CHAPTER 5: INDICES AND LOGARITHMS

Cloned SPM Question (2006, Paper 1)

Solve the equation $27^{3x-5} = \dfrac{1}{\sqrt{9^{3-x}}}$.

Solution

$$
\begin{aligned}
27^{3x-5} &= \frac{1}{\sqrt{9^{3-x}}} \\
3^{3(3x-5)} &= \frac{1}{\left[3^{2(3-x)}\right]^{\frac{1}{2}}} \\
&= \frac{1}{3^{(3-x)}} \\
&= 3^{-(3-x)}
\end{aligned}
$$

$3(3x-5) = -(3-x)$ ← Equate the indices.

$$
\begin{aligned}
9x - 15 &= x - 3 \\
8x &= 12 \\
x &= \frac{3}{2}
\end{aligned}
$$

Pointers

- Since 27 and 9 are both powers of 3, both sides of the equation can be expressed in index form as powers of 3.
- When the bases on both sides of the equation are the same, the indices on both sides of the equation must be the same.
- The equation is then solved by equating the indices.

Cloned SPM Question (2006, Paper 1)

Given that $\log_3 pq = 2 + 3\ \log_3 p - \log_3 q$, express p in terms of q.

Solution

$$\log_3 pq = 2 + 3\ \log_3 p - \log_3 q$$

$$\log_3 pq - \log_3 p^3 + \log_3 q = 2$$

$$\log_3 \frac{pq^2}{p^3} = 2$$

$$\frac{q^2}{p^2} = 3^2$$

$$p = \frac{q}{3}$$

> ***Pointers***
> - Since all the terms involving logarithms are to the same base, that is base 3, they can be combined as a single term using the laws of logarithms.
> - The equation is then changed into index form. Then make p the subject of the equation.

Cloned SPM Question (2006, Paper 1)

Solve the equation $3 + \log_2(x-1) = \log_2 x$.

Solution

$$3 + \log_2(x-1) = \log_2 x$$

$$3 = \log_2 x - \log_2(x-1)$$

$$\log_2 \frac{x}{x-1} = 3$$

$$\frac{x}{x-1} = 2^3$$

$$x = 8x - 8$$

$$7x = 8$$

$$x = \frac{8}{7}$$

> ***Pointers***
> - Combine all terms involving logarithms into a single term using the laws of logarithms.
> - Then, change the equation from the logarithmic form to the index form, which can be easily solved.