daripada 2 vokal $= {}^5C_3 \times {}^2C_1$

$ = 10 \times 2$

$ = 20$

6. (a) Bilangan cara $= {}^8C_6$

$ = 28$

(b) Bilangan cara $= {}^{10}C_5 \times {}^8C_1 + {}^{10}C_6$

$ = 2\,016 + 210$

$ = 2\,226$

7. (a) ${}^nC_n = \dfrac{n!}{n!\,0!}$

$\phantom{{}^nC_n} = 1$

(b) (i) Bilangan cara $= {}^6C_4$

$ = 15$

(ii) Bilangan cara $= {}^6C_1 \times {}^5C_3$

$ = 6 \times 10$

$ = 60$

8. (a) Bilangan cara $= {}^{12}C_4$

$ = 495$

(b) Bilangan cara untuk menyusun cawan-cawan itu dengan keadaan cawan biru dan cawan merah diletak bersebelahan $= 2 \times 6!$

$ = 1\,440$

Bilangan cara untuk menyusun cawan-cawan itu dengan keadaan cawan biru dan cawan merah tidak diletak bersebelahan

$= 7! - 2 \times 6!$

$= 5\,040 - 1\,440$

$= 3\,600$

FOKUS KBAT

1. ${}^7C_4 \times {}^5C_4 \times {}^4P_3 \times {}^5P_5 = 35 \times 5 \times 24 \times 120$

$\phantom{{}^7C_4 \times {}^5C_4 \times {}^4P_3 \times {}^5P_5} = 504\,000$

2. ${}^nC_2 \times {}^{n-2}C_{n-2} = 28$

$\dfrac{n!}{(n-2)!\,2!}(1) = 28$

$\dfrac{n(n-1)}{2} = 28$

$n^2 - n - 56 = 0$

$(n + 7)(n - 8) = 0$

$n = -7$ atau $n = 8$

Oleh sebab $n > 0$, maka $n = 8$.

BAB 7: KEBARANGKALIAN MUDAH

7.1

A

1. S = {Ahad, Isnin, Selasa, Rabu, Khamis, Jumaat, Sabtu}

2. S = {(G, G), (G, A), (A, G), (A, A)}

3. S = {(L, L, L), (L, L, P), (L, P, L), (P, L, L), (L, P, P), (P, L, P), (P, P, L), (P, P, P)}

B

1. Z = {April, Jun, September, November}
   $n(Z)$ = 4

2. Z = {K, P, R, T, F}
   $n(Z)$ = 5

3. Z = {(G, G), (G, A), (A, G)}
   $n(Z)$ = 3

C

1. $n(S)$ = 6
   n(nombor lebih besar daripada 4) = 2
   P(nombor lebih besar daripada 4) = $\dfrac{2}{6}$
   $\qquad\qquad = \dfrac{1}{3}$

2. $n(S)$ = 10
   n(huruf konsonan) = 6
   P(huruf konsonan) = $\dfrac{6}{10}$
   $\qquad\qquad = \dfrac{3}{5}$

3. S = {11, 12, 13, …, 40}
   $n(S)$ = 30
   A = {nombor kuasa dua sempurna}
   $\quad$ = {16, 25, 36}
   $n(A)$ = 3
   P(A) = $\dfrac{3}{30} = \dfrac{1}{10}$

4. $n(S)$ = 8 + 4 + 3 = 15
   n(nilai wang kertas lebih daripada RM1)
   = 4 + 3
   = 7
   P(nilai wang kertas lebih daripada RM1) = $\dfrac{7}{15}$

5. $n(S)$ = 11
   (a) n(kad dilabel dengan I) = 3
   $\qquad$ P(kad dilabel dengan I) = $\dfrac{3}{11}$

(b) n(kad dilabel dengan huruf konsonan)
   = 7
   P(kad dilabel dengan huruf konsonan)
   = $\dfrac{7}{11}$

D

1. $n(S)$ = 100, A = {suka kopi}, B = {suka teh}
   $n(A)$ = 45 + 15 = 60
   (a) P($A \cap B$) = $\dfrac{n(A \cap B)}{n(S)}$
   $\qquad\qquad = \dfrac{15}{100}$
   $\qquad\qquad = \dfrac{3}{20}$

   $n(B)$ = 15 + 20 = 35
   $n(A \cap B)$ = 15
   (b) P($A \cup B$) = P(A) + P(B) – P($A \cap B$)
   $\qquad\qquad = \dfrac{60}{100} + \dfrac{35}{100} - \dfrac{15}{100}$
   $\qquad\qquad = \dfrac{4}{5}$

2. S = {21, 22, 23, …, 40}, $n(S)$ = 20
   A = {gandaan 3}
   $\quad$ = {21, 24, 27, 30, 33, 36, 39}
   $n(A)$ = 7, $n(B)$ = 5
   (a) $A \cap B$ = {24, 36}
   $\qquad$ P($A \cap B$) = $\dfrac{n(A \cap B)}{n(S)}$
   $\qquad\qquad\quad = \dfrac{2}{20}$
   $\qquad\qquad\quad = \dfrac{1}{10}$
   $\qquad B$ = {gandaan 4} = {24, 28, 32, 36, 40}
   (b) P($A \cup B$) = P(A) + P(B) – P($A \cap B$)
   $\qquad\qquad = \dfrac{7}{20} + \dfrac{5}{20} - \dfrac{2}{20}$
   $\qquad\qquad = \dfrac{1}{2}$

7.2

1. (a) P(nombor genap) = $\dfrac{4}{8}$
   $\qquad$ P(nombor 5) = $\dfrac{1}{8}$
   $\qquad$ P(nombor genap atau nombor 5)
   $\qquad$ = $\dfrac{4}{8} + \dfrac{1}{8}$
   $\qquad$ = $\dfrac{5}{8}$

(b) P(nombor kurang daripada 3) = $\frac{2}{8}$

P(nombor lebih besar daripada 7) = $\frac{1}{8}$

P(nombor kurang daripada 3 atau nombor lebih besar daripada 7)

= $\frac{2}{8} + \frac{1}{8}$

= $\frac{3}{8}$

2. Jumlah bilangan pen = 5 + 3 + 6 = 14

P(pen merah) = $\frac{5}{14}$

P(pen biru) = $\frac{3}{14}$

P(pen hitam) = $\frac{6}{14}$

(a) P(pen merah atau pen hitam)

= $\frac{5}{14} + \frac{6}{14}$

= $\frac{11}{14}$

(b) P(pen biru atau pen hitam)

= $\frac{3}{14} + \frac{6}{14}$

= $\frac{9}{14}$

3. Jumlah bilangan huruf = 11

(a) P(huruf *I*) = $\frac{4}{11}$

P(huruf *P*) = $\frac{2}{11}$

P(huruf *I* atau huruf *P*)

= $\frac{4}{11} + \frac{2}{11}$

= $\frac{6}{11}$

(b) P(huruf *S*) = $\frac{4}{11}$

P(huruf vokal) = $\frac{4}{11}$

P(huruf *S* atau huruf vokal)

= $\frac{4}{11} + \frac{4}{11} = \frac{8}{11}$

7.3

A

1. P(angka) = $\frac{1}{2}$, P(nombor 6) = $\frac{1}{6}$

P(angka dan nombor 6)
= P(angka) × P(nombor 6)

= $\frac{1}{2} \times \frac{1}{6}$

= $\frac{1}{12}$

2. P(lulus Sains) = 0.75
P(gagal Matematik) = 0.2

P(lulus Sains, gagal Matematik)
= P(lulus Sains) × P(gagal Matematik)
= 0.75 × 0.2
= 0.15

3. (a) P(kedua-duanya terpilih)
= P(John dipilih) × P(Aisha dipilih)

= $\frac{2}{5} \times \frac{5}{9}$

= $\frac{2}{9}$

(b) P(hanya seorang dipilih)
= P(John dipilih, Aisha tidak dipilih) +
   P(John tidak dipilih, Aisha dipilih)

= $\frac{2}{5} \times \frac{4}{9} + \frac{3}{5} \times \frac{5}{9}$

= $\frac{23}{45}$

4. P(hujan akan turun pada suatu hari) = $\frac{3}{4}$

P(hujan tidak akan turun pada suatu hari) = $\frac{1}{4}$

(a) P(hujan turun pada ketiga-tiga hari)

= $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

= $\frac{27}{64}$

(b) P(hujan turun pada hari Isnin dan hari Selasa tetapi tidak pada hari Rabu)

= $\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

= $\frac{9}{64}$

5. Bilangan mentol rosak = 4
Bilangan mentol elok = 12 − 4 = 8

(a) P(kedua-dua mentol rosak)
= P(mentol pertama rosak) ×
   P(mentol kedua rosak)

= $\frac{4}{12} \times \frac{3}{11}$

= $\frac{1}{11}$

(b) P(sebiji daripada mentol rosak)
= P(mentol pertama rosak tetapi mentol kedua elok) + P(mentol pertama elok tetapi mentol kedua rosak)

= $\frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$

= $\frac{32}{132} + \frac{32}{132}$

= $\frac{16}{33}$

B

1. A = Azman, B = Bob, C = Chandran
   (a) P(mereka bertiga mengenai sasaran)
   = P(ABC)
   = P(A) × P(B) × P(C)
   = $\dfrac{3}{4} \times \dfrac{2}{3} \times \dfrac{2}{5}$
   = $\dfrac{1}{5}$

   (b) P(hanya seorang tidak mengenai sasaran)
   = P($A'BC$) + P($AB'C$) + P(ABC')
   = $\dfrac{1}{4} \times \dfrac{2}{3} \times \dfrac{2}{5} + \dfrac{3}{4} \times \dfrac{1}{3} \times \dfrac{2}{5} + \dfrac{3}{4} \times \dfrac{2}{3} \times \dfrac{3}{5}$
   = $\dfrac{4}{60} + \dfrac{6}{60} + \dfrac{18}{60}$
   = $\dfrac{7}{15}$

2. A = peristiwa memperoleh nombor 6
   A' = peristiwa tidak memperoleh nombor 6
   P(A) = $\dfrac{1}{6}$
   P(A') = $\dfrac{5}{6}$

   (a) P(nombor 6 diperoleh sekali)
   = P($AA'A'$) + P($A'AA'$) + P($A'A'A$)
   = $\dfrac{1}{6} \times \dfrac{5}{6} \times \dfrac{5}{6} + \dfrac{5}{6} \times \dfrac{1}{6} \times \dfrac{5}{6} + \dfrac{5}{6} \times \dfrac{5}{6} \times \dfrac{1}{6}$
   = $\dfrac{25}{216} + \dfrac{25}{216} + \dfrac{25}{216} = \dfrac{25}{72}$

   (b) P(nombor 6 diperoleh dua kali)
   = P(AAA') + P($AA'A$) + P($A'AA$)
   = $\dfrac{1}{6} \times \dfrac{1}{6} \times \dfrac{5}{6} + \dfrac{1}{6} \times \dfrac{5}{6} \times \dfrac{1}{6} + \dfrac{5}{6} \times \dfrac{1}{6} \times \dfrac{1}{6}$
   = $\dfrac{5}{216} + \dfrac{5}{216} + \dfrac{5}{216} = \dfrac{5}{72}$

3. R = peristiwa bola merah dipilih
   W = peristiwa bola putih dipilih
   P(R) = $\dfrac{6}{10}$ P(W) = $\dfrac{4}{10}$

   (a) P(bola pertama merah dan bola kedua putih)
   = P(RW)
   = P(R) × P(W)
   = $\dfrac{6}{10} \times \dfrac{4}{10} = \dfrac{6}{25}$

   (b) P(kedua-dua biji bola berwarna sama)
   = P(RR) + P(WW)
   = $\dfrac{6}{10} \times \dfrac{6}{10} + \dfrac{4}{10} \times \dfrac{4}{10}$
   = $\dfrac{13}{25}$

   (c) P(kedua-dua biji bola bukan berwarna merah)
   = 1 – P(kedua-dua biji bola berwarna merah)
   = 1 – P(RR)
   = $1 - \left(\dfrac{6}{10} \times \dfrac{6}{10} \right) = \dfrac{16}{25}$

Praktis Formatif: Kertas 1

1. (a) P[$(A \cup B)'$] = 1 – P($A \cup B$)
   = $1 - \dfrac{5}{6}$
   = $\dfrac{1}{6}$

   (b) P($A \cap B$) = P(A) + P(B) – P($A \cup B$)
   = $\dfrac{4}{7} + \dfrac{2}{3} - \dfrac{5}{6}$
   = $\dfrac{17}{42}$

2. (a) P(Azman atau Muthu dipilih)
   = P(Azman dipilih) + P(Muthu dipilih)
   $\dfrac{3}{8} = \dfrac{1}{4}$ + P(Muthu dipilih)
   P(Muthu dipilih) = $\dfrac{3}{8} - \dfrac{1}{4}$
   = $\dfrac{1}{8}$

