The diagram shows the graph of a quadratic function $y=f(x)$. The straight line $y=-9$ is a tangent to the curve $y=f(x)$.

(a) Write the equation of the axis of symmetry of the curve.
(b) Express $f(x)$ in the form $(x+h)^{2}+k$, where $h$ and $k$ are constants.

## Solution

(a) The axis of symmetry passes through the midpoint of the line joining $(1,0)$ and $(7,0)$. Thus, the equation of the axis of symmetry is

$$
\begin{aligned}
& x=\frac{1+7}{2} \\
& x=4
\end{aligned}
$$

(b) Minimum value of the function, $k=-9$

When $x=4, \quad 4+h=0$

$$
h=-4
$$

Thus, $f(x)=(x-4)^{2}-9$.

## Pointers

- The axis of symmetry must pass through the $x$-axis at the midpoint between the two roots, 1 and 7.
- As $(x+h)^{2}>0$, the minimum value of the function is $k$ when $x+h=0$.


## Cloned SPM Question (2006, Paper 1)

Find the range of values of $x$ for which $(3 x-1)(x+5)>5+x$.

## Solution

$$
\begin{aligned}
(3 x-1)(x+5) & >5+x \\
3 x^{2}+15 x-x-5 & >5+x \\
3 x^{2}+13 x-10 & >0 \\
(3 x-2)(x+5) & >0
\end{aligned}
$$

When $(3 x-2)(x+5)=0$,

$$
x=\frac{2}{3} \text { or } x=-5
$$



Thus, the range of values of $x$ which satisfies the inequality $(3 x-1)(x+5)>5+x$ is $x<-5$ or $x>\frac{2}{3}$.

## Pointers

- The quadratic inequality has to be rearranged into the form $a x^{2}+b x+c>0$ before factorising.
- A sketch of the graph is needed to determine the range of values of $x$ for $f(x)>0$.
- Remember not to make the mistake that if $(3 x-2)(x+5)>0$, then $x>\frac{2}{3}$ and $x>-5$.

