## (4) Cloned SPM Question (2006, Paper 1)

A set of positive integers consists of 1,4 and $k$. The variance for this set of integers is 6 . Find the value of $k$.

## Solution

Mean, $\bar{x}=\frac{\Sigma x}{N}$

$$
\begin{aligned}
& =\frac{1+4+k}{3} \\
& =\frac{5+k}{3}
\end{aligned}
$$

Variance, $\sigma^{2}=\frac{\Sigma x^{2}}{N}-\bar{x}^{2}$

$$
\begin{aligned}
& =\frac{1^{2}+4^{2}+k^{2}}{3}-\left(\frac{5+k}{3}\right)^{2} \\
& =\frac{17+k^{2}}{3}-\frac{25+10 k+k^{2}}{9} \\
& =\frac{51+3 k^{2}-25-10 k-k^{2}}{9} \\
& =\frac{2 k^{2}-10 k+26}{9}
\end{aligned}
$$

It is given $\sigma^{2}=6$, thus $\frac{2 k^{2}-10 k+26}{9}=6$

$$
\begin{aligned}
2 k^{2}-10 k+26 & =54 \\
2 k^{2}-10 k-28 & =0 \\
k^{2}-5 k-14 & =0 \\
(k+2)(k-7) & =0 \\
k=-2 \quad \text { or } \quad k & =7
\end{aligned}
$$

Since $k$ is positive, then $k=7$.

## Pointers

- Use the formula, $\sigma^{2}=\frac{\Sigma x^{2}}{N}-\bar{x}^{2}$ to find the variance of the data. Then equate it to 6 to solve for the value of $k$.
- Two values of $k$ will be obtained. Take the positive value of $k$.

Cloned SPM Question (2006, Paper 2)
The table shows the frequency distribution of the masses of books on a bookshelf.

| Mass (g) | Number of books |
| :---: | :---: |
| $30-39$ | 2 |
| $40-49$ | 3 |
| $50-59$ | 6 |
| $60-69$ | 10 |
| $70-79$ | $n$ |
| $80-89$ | 1 |

(a) It is given that the median mass of the books is 61.5 g . Calculate the value of $n$.
(b) Using a scale of 2 cm to 10 g on the horizontal axis and 2 cm to 2 books on the vertical axis, draw a histogram to represent the frequency distribution of the masses. Find the modal mass.
(c) What is the modal mass if the mass of each book is increased by 8 g .

## Solution

(a) Total frequency, $N=22+n$

It is given that the median is 61.5 g .
Hence, the median class is in the class $60-69$, where the lower boundary $L$ is 59.5 g .
Using the formula for median, $m=L+\left(\frac{\frac{N}{2}-F}{f_{m}}\right) C$
$61.5=59.5+\left(\frac{\frac{22+n}{2}-11}{10}\right) \times 10$
$61.5=59.5+\left(11+\frac{n}{2}-11\right)$
$2=\frac{n}{2}$
$n=4$
(b) The graph below is a histogram which represents the frequency distribution of the masses of the books.


From the histogram above, the mode is 63.5 g .
(c) If the mass of each book is increased by 8 g , then the modal mass of the books is also increased by the same amount, that is the new mode is 71.5 g .

## Pointers

- (a): Use the formula, $m=L+\left(\frac{\frac{N}{2}-F}{f_{m}}\right) C$ to find the median of the data and equate it to
61.5 to find the value of $n$.
- (b): A histogram is constructed by marking the boundaries of each class along the horizontal axis and the frequencies along the vertical axis. The mode of the data is situated in the rectangle of the modal class.
- (c): When each value of the data is increased uniformly with a value $c$, the mode of the data is also increased by the same value $c$.