   (b) P(Azman atau Muthu tidak dipilih)
   = 1 – P(Azman atau Muthu dipilih)
   = $1 - \dfrac{3}{8}$
   = $\dfrac{5}{8}$

3. (a) P(kedua-dua murid dipilih) = P(A) × P(B)
   = $\dfrac{5}{6} \times \dfrac{4}{7}$
   = $\dfrac{10}{21}$

   (b) P(hanya seorang murid dipilih)
   = P(A) × P(B') + P(A') × P(B)
   = $\dfrac{5}{6} \times \dfrac{3}{7} + \dfrac{1}{6} \times \dfrac{4}{7}$
   = $\dfrac{19}{42}$

4. $\dfrac{x}{x+4} \times \dfrac{2}{8} + \dfrac{4}{x+4} \times \dfrac{6}{8} = \dfrac{15}{28}$
   $7x + 84 = 15(x + 4)$
   $8x = 24$
   $x = 3$

5. (a) $\dfrac{4}{6} = \dfrac{2}{3}$

   (b) P(B atau C atau D)
   = P(B) + P(C) + P(D)
   = $\left(\dfrac{1}{3} \times \dfrac{2}{3} + \dfrac{2}{3} \times \dfrac{1}{3} \right) + 0 + \dfrac{2}{3} \times \dfrac{2}{3}$
   = $\dfrac{2}{9} + \dfrac{2}{9} + \dfrac{4}{9}$
   = $\dfrac{8}{9}$

6. (a) $P(AA) + P(BB) = \dfrac{5}{8} \times \dfrac{5}{8} + \dfrac{3}{8} \times \dfrac{3}{8}$

$\qquad\qquad\qquad\quad = \dfrac{17}{32}$

(b) $P(ABA) + P(BAA)$

$\quad = \dfrac{5}{8} \times \dfrac{3}{8} \times \dfrac{5}{8} + \dfrac{3}{8} \times \dfrac{5}{8} \times \dfrac{5}{8}$

$\quad = \dfrac{75}{256}$

7. P(dua nombor yang berlainan)

= 1 – P(dua nombor yang sama)

= 1 – [P(11) + P(22) + P(33) + P(44) + P(55) + P(66)]

$= 1 - \left(\dfrac{2}{11} \times \dfrac{2}{11} \times 5 + \dfrac{1}{11} \times \dfrac{1}{11} \right)$

$= 1 - \dfrac{21}{121}$

$= \dfrac{100}{121}$

FOKUS KBAT

Kebarangkalian mendapat selain daripada digit '1'

$= \left(1 - \dfrac{1}{5} \right) \times \dfrac{1}{8}$

$= \dfrac{1}{10}$

P(digit ganjil dan huruf vokal)

$= \left(\dfrac{1}{5} \times \dfrac{2}{6} \right) + \left[4 \left(\dfrac{1}{10} \right) \times \dfrac{2}{6} \right]$

$= \dfrac{1}{15} + \dfrac{2}{15}$

$= \dfrac{1}{5}$

JAWAPAN

BAB 8: TABURAN KEBARANGKALIAN

8.1

A

1. Katakan X mewakili bilangan perlawanan yang Kasim menang.

$p = \dfrac{3}{4}, q = \dfrac{1}{4}, n = 7$

$P(X = 4) = {}^7C_4\left(\dfrac{3}{4}\right)^4\left(\dfrac{1}{4}\right)^3$

$\qquad\qquad = 0.1730$

2. Katakan X mewakili bilangan hari bas lewat.
$p = 0.15, q = 0.85, n = 5$
$P(X = 2) = {}^5C_2(0.15)^2(0.85)^3$
$\qquad\qquad = 0.1382$

3. Katakan X mewakili bilangan tembakan yang mengenai sasaran.
$p = 0.8, q = 0.2, n = 9$
(a) $P(X = 8) = {}^9C_8(0.8)^8(0.2)^1$
$\qquad\qquad\quad = 0.3020$

(b) P(sekurang-kurangnya 8 das)
$\quad = P(X = 8) + P(X = 9)$
$\quad = 0.3020 + {}^9C_9(0.8)^9(0.2)^0$
$\quad = 0.3020 + 0.1342$
$\quad = 0.4362$

B

1. Katakan X ialah bilangan soalan yang dijawab betul.
$n = 3, p = 0.6, q = 0.4$
$X = \{0, 1, 2, 3\}$

$P(X = 0) = {}^3C_0(0.6)^0(0.4)^3 = 0.064$

$P(X = 1) = {}^3C_1(0.6)^1(0.4)^2 = 0.288$

$P(X = 2) = {}^3C_2(0.6)^2(0.4)^1 = 0.432$

$P(X = 3) = {}^3C_3(0.6)^3(0.4)^0 = 0.216$
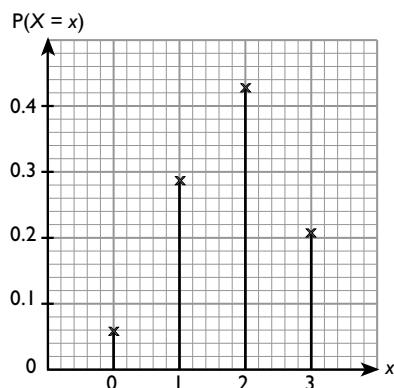
2. Katakan X ialah bilangan kali Henry mengenai sasaran.
$n = 4, p = 0.45, q = 0.55$
$X = \{0, 1, 2, 3, 4\}$

$P(X = 0) = {}^4C_0(0.45)^0(0.55)^4 = 0.092$

$P(X = 1) = {}^4C_1(0.45)^1(0.55)^3 = 0.299$

$P(X = 2) = {}^4C_2(0.45)^2(0.55)^2 = 0.368$

$P(X = 3) = {}^4C_3(0.45)^3(0.55)^1 = 0.200$

$P(X = 4) = {}^4C_4(0.45)^4(0.55)^0 = 0.041$
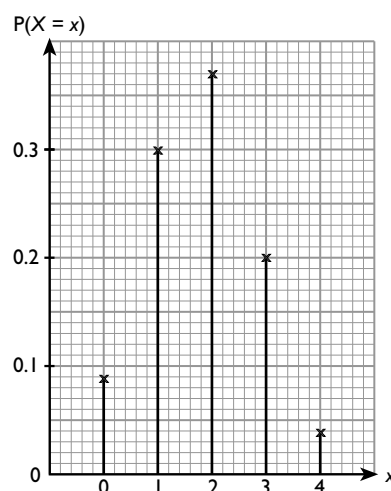
C

1. $n = 15, p = \dfrac{1}{6}, q = \dfrac{5}{6}$

Min $= 15\left(\dfrac{1}{6}\right) = 2.5$

Varians $= 15\left(\dfrac{1}{6}\right)\left(\dfrac{5}{6}\right)$

$\qquad\quad = 2.083$

Sisihan piawai $= \sqrt{2.083}$
$\qquad\qquad\qquad = 1.443$

2. $n = 48, p = 85\% = 0.85, q = 0.15$

Min $= 48(0.85) = 40.8$
Varians $= 48(0.85)(0.15)$
$\qquad\quad = 6.12$
Sisihan piawai $= \sqrt{6.12}$
$\qquad\qquad\qquad = 2.474$

3. $n = 30, p = 0.7, q = 0.3$

Min $= 30(0.7) = 21$
Varians $= 30(0.7)(0.3)$
$\qquad\quad = 6.3$
Sisihan piawai $= \sqrt{6.3}$
$\qquad\qquad\qquad = 2.510$

D

1. $p = \dfrac{2}{7}$, $np = 320$

$$n\left(\dfrac{2}{7}\right) = 320$$
$$n = 1\,120$$

Sisihan piawai $= \sqrt{1\,120 \times \dfrac{2}{7} \times \dfrac{5}{7}}$

$$= 15.12$$

Bilangan murid di sekolah itu ialah
1 120 orang.
Sisihan piawai bagi bilangan murid dalam
program itu ialah 15.12.

2. $np = 48$ ......①
$npq = 46.08$ ......②

② ÷ ①: $q = \dfrac{46.08}{48} = 0.96$
$$p = 0.04$$

Gantikan $p = 0.04$ ke dalam ①.
$$n(0.04) = 48$$
$$n = 1\,200$$

Bilangan calon yang menduduki ujian itu ialah
1 200 orang.
Kebarangkalian seorang calon lulus dalam ujian
itu ialah 0.04.

8.2

A

1. $P(Z > 1.022)$
$= 0.1539 - 0.0005$
$= 0.1534$

P(Z > 1.022)
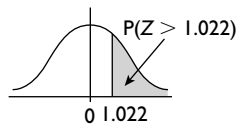
2. $P(Z \leqslant -0.635)$
$= P(Z \geqslant 0.635)$
$= 0.2643 - 0.0016$
$= 0.2627$

P(Z ≤ −0.635)
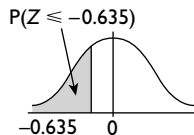
3. $P(Z \leqslant 0.839)$
$= 1 - P(Z > 0.839)$
$= 1 - 0.2008$
$= 0.7992$

P(Z ≤ 0.839)
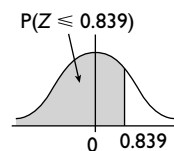
4. $P(-0.75 < Z < 1.2)$
$= 1 - P(Z < -0.75) - P(Z > 1.2)$
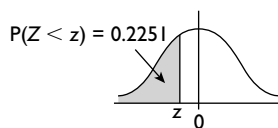$= 1 - P(Z > 0.75) - P(Z > 1.2)$
$= 1 - 0.2266 - 0.1151$
$= 0.6583$

P(−0.75 < Z < 1.2)
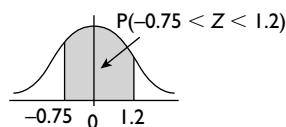
B

1. $P(Z < z) = 0.2251$

Skor-z = −0.755

P(Z < z) = 0.2251

2. $P(Z > z) = 0.7499$
$1 - P(Z < z) = 0.7499$
$P(Z < z) = 0.2501$

Skor-z = −0.674

P(Z > z) = 0.7499
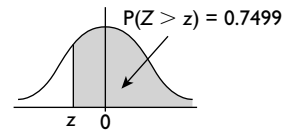
3. $P(z < Z < 0) = 0.342$
$0.5 - P(Z < z) = 0.342$
$P(Z < z) = 0.158$

Skor-z = −1.003

P(z < Z < 0) = 0.342
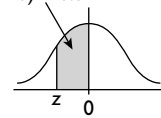
4. $P(0 < Z < z) = 0.363$
$0.5 - P(Z > z) = 0.363$
$P(Z > z) = 0.137$

Skor-z = 1.094

P(0 < Z < z) = 0.363
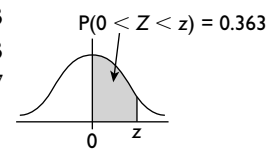
C

1. Diberi $\mu = 72$, $\sigma = 15$
Katakan X mewakili markah yang diperoleh
seorang murid.

(a) $P(X < 68) = P\left(Z < \dfrac{68 - 72}{15}\right)$
$$= P(Z < -0.267)$$
$$= 0.3947$$

(b) $P(69 < X < 78)$

$= P\left(\dfrac{69 - 72}{15} < Z < \dfrac{78 - 72}{15}\right)$
$= P(-0.2 < Z < 0.4)$
$= 1 - P(Z < -0.2) - P(Z > 0.4)$
$= 1 - 0.4207 - 0.3446$
$= 0.2347$

2. Diberi $\mu = 45$, $\sigma^2 = 36$, $\sigma = 6$
Katakan X mewakili umur seorang penduduk.

(a) $P(X > 48) = P\left(Z > \dfrac{48 - 45}{6}\right)$
$$= P(Z > 0.5)$$
$$= 0.3085$$

(b) $P(X > 40) = P\left(Z > \dfrac{40 - 45}{6}\right)$
$$= P(Z > -0.833)$$
$$= 1 - 0.2025$$
$$= 0.7975$$

Bilangan penduduk yang berumur melebihi
40 tahun = 80 000(0.7975)
$$= 63\,800$$

3. Diberi $\mu = 150$, $\sigma^2 = 25$, $\sigma = 5$
Katakan X mewakili jangka hayat bateri itu.

(a) $P(140 < X < 148)$
$= P\left(\dfrac{140 - 150}{5} < Z < \dfrac{148 - 150}{5}\right)$
$= P(-2 < Z < -0.4)$
$= P(Z > 0.4) - P(Z > 2)$
$= 0.3446 - 0.0228$
$= 0.3218$

(b)
$$P(X > x) = 0.1$$
$$P\left(Z > \frac{x - 150}{5}\right) = 0.1$$
$$\frac{x - 150}{5} = 1.281$$
$$x - 150 = 6.405$$
$$x = 156.405$$
$$= 156.4$$

Praktis Formatif: Kertas 1

1. (a) $P(X = 4) = {}^4C_4 p^4 q^0 = \dfrac{16}{81}$
$$p^4 = \left(\frac{2}{3}\right)^4$$
$$p = \frac{2}{3}$$

(b) $\left(1 - \dfrac{2}{3}\right) \times 60 = \dfrac{1}{3} \times 60$
$$= 20$$

2. (a) $P(X \leq 1) + P(X > 3)$
$$= 1 - P(X = 2) - P(X = 3)$$
$$= 1 - a - b$$

(b) $P(X = 4) = {}^4C_4 p^4 q^0$
$$p^4 = \frac{16}{625}$$
$$p = \frac{2}{5}$$

3. (a) (i) $\mu = 0$

(ii) $\sigma = 1$

(b) $P(-1 < Z < 1) = 1 - 2P(Z > 1)$
$$= 1 - 2(0.1587)$$
$$= 0.6826$$

4. (a) Skor-$z = \dfrac{33.2 - 32}{\sigma} = 1.5$
$$\sigma = \frac{1.2}{1.5}$$
$$= 0.8$$

(b)
$$P(X > k) = 0.4013$$
$$P\left(Z > \frac{k - 32}{0.8}\right) = 0.4013$$
$$\frac{k - 32}{0.8} = 0.25$$
$$k - 32 = 0.2$$
$$k = 32.2$$

5. (a) $P(k < z < 0) = 0.3849$

$$P(z < k) = 0.5 - 0.3849$$
$$= 0.1151$$

$$P(z > -k) = 0.1151$$
$$-k = 1.2$$
$$k = -1.2$$

(b)
$$Z = \frac{61.4 - \mu}{5}$$
$$\frac{61.4 - \mu}{5} = -1.2$$
$$61.4 - \mu = -6$$
$$\mu = 67.4$$

Praktis Formatif: Kertas 2

1. (a) (i) $X \sim B(6, p)$
$$P(X = 6) = {}^6C_6 p^6 = 0.262144$$
$$p = \sqrt[6]{0.262144}$$
$$= 0.8$$

(ii) $P(X > 4) = P(X = 5) + P(X = 6)$
$$= {}^6C_5 (0.8)^5 (0.2) + 0.262144$$
$$= 0.393216 + 0.262144$$
$$= 0.65536$$

(b) $X \sim N(450, 225)$

(i)
$$P(X > V) = 0.2611$$
$$P\left(Z > \frac{V - 450}{\sqrt{225}}\right) = 0.2611$$
$$\frac{V - 450}{15} = 0.64$$
$$V = 459.6$$

(ii) $P(420 < X < 470)$
$$= P\left(\frac{420 - 450}{\sqrt{225}} < Z < \frac{470 - 450}{\sqrt{225}}\right)$$
$$= P\left(-2 < Z < \frac{4}{3}\right)$$
$$= 1 - P(Z > 2) - P\left(Z > \frac{4}{3}\right)$$
$$= 1 - 0.0228 - 0.0913$$
$$= 0.8859$$

2. (a) $X \sim B(10, 0.25)$
$$P(X = 4) = {}^{10}C_4 (0.25)^4 (0.75)^6$$
$$= 0.1460$$

(b) $X \sim N(2.4, k^2)$

(i)
$$P(X > 3) = 0.1056$$
$$P\left(Z > \frac{3 - 2.4}{k}\right) = 0.1056$$
$$\frac{0.6}{k} = 1.25$$
$$k = 0.48$$

(ii) $P(2.0 < X < 3.0)$

$$= P\left(\frac{2.0 - 2.4}{0.48} < Z < \frac{3.0 - 2.4}{0.48}\right)$$
$$= P(-0.8333 < Z < 1.25)$$
$$= 1 - 0.2025 - 0.1056$$
$$= 0.6919$$

Bilangan ikan $= 1\,800 \times 0.6919$
$$= 1\,245$$

3. (a) (i) $P(X > 2.8) = P\left(Z > \dfrac{2.8 - 2.2}{0.8}\right)$

$$= P(Z > 0.75)$$
$$= 0.2266$$

(ii) $\qquad P(X < m) = 15\%$

$$P\left(Z < \dfrac{m - 2.2}{0.8}\right) = 0.15$$

$$\dfrac{m - 2.2}{0.8} = -1.036$$
$$m - 2.2 = -0.8288$$
$$m = 1.3712$$
$$m \approx 1.37$$

Jisim minimum tembikai gred II ialah 1.37 kg.

(b) (i) $\qquad P(Y = 1) = 8P(Y = 0)$
$${}^nC_1(0.2)(0.8)^{n-1} = 8(0.2)^0(0.8)^n$$
$$n(0.2)(0.8)^n(0.8)^{-1} = 8(0.8)^n$$

$$n = \dfrac{8}{0.2}(0.8)$$
$$= 32$$

(ii) Sisihan piawai $= \sqrt{npq}$
$$= \sqrt{32(0.2)(0.8)}$$
$$= 2.263$$

FOKUS KBAT

(a) $p - q = 0.15$ dan $p + q = 1$
Maka, $p = 0.575$, $q = 0.425$

$$P(X \geqslant 1) < 0.98$$
$$1 - P(X = 0) < 0.98$$
$$1 - {}^nC_0(0.575)^0(0.425)^n < 0.98$$
$$1 - (1)(1)(0.425)^n < 0.98$$
$$0.425^n > 0.02$$
$$\log_{10} 0.425^n > \log_{10} 0.02$$
$$n \log_{10} 0.425 > \log_{10} 0.02$$
$$n < \dfrac{\log_{10} 0.02}{\log_{10} 0.425}$$
$$n < 4.57$$

Bilangan maksimum buah-buahan = 4 biji.

(b) Min, $\mu = 40$

$$P(32 < X < 38)$$
$$= P\left(\dfrac{32 - 40}{8} < Z < \dfrac{38 - 40}{8}\right)$$
$$= P(-1 < Z < -0.25)$$
$$= P(Z > 0.25) - P(Z > 1)$$
$$= 0.4013 - 0.1587$$
$$= 0.2426$$

Jumlah bilangan peserta $= \dfrac{20}{0.2426}$
$$= 82$$

Terdapat seorang pemenang sahaja.

$$P(X \leqslant x) = \dfrac{1}{82}$$

$$P\left(Z < \dfrac{x - 40}{8}\right) = 0.0122$$

Daripada jadual taburan normal,
$P(Z > 2.25) = 0.0122$.

Maka, $\dfrac{x - 40}{8} = -2.25$

$$x = 22$$

Masa minimum = 22 minit.

BAB 9: GERAKAN PADA GARIS LURUS

9.1

A

1. (a) $s = 3^2 + 3 = 12$

(b) $\quad t^2 + t = 6$
$t^2 + t - 6 = 0$
$(t - 2)(t + 3) = 0$
$t = 2$ atau $t = -3$

Oleh sebab $t \geqslant 0$, maka $t = 2$.

2. (a) $s = 5^2 + 3(5) = 40$

(b) $\quad t^2 + 3t = 10$
$t^2 + 3t - 10 = 0$
$(t - 2)(t + 5) = 0$
$t = 2$ atau $t = -5$

Oleh sebab $t \geqslant 0$, maka $t = 2$.

3. (a) $s = 2(4)^2 - 5(4) + 6 = 18$

(b) $\quad 2t^2 - 5t + 6 = 3$
$2t^2 - 5t + 3 = 0$
$(t - 1)(2t - 3) = 0$
$t = 1$ atau $t = \dfrac{3}{2}$

4. (a) $s = 6(3) - 2(3)^2 = 0$

(b) $\quad 6t - 2t^2 = 4$
$2t^2 - 6t + 4 = 0$
$t^2 - 3t + 2 = 0$
$(t - 1)(t - 2) = 0$
$t = 1$ atau $t = 2$

5. (a) $s = 4(5) - 3(5)^2 + 2 = -53$

(b) $\quad 4t - 3t^2 + 2 = -2$
$3t^2 - 4t - 4 = 0$
$(t - 2)(3t + 2) = 0$
$t = 2$ atau $t = -\dfrac{2}{3}$

Oleh sebab $t \geqslant 0$, maka $t = 2$.

B

1. Apabila $t = 0$, $s = 0$ m
Apabila $t = 1$, $s = 7(1) - 3(1)^2$
$= 4$ m
Jarak yang dilalui $= 4 - 0$
$= 4$ m

2. Apabila $t = 3$, $s = 7(3) - 3(3)^2$
$= 21 - 27$
$= -6$ m
Apabila $t = 4$, $s = 7(4) - 3(4)^2$
$= 28 - 48$
$= -20$ m
Jarak yang dilalui $= |-20 - (-6)|$
$= 14$ m

3. Apabila $t = 4$, $s = 7(4) - 3(4)^2$
$= 28 - 48$
$= -20$ m
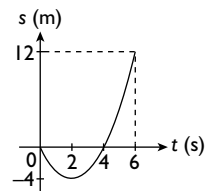Apabila $t = 5$, $s = 7(5) - 3(5)^2$
$= 35 - 75$
$= -40$ m
Jarak yang dilalui $= |-40 - (-20)|$
$= 20$ m

C

1. $t = 2$, $\quad s = 2^2 - 4(2)$
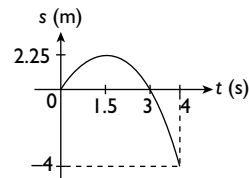$= -4$ m
$t = 6$, $\quad s = 6^2 - 4(6)$
$= 12$ m
Jumlah jarak yang dilalui
$= 4 + 4 + 12$
$= 20$ m

2. $t = 1.5$, $\quad s = 3(1.5) - 1.5^2$
$= 2.25$ m
$t = 4$, $\quad s = 3(4) - 4^2$
$= -4$ m
Jumlah jarak yang dilalui
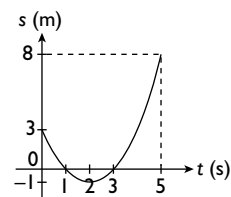$= 2.25 + 2.25 + 4$
$= 8.5$ m

3. $t = 0$, $\quad s = (-1)(-3)$
$= 3$ m
$t = 2$, $\quad s = (2 - 1)(2 - 3)$
$= -1$ m
$t = 5$, $\quad s = (5 - 1)(5 - 3)$
$= 8$ m
Jumlah jarak yang dilalui
$= 3 + 1 + 1 + 8$
$= 13$ m

9.2

A

1. $v = \dfrac{ds}{dt} = 4t - 6$

Apabila $t = 3$, $v = 4(3) - 6$
$= 6$ m s^{-1}

2. $v = \dfrac{ds}{dt} = 5 - 2t$

Apabila $t = 3$, $\quad v = 5 - 2(3)$
$$\qquad\qquad\qquad = -1 \text{ m s}^{-1}$$

3. $v = \dfrac{ds}{dt} = 4 - 10t$

Apabila $t = 3$, $\quad v = 4 - 10(3)$
$$\qquad\qquad\qquad = -26 \text{ m s}^{-1}$$

4. $v = \dfrac{ds}{dt} = 3t^2 - 4t + 3$

Apabila $t = 3$,
$v = 3(3)^2 - 4(3) + 3$
$\quad = 18 \text{ m s}^{-1}$

5. $v = \dfrac{ds}{dt} = 2t - 12t^2 + 6$

Apabila $t = 3$,
$v = 2(3) - 12(3)^2 + 6$
$\quad = -96 \text{ m s}^{-1}$

B

1. $s = 3t^2 - 12t - 4$

$v = \dfrac{ds}{dt} = 6t - 12$

(a) Apabila zarah itu berada dalam keadaan pegun, $v = 0$.
$6t - 12 = 0$
$\qquad 6t = 12$
$\qquad\ t = 2$

(b) Apabila zarah itu bergerak ke arah positif, $v > 0$.
$6t - 12 > 0$
$\qquad 6t > 12$
$\qquad\ t > 2$

2. $s = \dfrac{1}{3}t^3 - \dfrac{3t^2}{2} + 2t + 4$

$v = \dfrac{ds}{dt} = t^2 - 3t + 2$

(a) Apabila zarah itu berhenti seketika, $v = 0$.
$t^2 - 3t + 2 = 0$
$(t - 1)(t - 2) = 0$
$t = 1$ atau $t = 2$

(b) Apabila zarah itu bergerak dengan halaju negatif, $v < 0$.
$t^2 - 3t + 2 < 0$
$(t - 1)(t - 2) < 0$
$1 < t < 2$

3. $s = 2t^3 - 9t^2 - 24t + 8$

$v = \dfrac{ds}{dt} = 6t^2 - 18t - 24$

(a) Apabila zarah itu berhenti seketika, $v = 0$.
$6t^2 - 18t - 24 = 0$
$t^2 - 3t - 4 = 0$
$(t + 1)(t - 4) = 0$
$t = -1$ atau $t = 4$

Oleh sebab $t \geqslant 0$, maka $t = 4$.

(b) Apabila zarah itu bergerak ke arah kanan, $v > 0$.
$t^2 - 3t - 4 > 0$
$(t + 1)(t - 4) > 0$
$t < -1$ atau $t > 4$

Oleh sebab $t \geqslant 0$, maka $t > 4$.

C

1. $s = \int (3t^2 - 8t + 5)\, dt$
$\quad = t^3 - 4t^2 + 5t + c$

Pada $t = 0$, $s = 0$, $c = 0$.
Maka, $s = t^3 - 4t^2 + 5t$

Pada $t = 2$,
$s = 2^3 - 4(2)^2 + 5(2)$
$\quad = 2 \text{ m}$

2. $s = \int (4t - 6t^2 - 5)\, dt$
$\quad = 2t^2 - 2t^3 - 5t + c$

Pada $t = 0$, $s = 0$, $c = 0$.
Maka, $s = 2t^2 - 2t^3 - 5t$

Pada $t = 2$,
$s = 2(2)^2 - 2(2)^3 - 5(2)$
$\quad = -18 \text{ m}$

D

1. (a) Apabila zarah itu bertukar arah gerakannya, $v = 0$.
$15 - 3t = 0$
$\qquad t = 5$
$s = \int (15 - 3t)\, dt = 15t - \dfrac{3t^2}{2} + c$
Pada $t = 0$, $s = 0$, $c = 0$.
Maka, $s = 15t - \dfrac{3t^2}{2}$
Pada $t = 5$, $s = 15(5) - \dfrac{3}{2}(5)^2 = 37.5 \text{ m}$

(b) Apabila zarah itu kembali ke O semula, $s = 0$.
$15t - \dfrac{3t^2}{2} = 0$
$t\left(15 - \dfrac{3}{2}t\right) = 0$
$t = 0$ atau $t = 10$
Pada $t = 10$, $v = 15 - 3(10)$
$$\qquad\qquad\qquad = -15 \text{ m s}^{-1}$$

2. (a) Apabila $v = -3$,
$7 - 2t = -3$
$\qquad 2t = 10$
$\qquad\ t = 5$
$s = \int (7 - 2t)\, dt = 7t - t^2 + c$
Pada $t = 0$, $s = 5$, $c = 5$.
Maka, $s = 7t - t^2 + 5$
Pada $t = 5$, $s = 7(5) - 5^2 + 5 = 15 \text{ m}$

(b) Pada $t = 4$, $s = 7(4) - 4^2 + 5 = 17 \text{ m}$
Pada $t = 5$, $s = 15 \text{ m}$
Jarak yang dilalui $= |15 - 17|$
$$\qquad\qquad\qquad = 2 \text{ m}$$

A

1. $a = \dfrac{dv}{dt} = 8t - 3$

  (a) Pada $t = 0$, $a = 8(0) - 3$
$$= -3 \text{ m s}^{-2}$$

  (b) Pada $t = 3$, $a = 8(3) - 3$
$$= 21 \text{ m s}^{-2}$$

2. $a = \dfrac{dv}{dt} = 3t^2 - 12t + 2$

  (a) Pada $t = 0$, $a = 3(0)^2 - 12(0) + 2$
$$= 2 \text{ m s}^{-2}$$

  (b) Pada $t = 3$, $a = 3(3)^2 - 12(3) + 2$
$$= -7 \text{ m s}^{-2}$$

3. $a = \dfrac{dv}{dt} = 12 - 12t^2$

  (a) Pada $t = 0$, $a = 12 - 12(0)^2$
$$= 12 \text{ m s}^{-2}$$

  (b) Pada $t = 3$, $a = 12 - 12(3)^2$
$$= -96 \text{ m s}^{-2}$$

B

1. (a) $a = 4t - 3$
$$v = \int (4t - 3)\, dt$$
$$= 2t^2 - 3t + c$$

    Pada $t = 0$, $v = 5$, $c = 5$.
    Maka, $v = 2t^2 - 3t + 5$

    Pada $t = 2$, $v = 2(2)^2 - 3(2) + 5$
$$= 7 \text{ m s}^{-1}$$

  (b) Apabila v maksimum atau minimum, $a = 0$.
$$4t - 3 = 0$$
$$t = \dfrac{3}{4}$$
$$\dfrac{d^2 v}{dt^2} = \dfrac{da}{dt} = 4 > 0$$

    $\Rightarrow v$ adalah minimum apabila $t = \dfrac{3}{4}$.

    Halaju minimum $= 2\left(\dfrac{3}{4}\right)^2 - 3\left(\dfrac{3}{4}\right) + 5$
$$= 3\dfrac{7}{8} \text{ m s}^{-1}$$

2. (a) $a = 32 - 8t$
$$v = \int (32 - 8t)\, dt$$
$$= 32t - 4t^2 + c$$

    Pada $t = 0$, $v = 6$, $c = 6$.
    Maka, $v = 32t - 4t^2 + 6$

    Pada $t = 2$, $v = 32(2) - 4(2)^2 + 6$
$$= 54 \text{ m s}^{-1}$$

  (b) Apabila v maksimum atau minimum, $a = 0$.
$$32 - 8t = 0$$
$$t = 4$$
$$\dfrac{d^2 v}{dt^2} = \dfrac{da}{dt} = -8 < 0$$

    $\Rightarrow v$ adalah maksimum apabila $t = 4$.

    Halaju maksimum $= 32(4) - 4(4)^2 + 6$
$$= 70 \text{ m s}^{-1}$$

C

1. $a = 6t + 4$
$$v = \int (6t + 4)\, dt$$
$$= 3t^2 + 4t + c$$

Pada $t = 0$, $v = 5$, $c = 5$.
Maka, $v = 3t^2 + 4t + 5$
$$s = \int (3t^2 + 4t + 5)\, dt$$
$$= t^3 + 2t^2 + 5t + k$$

Pada $t = 0$, $s = 0$, $k = 0$.
Maka, $s = t^3 + 2t^2 + 5t$

Pada $t = 3$, $s = 3^3 + 2(3)^2 + 5(3)$
$$= 60 \text{ m}$$

2. $a = 3 - 4t$
$$v = \int (3 - 4t)\, dt$$
$$= 3t - 2t^2 + c$$

Pada $t = 0$, $v = 5$, $c = 5$.
Maka, $v = 3t - 2t^2 + 5$
$$s = \int (3t - 2t^2 + 5)\, dt$$
$$= \dfrac{3t^2}{2} - \dfrac{2t^3}{3} + 5t + k$$

Pada $t = 0$, $s = 0$, $k = 0$.

Maka, $s = \dfrac{3t^2}{2} - \dfrac{2t^3}{3} + 5t$

Pada $t = 2$, $s = \dfrac{3}{2}(2)^2 - \dfrac{2}{3}(2)^3 + 5(2)$
$$= 10\dfrac{2}{3} \text{ m}$$

3. $a = 2 - 6t$
$$v = \int (2 - 6t)\, dt$$
$$= 2t - 3t^2 + c$$

Pada $t = 0$, $v = 5$, $c = 5$.
Maka, $v = 2t - 3t^2 + 5$

Apabila zarah itu berhenti seketika, $v = 0$.
$$2t - 3t^2 + 5 = 0$$
$$3t^2 - 2t - 5 = 0$$
$$(3t - 5)(t + 1) = 0$$

Oleh sebab $t \geqslant 0$, maka $t = \dfrac{5}{3}$.
$$s = \int (2t - 3t^2 + 5)\, dt$$
$$= t^2 - t^3 + 5t + k$$
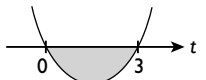
Pada $t = 0$, $s = 0$, $k = 0$.
Maka, $s = t^2 - t^3 + 5t$

Pada $t = \dfrac{5}{3}$, $s = \left(\dfrac{5}{3}\right)^2 - \left(\dfrac{5}{3}\right)^3 + 5\left(\dfrac{5}{3}\right)$

$\qquad\qquad = 6\dfrac{13}{27}$ m

Praktis Formatif: Kertas 2

1. (a) $a = 4t - 12$
Apabila $t = 0$, $a = -12$
Pecutan awal zarah itu ialah -12 m s^{-2}.

(b) $v = \int (4t - 12)\, dt$
$\qquad = 2t^2 - 12t + c$
Pada $t = 0$, $v = 10$, $c = 10$.
Maka, $v = 2t^2 - 12t + 10$
Apabila halaju minimum, $\dfrac{dv}{dt} = a = 0$.
$\qquad 4t - 12 = 0$
$\qquad\qquad t = 3$
$\dfrac{d^2v}{dt^2} = 4 > 0$
v adalah minimum apabila $t = 3$.
Halaju minimum $= 2(3)^2 - 12(3) + 10$
$\qquad\qquad\qquad = -8$ m s^{-1}

(c) Apabila zarah itu berhenti seketika, $v = 0$.
$\qquad 2t^2 - 12t + 10 = 0$
$\qquad\quad t^2 - 6t + 5 = 0$
$\qquad (t - 1)(t - 5) = 0$
$\qquad t = 1$ atau $t = 5$

(d) $s = \int (2t^2 - 12t + 10)\, dt$
$\qquad = \dfrac{2t^3}{3} - 6t^2 + 10t + c$
Pada $t = 0$, $s = 0$, $c = 0$.
Maka, $s = \dfrac{2}{3}t^3 - 6t^2 + 10t$.
Apabila $t = 1$, $s = \dfrac{2}{3} - 6 + 10$
$\qquad\qquad\qquad = 4\dfrac{2}{3}$ m
Apabila $t = 4$, $s = \dfrac{2}{3}(64) - 6(16) + 10(4)$
$\qquad\qquad\qquad = -13\dfrac{1}{3}$ m
Jumlah jarak yang dilalui dalam 4 saat pertama
$= 2\left(4\dfrac{2}{3}\right) + 13\dfrac{1}{3}$
$= 22\dfrac{2}{3}$ m

2. (a) $v = at^2 + bt$
Apabila $t = 3$, $v = 0$.
$a(3)^2 + b(3) = 0$
$\qquad 9a + 3b = 0$
$\qquad 3a + b = 0 \qquad$ .... ①
pecutan $= \dfrac{dv}{dt} = 2at + b$
Apabila $t = 1$, $a = -3$.
$\qquad 2a + b = -3 \quad$ .... ②

① – ②: $\qquad a = 3$
Dari ①, $\qquad b = -3a = -9$

(b) $v = 3t^2 - 9t$
Apabila zarah itu bergerak ke arah kiri, $v < 0$.
$\qquad 3t^2 - 9t < 0$
$\qquad\quad t(t - 3) < 0$
$\qquad\quad 0 < t < 3$

(c) $s = \int (3t^2 - 9t)\, dt$
$\qquad = t^3 - \dfrac{9t^2}{2} + c$
Apabila $t = 0$, $s = 0$, $c = 0$.
Maka, $s = t^3 - \dfrac{9}{2}t^2$
Apabila $t = 2$, $s = 8 - \dfrac{9}{2}(4) = -10$
Apabila $t = 3$, $s = 27 - \dfrac{9}{2}(9) = -13.5$
Jarak yang dilalui oleh zarah itu dalam saat ketiga $= |-13.5 - (-10)|$
$\qquad\qquad = 3.5$ m

3. (a) $v = 12 + 4t - t^2$
Apabila $t = 0$, $v = 12$
Halaju awal $= 12$ m s^{-1}

(b) Apabila zarah itu berhenti seketika, $v = 0$.
$\qquad 12 + 4t - t^2 = 0$
$\qquad\quad t^2 - 4t - 12 = 0$
$\qquad (t - 6)(t + 2) = 0$
Oleh sebab $t > 0$, maka $t = 6$.

(c) $\dfrac{dv}{dt} = 4 - 2t$
Apabila halaju maksimum, $\dfrac{dv}{dt} = 0$
$\qquad\qquad 4 - 2t = 0$
$\qquad\qquad\qquad t = 2$
$\qquad\qquad \dfrac{d^2v}{dt^2} = -2 < 0$
Maka, v adalah maksimum apabila $t = 2$.
Halaju maksimum $= 12 + 4(2) - 2^2 = 16$ m s^{-1}

(d) $s = \int (12 + 4t - t^2)\, dt$
$\qquad = 12t + 2t^2 - \dfrac{t^3}{3} + c$
Apabila $t = 0$, $s = 0$, maka $c = 0$.
$s = 12t + 2t^2 - \dfrac{t^3}{3}$
Apabila $t = 6$,
$s = 12(6) + 2(6)^2 - \dfrac{6^3}{3}$
$\quad = 72$
Apabila $t = 10$,
$s = 12(10) + 2(10)^2 - \dfrac{10^3}{3}$
$\quad = -13\dfrac{1}{3}$
Jumlah jarak yang dilalui
$= 2(72) + \left|-13\dfrac{1}{3}\right|$
$= 157\dfrac{1}{3}$ m

4. (a) $v = ht^2 - 8t$

$\quad a = \dfrac{dv}{dt} = 2ht - 8$

Apabila $t = 3$, $a = 4$.
$2h(3) - 8 = 4$
$\quad\quad 6h = 12$
$\quad\quad\ \ h = 2$

(b) $v = 2t^2 - 8t$

Apabila halaju zarah itu menyusut, $v < 0$.
$2t^2 - 8t < 0$
$\ t^2 - 4t < 0$
$t(t - 4) < 0$
Maka, julat masa ialah $0 < t < 4$.

(c) Apabila zarah itu berhenti seketika, $v = 0$.
$2t^2 - 8t = 0$
$t(t - 4) = 0$
$t = 0$ atau $t = 4$
Maka, $t = 4$ s.

(d) Jumlah jarak yang dilalui

$= \left| \displaystyle\int_0^4 \left(2t^2 - 8t\right) dt \right| + \displaystyle\int_4^5 \left(2t^2 - 8t\right) dt$

$= -\left[\dfrac{2t^3}{3} - 4t^2 \right]_0^4 + \left[\dfrac{2t^3}{3} - 4t^2 \right]_4^5$

$= -\left[\dfrac{2}{3}(64) - 4(16) \right] + \Big[\dfrac{2}{3}(125) - 4(25) -$

$\quad\quad \dfrac{2}{3}(64) + 4(16) \Big]$

$= \dfrac{64}{3} + \dfrac{14}{3}$

$= 26$ m

5. (a) $s_B = 4t^3 - 3t$

$\quad v_B = \dfrac{ds}{dt}$

$\quad\quad = 12t^2 - 3$

Apabila $t = 0$, $v_B = -3$ m s^{-1}.
Maka, halaju awal zarah B ialah -3 m s^{-1}.

(b) Apabila zarah B berhenti seketika, $v_B = 0$.
$\quad\quad 12t^2 - 3 = 0$

$\quad\quad\quad\quad t^2 = \dfrac{1}{4}$

$\quad\quad\quad\quad t = \dfrac{1}{2} \ (t > 0)$

Apabila $t = 0$, $s_B = 0$.

$t = \dfrac{1}{2}$, $s_B = 4\left(\dfrac{1}{2}\right)^3 - 3\left(\dfrac{1}{2}\right) = -1$

$t = 3$, $s_B = 4(3)^3 - 3(3) = 99$

Jumlah jarak yang dilalui oleh zarah B dalam 3 saat pertama
$= 1 + 1 + 99$
$= 101$ m

(c) $s_A = \displaystyle\int (12t^2 + 6)\, dt$

$\quad\ = 4t^3 + 6t + c$

Apabila $t = 0$, $s_A = -18$ (merujuk titik P)
$c = -18$
$\therefore s_A = 4t^3 + 6t - 18$
Apabila kedua-dua zarah berhenti, $s_A = s_B$.
$\quad 4t^3 + 6t - 18 = 4t^3 - 3t$
$\quad\quad\quad\quad 9t = 18$
$\quad\quad\quad\quad\ t = 2$
Apabila $t = 2$, $s_A = s_B$
$\quad\quad\quad\quad\quad\quad = 4(2)^3 - 3(2)$
$\quad\quad\quad\quad\quad\quad = 26$

Jarak bagi zarah-zarah itu dari titik P apabila zarah A dan zarah B bertemu ialah 26 m.

FOKUS KBAT

(a) $a = \dfrac{dv}{dt}$

$\quad\ = 2t - 10$

Pada halaju minimum, $a = 0$.
$\quad\quad 2t - 10 = 0$
$\quad\quad\quad\quad t = 5$ s

Halaju minimum, $v = 5^2 - 10(5) + 32$
$\quad\quad\quad\quad\quad\quad = 7$ m s^{-1}

Kereta mainan itu tidak bergerak songsang kerana halaju minimum > 0.
Maka, kereta mainan itu hanya bergerak ke hadapan.

(b) $s = \displaystyle\int_0^5 (t^2 - 10t + 32)\, dt$

$\quad = \left[\dfrac{t^3}{3} - 5t^2 + 32t \right]_0^5$

$\quad = \left[\dfrac{(5)^3}{3} - 5(5)^2 + 32(5) \right] - 0$

$\quad = 76\dfrac{2}{3}$ m

$\sin 10° = \dfrac{h}{76\dfrac{2}{3}}$

$\quad\quad h = 76\dfrac{2}{3} \sin 10°$

$\quad\quad\ \ = 13.31$ m

BAB 10: PENGATURCARAAN LINEAR

10.1

A

1.

2.

B

1. Ketaksamaan linear:

$y \leq x$

$x + y \geq 4$

$x \leq 4$

2. Persamaan garis lurus:

$x = 1$

$y = x$

$x + y = 5$

Ketaksamaan linear:

$x > 1$

$y \geq x$

$x + y \leq 5$

3. Persamaan garis lurus:

$x + y = 4$

$y = \dfrac{1}{2}x$

$2x + 3y = 12$

Ketaksamaan linear:

$x + y > 4$

$y \geq \dfrac{1}{2}x$

$2x + 3y \leq 12$

4. Persamaan garis lurus:

$y = x + 2$

$x + y = 6$

$x + 3y = 6$

Ketaksamaan linear:

$y \leq x + 2$

$x + y \leq 6$

$x + 3y > 6$

5. Persamaan garis lurus:

$x = 0$

$y = x + 4$

$y = \dfrac{1}{2}x,\ x + y = 8$

Ketaksamaan linear:

$x \geq 0$

$y \leq x + 4$

$y > \dfrac{1}{2}x,\ x + y \leq 8$

10.2

A

1. I : $x \geq 8$

II : $x + y \leq 20$

III : $y > 2x$

IV : $180x + 120y \leq 3\ 000$
$3x + 2y \leq 50$

2. I : $1.8x \leq 80$
$9x \leq 400$

II : $x + y < 450$

III : $1.8x + 1.5y \geq 200$
$18x + 15y \geq 2\ 000$

3. I : $20x \geq 1\ 500$
$x \geq 75$

II : $50y \geq 4\ 000$
$y \geq 80$

III : $y > 2x$

IV : $20x + 50y \leq 9\ 000$
$2x + 5y \leq 900$

B

1. Harga bagi x buah kalkulator = RM35x

Harga bagi y buah jam = RM45y

Fungsi objektif, $k = 35x + 45y$

2. Kos bagi x unit robot model A = RM80x

Kos bagi y unit robot model B = RM120y

Fungsi objektif, $k = 80x + 120y$

3. Kos operasi sehari bagi x unit mesin P = RM40x

Kos operasi sehari bagi y unit mesin Q = RM60y
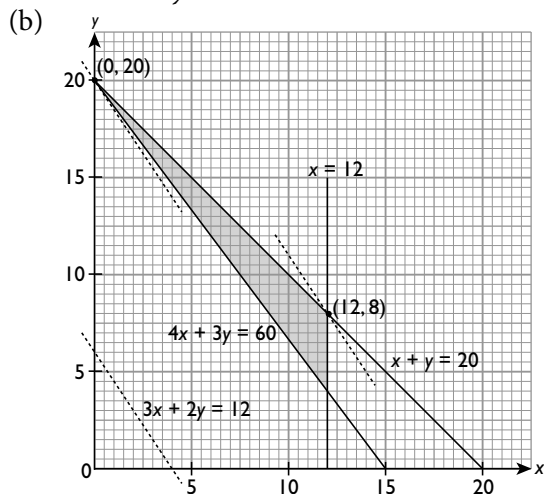
Fungsi objektif, $k = 40x + 60y$

4. Upah harian bagi x orang pekerja mahir
$= RM80x$

Upah harian bagi y orang pekerja tidak mahir
$= RM45y$

Fungsi objektif, $k = 80x + 45y$

5. Keuntungan daripada jualan x unit peti sejuk
$= RM250x$

Keuntungan daripada jualan y unit televisyen
$= RM200y$

Fungsi objektif, $k = 250x + 200y$

C

1. (a) I   : $40x + 30y \geqslant 600$
$\qquad 4x + 3y \geqslant 60$

II   : $x \leqslant 12$

III : $x + y \leqslant 20$

(b)

(c) Kos operasi ialah $k = 300x + 200y$.
Katakan $k = 1\,200$
$300x + 200y = 1\,200$
$\qquad 3x + 2y = 12$

Lukis garis $3x + 2y = 12$ sebagai rujukan.

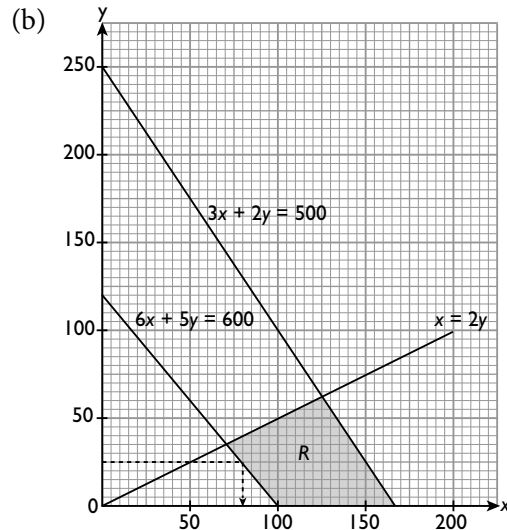Daripada graf, titik optimum ialah $(0, 20)$ dan $(12, 8)$.

Kos operasi minimum
$= 300(0) + 200(20)$
$= RM4\,000$

Kos operasi maksimum
$= 300(12) + 200(8)$
$= RM5\,200$

2. (a) I   : $12x + 8y \leqslant 2\,000$
$\qquad 3x + 2y \leqslant 500$

II   : $x \geqslant 2y$

III : $(18 - 12)x + (13 - 8)y \geqslant 600$
$\qquad 6x + 5y \geqslant 600$

(b)

(c) (i) Bilangan maksimum kemeja-$T = 62$

(ii) Apabila $y = 25$, nilai minimum $x = 80$
Jumlah keuntungan minimum
$= 6(80) + 5(25)$
$= RM605$

Praktis Formatif: Kertas 2

1. (a) I   : $80x + 60y \leqslant 4\,800$
$\qquad 4x + 3y \leqslant 240$

II   : $x + y \geqslant 40$

III : $y \geqslant \dfrac{3}{4}x$

(b)
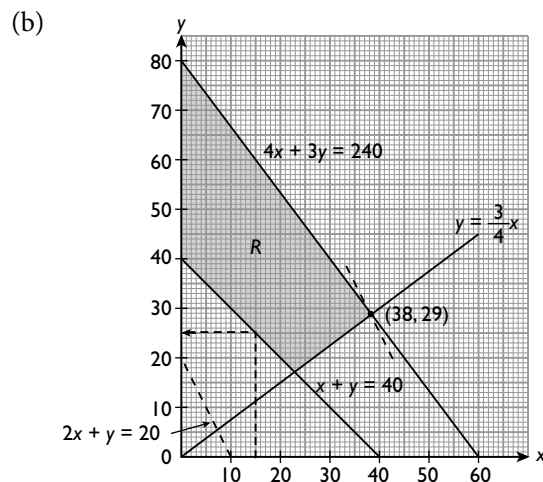
(c) (i) Daripada graf, apabila $x = 15$, $y = 25$.
Bilangan minimum kerusi ialah 25 buah.

(ii) Fungsi objektif ialah $50x + 25y$.
Katakan $50x + 25y = 500$
$\qquad\qquad 2x + y = 20$

Daripada graf, titik optimum ialah $(38, 29)$.

Jumlah keuntungan maksimum
$= 50(38) + 25(29)$
$= RM2\,625$

2. (a) I : $x + y \leqslant 18$

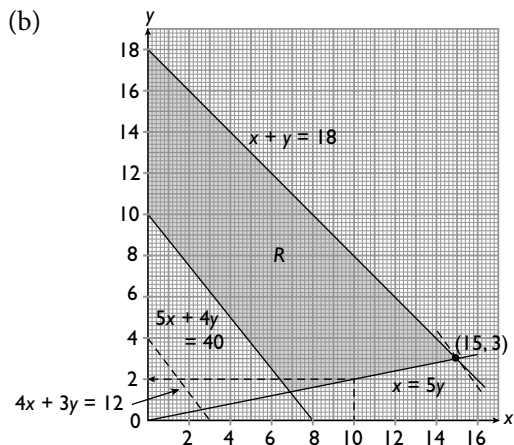II : $x \leqslant 5y$

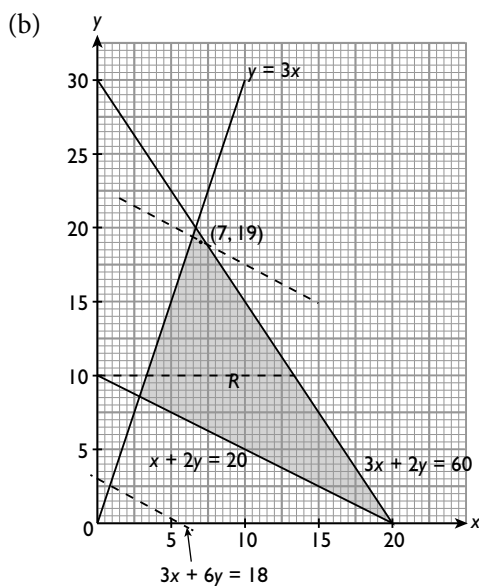III : $50x + 40y \geqslant 400$

$5x + 4y \geqslant 40$

(b)

(c) (i) Apabila $x = 10$, nilai minimum $y = 2$. Bilangan minimum perjalanan ke Pulau Q ialah 2.

(ii) Fungsi objektif ialah RM$(40x + 30y)$. Katakan $40x + 30y = 120$

$4x + 3y = 12$

Daripada graf, titik optimum ialah $(15, 3)$.

Keuntungan maksimum
$= 40(15) + 30(3)$
$= $ RM690

3. (a) I : $2x + 4y \geqslant 40$

$x + 2y \geqslant 20$

II : $300x + 200y \leqslant 6\ 000$

$3x + 2y \leqslant 60$

III : $y \leqslant 3x$

(b)

(c) (i) Apabila $y = 10$, $3 \leqslant x \leqslant 13$

(ii) Fungsi objektif ialah $3x + 6y$. Katakan $3x + 6y = 18$.

Daripada graf, titik optimum ialah $(7, 19)$.

Bilangan maksimum murid
$= 3(7) + 6(19)$
$= 135$ orang

4. (a) I : $x + y \leqslant 180$

II : $y \geqslant \dfrac{4}{5}x$

III : $8x + 6y \geqslant 480$

$4x + 3y \geqslant 240$

(b)
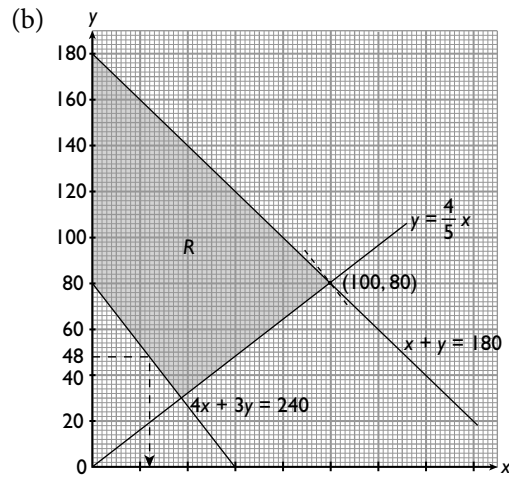
(c) (i) Apabila $y = 48$, nilai minimum $x = 24$. Maka, bilangan minimum kipas angin jenis P yang dijual $= 24$ unit.

(ii) Daripada graf, titik optimum ialah $(100, 80)$.

Keuntungan maksimum
$= 8(100) + 6(80)$
$= $ RM1 280

5. (a) I : $40x + 30y \leqslant 2\ 400$

$4x + 3y \leqslant 240$

II : $y \leqslant 3x$

(b) Bilangan kalkulator tidak melebihi 2 kali bilangan kotak geometri.

(c)

(d) (i) Apabila $x = 25$, nilai minimum $y = 13$.
Bilangan minimum kotak geometri ialah 13.

(ii) Daripada graf, titik optimum ialah $(19, 54)$.

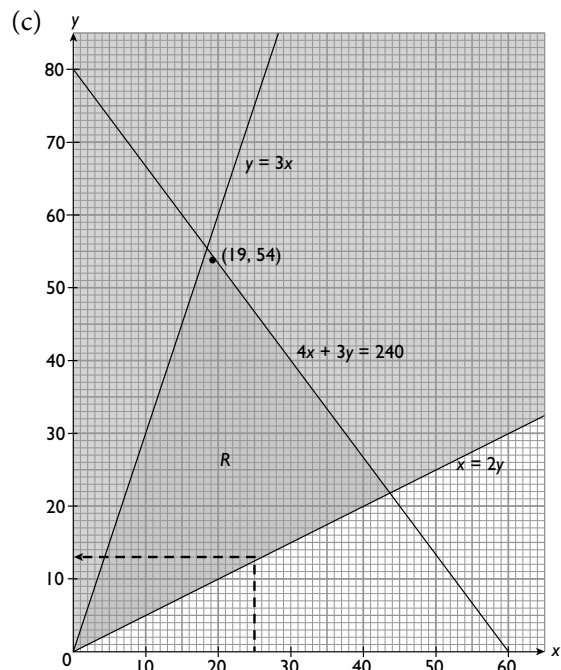Maka, jumlah maksimum bilangan kalkulator dan kotak geometri yang boleh dibeli
$= 19 + 54$
$= 73$

6. (a) I : $x + y > 60$
II : $8x + 6y \leq 960$
$4x + 3y \leq 480$

III : $x : y \leq 4 : 5$
$\dfrac{x}{y} \leq \dfrac{4}{5}$
$y \geq \dfrac{5}{4}x$

(b)
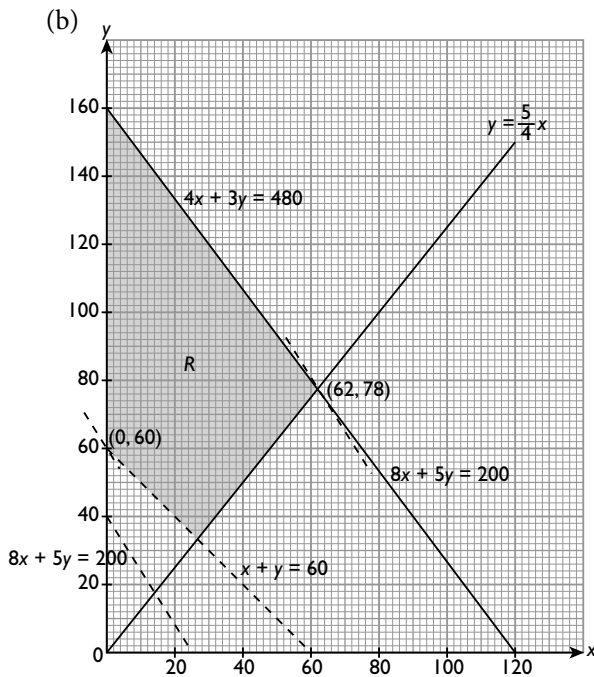
(c) Jumlah jualan = RM$(8x + 5y)$
Katakan $8x + 5y = 200$.

Lukis $8x + 5y = 200$ sebagai rujukan.

Titik optimum = $(0, 60)$ dan $(62, 78)$

Jumlah jualan minimum
$= $ RM$[8(0) + 5(60)]$
$= $ RM300

Jumlah jualan maksimum
$= $ RM$[8(62) + 5(78)]$
$= $ RM886

RM300 $\leq$ Jumlah jualan $\leq$ RM886

FOKUS KBAT

(a) I : $x < 3y$
II : $80x + 60y \leq 1\,200$

(b) Bilangan bangku melebihi bilangan meja tidak lebih daripada 10 buah.

(c)
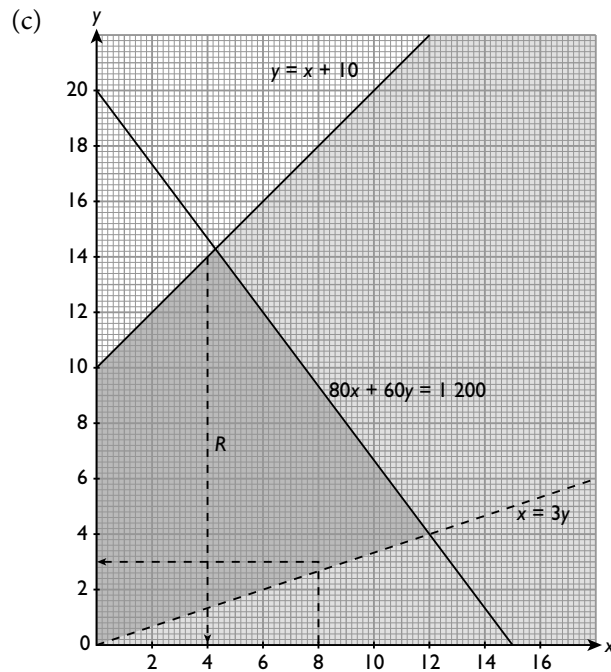
(d) (i) Integer maksimum bagi $y = 14$.
Bilangan meja yang boleh dibeli = 4

(ii) Apabila $x = 8$, nilai minimum bagi $y = 3$.
Baki peruntukkan maksimum
$= $ RM1 200 – 8(RM80) – 3(RM60)
$= $ RM380

KERTAS MODEL SPM

KERTAS 1

1. (a) R (b) P

2. Katakan α ialah punca yang lain.

Maka, $k + \alpha = -(-4)$

$\qquad \alpha = 4 - k$

$p = k(4 - k)$

$p = 4k - k^2$

3. Pada titik maksimum $(-4, 6)$, fungsi kuadratik ialah $f(x) = a(x + 4)^2 + 6$.

Pada $(0, -2)$,

$\qquad -2 = a(0 + 4)^2 + 6$

$\qquad -8 = 16a$

$\qquad a = -\dfrac{1}{2}$

$\quad f(x) = -\dfrac{1}{2}(x + 4)^2 + 6$

$\quad f(x) = -\dfrac{1}{2}x^2 - 4x - 2$

4. $\sin x = \dfrac{h}{5}$

$\cos 2x = 1 - 2\sin^2 x$

$\qquad r = 1 - 2\left(\dfrac{h}{5}\right)^2$

$\qquad h = 5\sqrt{\dfrac{1 - r}{2}}$

5. $\qquad 4 \sek^2 x + \tan x - 7 = 0$

$4(\tan^2 x + 1) + \tan x - 7 = 0$

$\qquad 4\tan^2 x + \tan x - 3 = 0$

$\quad (4\tan x - 3)(\tan x + 1) = 0$

$\tan x = \dfrac{3}{4}$

$\qquad x = 36.87°, 216.87°$

atau

$\tan x = -1$

$\qquad x = 135°, 315°$

Maka, $x = 36.87°, 135°, 216.87°, 315°$

6. $\tan 30° = \dfrac{5}{AQ}$

$\qquad AQ = 8.66$ cm

$60° = 60° \times \dfrac{3.142}{180°}$

$\qquad = 1.0473$ radian

Luas kawasan berlorek

$= \dfrac{1}{2}(10)^2 \sin 60° - \dfrac{1}{2}(8.66)^2(1.0473)$

$= 4.03$ cm^2

7. $|4(p + 1) - 9| = 3$

$4p - 5 = 3 \qquad$ atau $\qquad 4p - 5 = -3$

$\qquad p = 2 \qquad\qquad\qquad\qquad p = \dfrac{1}{2}$

8. $g(x) = h^{-1}[hg(x)]$

$\qquad = 15\left(\dfrac{x + 8}{3}\right) - 19$

$\qquad = 5x + 21$

9. $y = 2x - 5 \qquad\qquad$ …… ①

$y = x^2 + kx + 4 \quad$ …… ②

Gantikan ② ke dalam ①.

$\qquad x^2 + kx + 4 = 2x - 5$

$x^2 + (k - 2)x + 9 = 0$

Diberi garis lurus itu menyilang lengkung itu pada dua titik yang berlainan, maka $b^2 - 4ac > 0$.

$(k - 2)^2 - 4(1)(9) > 0$

$k^2 - 4k + 4 - 36 > 0$

$\qquad k^2 - 4k - 32 > 0$

$(k + 4)(k - 8) > 0$
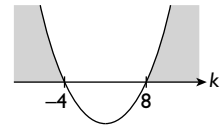
Oleh sebab k adalah negatif, maka julat nilai k ialah $k < -4$.

10. $\qquad \dfrac{12^n \times 2^{n-1}}{3^n} = 16$

$\dfrac{(3 \times 2^2)^n \times 2^{n-1}}{3^n} = 2^4$

$\dfrac{3^n \times 2^{2n} \times 2^{n-1}}{3^n} = 2^4$

$\qquad 2^{2n + n - 1} = 2^4$

$\qquad\qquad 3n - 1 = 4$

$\qquad\qquad\qquad n = \dfrac{5}{3}$

11. $\dfrac{2\log_x 10 - \log_x 0.8}{\log_x 625} = \dfrac{\log_x 10^2 - \log_x 0.8}{\log_x 625}$

$\qquad = \dfrac{\log_x\left(\dfrac{100}{0.8}\right)}{\log_x 625}$

$\qquad = \dfrac{\log_x 125}{\log_x 625}$

$\qquad = \dfrac{\log_x 5^3}{\log_x 5^4}$

$\qquad = \dfrac{3\log_x 5}{4\log_x 5}$

$\qquad = \dfrac{3}{4}$

12. $a = -8$, $d = 5$

$$\frac{n}{2}[2(-8) + (n-1)(5)] = 544$$
$$5n^2 - 21n - 1\,088 = 0$$
$$(5n + 64)(n - 17) = 0$$

Oleh sebab n mesti integer positif, maka $n = 17$.
$$p = T_{17}$$
$$= -8 + (17 - 1)(5)$$
$$= 72$$

13. $r = \frac{1}{2}$

$$S_\infty = \frac{120}{1 - \frac{1}{2}}$$
$$= 240 \text{ cm}$$

14. $\quad \text{P}(X \geqslant 1) = 0.98976$
$$1 - \text{P}(X = 0) = 0.98976$$
$$1 - {}^5C_0 p^0 q^5 = 0.98976$$
$$1 - 0.98976 = (1)(1)q^5$$
$$0.01024 = q^5$$
$$(0.4)^5 = q^5$$
$$q = 0.4$$
$$p = 1 - 0.4 = 0.6$$

Peratus murid perempuan di sekolah itu ialah 60%.

15. $\text{P}(Z > k) = 0.5 - 0.258$
$$= 0.2420$$
Daripada buku sifir, $k = 0.7$.

Luas kawasan berlorek
$$= \text{P}[-2(0.7) < Z < 0.7]$$
$$= 1 - \text{P}(Z > 1.4) - \text{P}(Z > 0.7)$$
$$= 1 - 0.0808 - 0.2420$$
$$= 0.6772$$

16. (a) Kebarangkalian $= 1 - \frac{1}{4} - \frac{2}{5}$
$$= \frac{7}{20}$$

(b) Jumlah bilangan pakaian di dalam almari
$$= 20$$
Kebarangkalian $= \left(\frac{1}{4} \times \frac{15}{19}\right) + \left(\frac{15}{20} \times \frac{5}{19}\right)$
$$= \frac{15}{38}$$

17. $\quad y = \frac{p}{x^n}$

$\log_2 y = \log_2 \frac{p}{x^n}$

$\log_2 y = \log_2 p - \log_2 x^n$

$\log_2 y = -n \log_2 x + \log_2 p$

$-n = \dfrac{9 - 3}{-8 - 0}$

$\quad = -\dfrac{3}{4}$

$n = \dfrac{3}{4}$

$\log_2 p = 3$
$\quad\quad p = 8$

18. $I = \frac{4}{3}\pi j^3$

Apabila $I = 36\pi$ cm^3,
$$36\pi = \frac{4}{3}\pi j^3$$
$$j = 3$$

$$\frac{dI}{dj} = 4\pi j^2$$

$$\delta I = 38\pi - 36\pi$$
$$= 2\pi$$

$$\delta I = \frac{dI}{dj} \times \delta j$$
$$2\pi = 4\pi(3)^2 \times \delta j$$
$$\delta j = \frac{1}{18} \text{ cm}$$

19.
$$\int_{-2}^{1} [f(x) + kx]\, dx = 5$$

$$\int_{-2}^{0} [f(x) + kx]\, dx + \int_{0}^{1} [f(x) + kx]\, dx = 5$$

$$\int_{-2}^{0} f(x)\, dx + \int_{-2}^{0} kx\, dx + (-6) = 5$$

$$8 + \left[\frac{kx^2}{2}\right]_{-2}^{0} - 6 = 5$$

$$\left[0 - \frac{k(-2)^2}{2}\right] = 3$$

$$k = -\frac{3}{2}$$

20. Bilangan susunan
$$= ({}^3C_2 \times {}^5P_2 \times {}^4P_1 \times {}^6P_6) + ({}^3C_1 \times {}^5P_1 \times {}^4P_2 \times {}^6P_6)$$
$$= 172\,800 + 129\,600$$
$$= 302\,400$$

21. $\sum x = (8 \times 11.5) - 17.2 + 12.4$
$$= 87.2$$

Masa lari pecut purata sebenar $= \dfrac{87.2}{8}$
$$= 10.9 \text{ s}$$

22. $\quad \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$\overrightarrow{AD} = \dfrac{1}{n}\overrightarrow{PQ} + \overrightarrow{BD}$

$m\underset{\sim}{x} + \underset{\sim}{y} = \dfrac{1}{n}(2\underset{\sim}{x} + 9\underset{\sim}{y}) + (\underset{\sim}{x} - 5\underset{\sim}{y})$

$m\underset{\sim}{x} + \underset{\sim}{y} = \left(\dfrac{2}{n} + 1\right)\underset{\sim}{x} + \left(\dfrac{9}{n} - 5\right)\underset{\sim}{y}$

$\dfrac{9}{n} - 5 = 1 \quad\quad\quad m = \dfrac{2}{n} + 1$

$\quad n = \dfrac{3}{2} \quad\quad\quad\quad = \dfrac{2}{\frac{3}{2}} + 1$

$\quad\quad\quad\quad\quad\quad\quad\quad = \dfrac{7}{3}$

23. $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BA}$

$\qquad = 3\underset{\sim}{i} + 13\underset{\sim}{j} - (11\underset{\sim}{i} - 2\underset{\sim}{j})$

$\qquad = -8\underset{\sim}{i} + 15\underset{\sim}{j}$

$|\overrightarrow{OA}| = \sqrt{(-8)^2 + 15^2}$

$\qquad = 17$ unit

24. $(5, h) = \left[\dfrac{2(k) + 3(-3)}{2 + 3}, \dfrac{2(7) + 3(2)}{2 + 3}\right]$

$(5, h) = \left(\dfrac{2k - 9}{5}, 4\right)$

$\qquad 5 = \dfrac{2k - 9}{5}$

$\qquad k = 17$

$\qquad h = 4$

25.
$$PT = PQ$$
$$\sqrt{[x - (-1)]^2 + (y - 6)^2} = \sqrt{[3 - (-1)]^2 + (4 - 6)^2}$$
$$(x + 1)^2 + (y - 6)^2 = 4^2 + (-2)^2$$
$$x^2 + 2x + 1 + y^2 - 12y + 36 = 16 + 4$$
$$x^2 + y^2 + 2x - 12y + 17 = 0$$

KERTAS 2

1. $\qquad x + 3y - 5 = 0$

$\qquad\qquad x = 5 - 3y \quad \ldots\ldots ①$

$x^2 - 2y^2 + 4xy = 9 \quad \ldots\ldots ②$

Gantikan ① ke dalam ②.

$\qquad (5 - 3y)^2 - 2y^2 + 4(5 - 3y)y = 9$

$25 - 30y + 9y^2 - 2y^2 + 20y - 12y^2 = 9$

$\qquad\qquad\qquad 5y^2 + 10y - 16 = 0$

$$y = \dfrac{-10 \pm \sqrt{10^2 - 4(5)(-16)}}{2(5)}$$

$y = 1.049$ atau -3.049

Apabila $y = 1.049$, $x = 5 - 3(1.049)$

$\qquad\qquad\qquad = 1.853$

Apabila $y = -3.049$, $x = 5 - 3(-3.049)$

$\qquad\qquad\qquad = 14.147$

Penyelesaian ialah $x = 1.853$, $y = 1.049$ dan $x = 14.147$, $y = -3.049$.

2. (a) $j^2 = 5^2 + x^2$

$\qquad j = \sqrt{25 + x^2}$

$L = \dfrac{1}{2}\pi j^2 - 10x$

$\quad = \dfrac{1}{2}\pi(25 + x^2) - 10x$

$\quad = \dfrac{1}{2}\pi x^2 - 10x + \dfrac{25}{2}\pi$

(b) $\dfrac{dL}{dx} = \pi x - 10$

Apabila luas minimum, $\dfrac{dL}{dx} = 0$.

Maka, $\pi x - 10 = 0$

$\qquad\qquad x = \dfrac{10}{\pi}$

Luas minimum $= \dfrac{1}{2}\pi\left(\dfrac{10}{\pi}\right)^2 - 10\left(\dfrac{10}{\pi}\right) + \dfrac{25}{2}\pi$

$\qquad\qquad\qquad = \dfrac{25}{2}\pi - \dfrac{50}{\pi}$

3. (a) $\dfrac{y - 3}{x - 8} = \dfrac{9 - 3}{-4 - 8}$

$\qquad y - 3 = -\dfrac{1}{2}(x - 8)$

$\qquad\quad y = -\dfrac{1}{2}x + 7$

(b) $m_1 = -\dfrac{1}{2}$, $m_2 = 2$

Melalui $(7, 11)$, maka

$y - 11 = 2(x - 7)$

$\qquad y = 2x - 3 \quad \ldots\ldots ①$

Gantikan ① ke dalam $y = -\dfrac{1}{2}x + 7$.

$2x - 3 = -\dfrac{1}{2}x + 7$

$\dfrac{5}{2}x = 10$

$\quad x = 4$

Daripada ①, $y = 2(4) - 3$

$\qquad\qquad = 5$

Kedudukan bagi perhentian bas itu ialah $(4, 5)$.

(c) Katakan perhentian bas $= S$,
maka $AS : SC = p : q$.

$\left[\dfrac{8p + (-4q)}{p + q}, \dfrac{3p + 9q}{p + q}\right] = (4, 5)$

$\qquad\qquad \dfrac{8p - 4q}{p + q} = 4$

$\qquad\qquad 8p - 4q = 4p + 4q$

$\qquad\qquad\qquad 4p = 8q$

$\qquad\qquad\qquad \dfrac{p}{q} = \dfrac{2}{1}$

$\qquad\qquad p : q = 2 : 1$

Maka, $p = 2$, $q = 1$.

4. Bagi bandar A, $r = 1.07$

Bagi bandar B, $r = 1.1$ dan $T_3 = 65\,000$

$a(1.1)^{3 - 1} = 65\,000$

$\qquad a = \dfrac{65\,000}{1.1^2}$

$\qquad\qquad\qquad T_B > T_A$

$\dfrac{65\,000}{1.1^2}(1.1)^{n - 1} > 80\,000(1.07)^{n - 1}$

$65\,000(1.1)^{n - 3} > 80\,000(1.07)^{n - 1}$

$\log_{10} 65\,000(1.1)^{n - 3} > \log_{10} 80\,000\,(1.07)^{n - 1}$

$\log_{10} 65\,000 + (n - 3)\log_{10} 1.1 > \log_{10} 80\,000 +$
$\qquad\qquad\qquad\qquad\qquad (n - 1)\log_{10} 1.07$

$n \log_{10} 1.1 - n \log_{10} 1.07 > \log_{10} 80\,000 - $
$\qquad\qquad\qquad\log_{10} 1.07 - \log_{10} 65\,000$
$\qquad\qquad\qquad\qquad + 3 \log_{10} 1.1$
$n(\log_{10} 1.1 - \log_{10} 1.07\,) > 0.185$
$\qquad\qquad\qquad n(0.012) > 0.185$
$\qquad\qquad\qquad\qquad\quad n > 15.42$

$n = 16$

Pada tahun 2016, populasi bagi bandar B akan mula melebihi populasi bagi bandar A.

5. (a) $2 \tan x - 2 \tan x \sin^2 x = 2 \tan x(1 - \sin^2 x)$

$\qquad\qquad\qquad\qquad\qquad = 2\left(\dfrac{\sin x}{\text{kos } x}\right)(\text{kos}^2\, x)$

$\qquad\qquad\qquad\qquad\qquad = 2 \sin x\, \text{kos } x$

$\qquad\qquad\qquad\qquad\qquad = \sin 2x$

(b) (i), (ii)

$10\pi \tan x - 10\pi \tan x \sin^2 x = x - 2\pi$

$5\pi(2 \tan x - 2 \tan x \sin^2 x) = x - 2\pi$

$\qquad\qquad\qquad 5\pi \sin 2x = x - 2\pi$

$\qquad\qquad\qquad 5 \sin 2x = \dfrac{x}{\pi} - 2$

$\qquad\qquad\qquad\qquad y = \dfrac{x}{\pi} - 2$

Garis lurus yang perlu dilukis ialah $y = \dfrac{x}{\pi} - 2$.

Bilangan penyelesaian = 2

6. (a) $59.5 + \left[\dfrac{\frac{3}{4}(3 + n + 6 + 8 + 2) - (3 + n + 6)}{8}\right](5) = 62$

$\qquad\qquad\left[\dfrac{\frac{3}{4}(n + 19) - n - 9}{8}\right](5) = 2.5$

$\qquad\qquad\qquad \dfrac{3}{4}(n + 19) - n - 9 = 4$

$\qquad\qquad\qquad \dfrac{3}{4}(n + 19) - n = 13$

$\qquad\qquad\qquad -\dfrac{1}{4}n + \dfrac{57}{4} = 13$

$\qquad\qquad\qquad\qquad \dfrac{1}{4}n = \dfrac{5}{4}$

$\qquad\qquad\qquad\qquad\quad n = 5$

(b)

Jisim (kg)	Kekerapan (f)	Titik tengah (x)	fx	fx^2
45 – 49	3	47	141	6 627
50 – 54	5	52	260	13 520
55 – 59	6	57	342	19 494
60 – 64	8	62	496	30 752
65 – 69	2	67	134	8 978
	$\sum f = 24$		$\sum fx = 1\,373$	$\sum fx^2 = 79\,371$

Sisihan piawai $= \sqrt{\dfrac{79\,371}{24} - \left(\dfrac{1\,373}{24}\right)^2} = 5.859$ kg

7. (a) $CD = BC = \dfrac{2}{3}(60) = 40$ m

$\qquad \text{kos } \angle BCD = \dfrac{40}{60}$

$\qquad\qquad \angle BCD = 0.841$ radian

$\qquad S_{BD} = 40(0.841)$

$\qquad\qquad = 33.64$ m

$\qquad S_{AC} = 30(3.142)$

$\qquad\qquad = 94.26$ m

$\qquad \text{Perimeter} = 33.64 + 94.26 + 40 + 20$

$\qquad\qquad\qquad\quad = 187.9$ m

(b) Luas $= \dfrac{1}{2}(30)^2(3.142) - \dfrac{1}{2}(40)^2(0.841)$

$\qquad\qquad = 741.1$ m^2

8. (a) $\dfrac{dy}{dx} = k - 2x$

$\qquad$ Pada titik A, $k - 2(3) = 2$

$\qquad\qquad\qquad\qquad k = 8$

$\qquad n = 8(3) - 3^2$

$\qquad\quad = 15$

(b) Pada $(3, 15)$, $\quad 15 = 2(3) + c$
$$c = 9$$
Luas rantau berlorek
$$= \int_0^3 [2x + 9 - (8x - x^2)]\, dx$$
$$= \int_0^3 (x^2 - 6x + 9)\, dx$$
$$= \left[\frac{x^3}{3} - 3x^2 + 9x\right]_0^3$$
$$= \left[\frac{3^3}{3} - 3(3)^2 + 9(3)\right] - (0)$$
$$= 9 \text{ unit}^2$$

(c) Apabila $y = 0$, $\quad 8x - x^2 = 0$
$$x(8 - x) = 0$$
$$x = 0 \text{ atau } x = 8$$
Isi padu $= \pi \int_0^8 y^2\, dx$
$$= \pi \int_0^8 (8x - x^2)^2\, dx$$
$$= \pi \int_0^8 (64x^2 - 16x^3 + x^4)\, dx$$
$$= \pi \left[\frac{64}{3}x^3 - 4x^4 + \frac{1}{5}x^5\right]_0^8$$
$$= \pi \left\{\left[\frac{64}{3}(8)^3 - 4(8)^4 + \frac{1}{5}(8)^5\right] - [0]\right\}$$
$$= 1\,092\frac{4}{15}\pi \text{ unit}^3$$

9. (a) $\overrightarrow{OA} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$
$$= (\overrightarrow{AO} + \overrightarrow{OB}) + \overrightarrow{AD}$$
$$= (3\underset{\sim}{i} - 4\underset{\sim}{j} + 6\underset{\sim}{i}) + (\overrightarrow{AO} + \overrightarrow{OD})$$
$$= 9\underset{\sim}{i} - 4\underset{\sim}{j} + (3\underset{\sim}{i} - 4\underset{\sim}{j} + 10\underset{\sim}{j})$$
$$= 12\underset{\sim}{i} + 2\underset{\sim}{j}$$

(b) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$
$$= (-3\underset{\sim}{i} + 4\underset{\sim}{j}) + (12\underset{\sim}{i} + 2\underset{\sim}{j})$$
$$= 9\underset{\sim}{i} + 6\underset{\sim}{j}$$
Koordinat C ialah $(9, 6)$.

(c) $\overrightarrow{CD} = \overrightarrow{BA}$
$$= \overrightarrow{BO} + \overrightarrow{OA}$$
$$= -6\underset{\sim}{i} + (-3\underset{\sim}{i} + 4\underset{\sim}{j})$$
$$= -9\underset{\sim}{i} + 4\underset{\sim}{j}$$
$$|\overrightarrow{CD}| = \sqrt{(-9)^2 + 4^2}$$
$$= \sqrt{97} \text{ unit}$$
Vektor unit $= -\dfrac{9}{\sqrt{97}}\underset{\sim}{i} + \dfrac{4}{\sqrt{97}}\underset{\sim}{j}$

10. (a) (i) $P(0 < X < 3) = 1 - h - k$

(ii) $P(X = 1) = \dfrac{54}{125}$
$${}^3C_1 p^1 q^2 = \frac{54}{125}$$
$$3pq^2 = \frac{54}{125}$$
$$pq^2 = \frac{18}{125} \qquad \text{......} \ ①$$
$$P(X = 2) = \frac{36}{125}$$
$${}^3C_2 p^2 q = \frac{36}{125}$$
$$3p^2 q = \frac{36}{125}$$
$$q = \frac{12}{125p^2} \qquad \text{......} \ ②$$
Gantikan ② ke dalam ①.
$$p\left(\frac{12}{125p^2}\right)^2 = \frac{18}{125}$$
$$\frac{144}{15\,625p^3} = \frac{18}{125}$$
$$p^3 = \frac{8}{125}$$
$$p = \frac{2}{5}$$

(b) (i) $P(4\,150 < X < 4\,280)$
$$= \left(\frac{4\,150 - 4\,000}{200} < Z < \frac{4\,280 - 4\,000}{200}\right)$$
$$= P(0.75 < Z < 1.40)$$
$$= P(Z > 0.75) - P(Z > 1.40)$$
$$= 0.2266 - 0.0808$$
$$= 0.1458$$

(ii) $\qquad P(X > x) = \dfrac{48}{60}$
$$P\left(Z > \frac{x - 4\,000}{200}\right) = 0.8$$
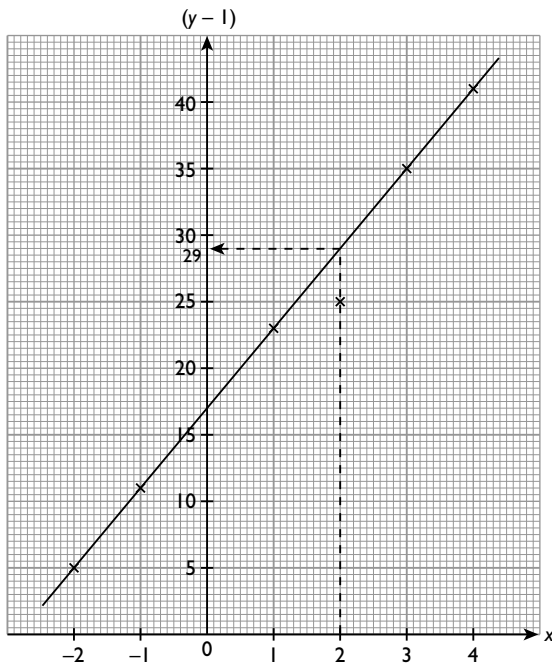Daripada jadual taburan normal,
$P(Z > 0.842) = 0.2$.
$$-\frac{x - 4\,000}{200} = 0.842$$
$$x = 3\,831.60$$

11. (a)

x	-2	-1	1	2	3	4
$y - 1$	5	11	23	25	35	41

(b)
$$py = x + p + q$$
$$py - p = x + q$$
$$p(y - 1) = x + q$$
$$y - 1 = \frac{1}{p}x + \frac{q}{p}$$

(i) $\dfrac{1}{p}$ = Kecerunan graf

$$\frac{1}{p} = 6$$

$$p = \frac{1}{6}$$

$\dfrac{q}{p}$ = Pintasan-$(y - 1)$

$$\frac{q}{\frac{1}{6}} = 17$$

$$q = 2.83$$

(ii) Daripada graf, terdapat satu nilai yang tidak betul bagi $(y - 1)$ apabila $x = 2$.

Nilai yang betul bagi $(y - 1)$ ialah 29. Maka, $y = 30$.

12. (a) $\frac{1}{2}(2.8)(2.8) \sin \angle RQS = 3$

$$\angle RQS = 49.93°$$
$$RS^2 = 2.8^2 + 2.8^2 - 2(2.8)(2.8) \text{ kos } 49.93°$$
$$RS^2 = 5.586$$
$$RS = 2.36 \text{ m}$$

(b) Katakan M ialah titik tengah RS.
$$PM^2 = PR^2 - MR^2$$
$$= 2^2 - 1.18^2$$
$$PM = 1.615 \text{ m}$$
$$\frac{PQ}{\sin 55°} = \frac{1.615}{\sin 24°}$$
$$PQ = 3.25 \text{ m}$$

(c)
$$2^2 = 3.25^2 + 2.8^2 - 2(3.25)(2.8)$$
$$\text{kos } \angle PQR$$
$$\text{kos } \angle PQR = 0.7913$$
$$PQR = 37.69°$$

$$\text{Luas} = \frac{1}{2}(3.25)(2.8) \sin 37.69°$$
$$= 2.782 \text{ m}^2$$

13. (a) $a = \dfrac{dv}{dt}$

$$= -2t + 7$$
Pada halaju maksimum, $a = 0$.
$$-2t + 7 = 0$$
$$t = 3.5$$

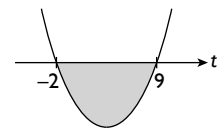Halaju maksimum, $v = -(3.5)^2 + 7(3.5) + 18$
$$= 30.25 \text{ m s}^{-1}$$

(b) Apabila zarah itu bergerak ke arah kanan, $v > 0$.

$$-t^2 + 7t + 18 > 0$$
$$t^2 - 7t - 18 < 0$$
$$(t + 2)(t - 9) < 0$$
$$-2 < t < 9$$

Oleh sebab $t \geqslant 0$, julat nilai t ialah $0 \leqslant t < 9$.
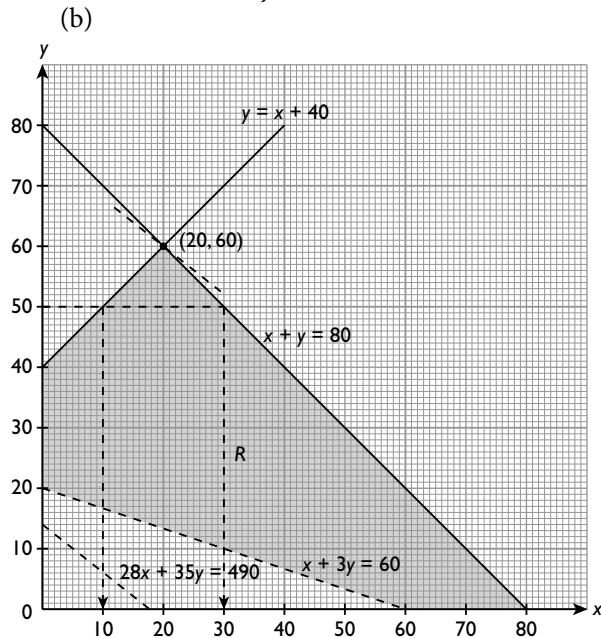
(c) Jumlah jarak yang dilalui
$$= \int_0^4 (-t^2 + 7t + 18) \, dt$$
$$= \left[-\frac{1}{3}t^3 + \frac{7}{2}t^2 + 18x \right]_0^4$$
$$= \left[-\frac{1}{3}(4)^3 + \frac{7}{2}(4)^2 + 18(4) \right] - 0$$
$$= 106\frac{2}{3} \text{ m}$$

14. (a) I   : $x + y \leqslant 80$

II : $y \leqslant x + 40$

III : $10x + 30y > 10 \times 60$

$x + 3y > 60$

(b)

(c) (i) Apabila $y = 50$, nilai minimum integer bagi $x = 10$.

Bilangan minimum jawatan kosong untuk bahagian teknikal = 10.

(ii) Jumlah gaji yang perlu dibayar,

$P = 2\,800x + 3\,500y$

Katakan $2\,800x + 3\,500y = 49\,000$.

Lukis garis lurus $28x + 35y = 490$ sebagai rujukan.

Titik optimum = (20, 60)

Jumlah maksimum gaji yang perlu dibayar

= RM$[2\,800(20) + 3\,500(60)]$

= RM266 000

15. (a) $k = 125 \times \dfrac{140}{100}$

$= 175$

(b) Sudut yang mewakili bahan mentah A dan B

$= \dfrac{180° - 72°}{2}$

$= 54°$

Untuk bahan mentah A,

$120 \times \dfrac{I_A}{100} = 138$

$I_A = 115$

Untuk bahan mentah C, $I_C = 100$

Untuk bahan mentah D,

$110 \times \dfrac{I_D}{100} = 165$

$I_D = 150$

Indeks gubahan, $\bar{I}$

$= \dfrac{54(115) + 54(140) + 72(100) + 180(150)}{360}$

$= 133.25$

$\therefore x = 33.25$

(c) $\dfrac{18}{P_{2016}} \times 100 = 133.25$

$P_{2016} = $ RM13.51