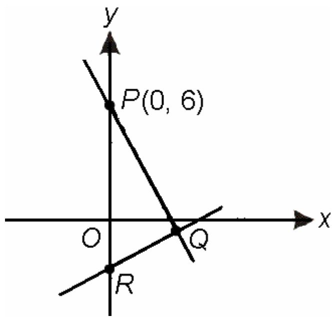
# CHAPTER 6: COORDINATE GEOMETRY

## Cloned SPM Question (2006, Paper 1)

The diagram shows the straight line *PQ* which is perpendicular to the straight line *QR* at the point *Q*.




The equation of the straight line *RQ* is $2y = x - 5$. Find the coordinates of *Q*.

***Solution***

Equation of *QR*: $2y = x - 5$ …… (1)

Gradient of $QR = \frac{1}{2}$

Since *PQ* is perpendicular to *QR*, then the gradient of *PQ* is $-2$.

Equation of *PQ*: $y = -2x + 6$ …… (2)

Substitute (2) into (1).

$$2(-2x + 6) = x - 5$$
$$-4x + 12 = x - 5$$
$$17 = 5x$$
$$x = 3.4$$

Substitute $x = 3.4$ into (2).

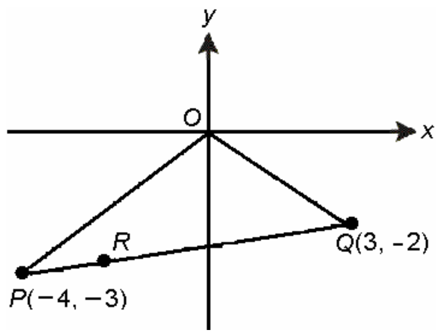
$$y = -2(3.4) + 6$$
$$= -6.8 + 6$$
$$= -0.8$$

Thus, the coordinates of *Q* are (3.4, –0.8).

***Pointers***
- If *PQ* is perpendicular to *QR*, then the product of their gradients = –1.
- From the gradient of *QR*, find the gradient of *PQ*. Write the equation of *PQ* and solve the two equations to find the point of intersection, *Q*.

## Cloned SPM Question (2006, Paper 2)

The diagram shows the triangle *OPQ*, where *O* is the origin. Point *R* lies on the straight line *PQ*.




(a) Calculate the area, in unit$^2$, of triangle *OPQ*.
(b) Given that *PR* : *RQ* = 1 : 3, find the coordinates of *R*.
(c) A point *S* moves such that its distance from *P* is always twice its distance from *Q*.
    (i) Find the equation of the locus of *P*.
    (ii) Hence, determine whether or not this locus intersects the *y*-axis.

***Solution***

(a) Area of $\Delta OPQ = \frac{1}{2}\begin{vmatrix} 0 & -4 & 3 & 0 \\ 0 & -3 & -2 & 0 \end{vmatrix}$

$$= \frac{1}{2}\left|8-(-9)\right|$$
$$= 8.5 \text{ unit}^2$$

(b) Given *PR* : *RQ* = 1 : 3.

Coordinates of $R = \left(\frac{3(-4)+1(3)}{4}, \frac{3(-3)+1(-2)}{4}\right)$

$$= \left(-\frac{9}{4}, -\frac{11}{4}\right)$$

(c) (i) Let the coordinates of *S* be (*x*, *y*).

$$SP = 2SQ$$
$$\sqrt{(x+4)^2+(y+3)^2} = 2\sqrt{(x-3)^2+(y+2)^2}$$
$$(x+4)^2+(y+3)^2 = 4[(x-3)^2+(y+2)^2]$$
$$x^2+8x+16+y^2+6y+9 = 4(x^2-6x+9+y^2+4y+4)$$
$$3x^2+3y^2-32x+10y+27=0$$

Thus, the equation of the locus of *S* is $3x^2+3y^2-32x+10y+27=0$.

(ii) When the locus of $P$ intersects the $y$-axis, $x = 0$.
Hence, $3y^2 + 10y + 27 = 0$.
$b^2 - 4ac = 10^2 - 4(3)(27)$
$= -224$

Since $b^2 - 4ac < 0$, the equation has no roots.
Thus, the locus of $P$ does not intersect the $y$-axis.

***Pointers***

- Use the formula $\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$ to find the area of the triangle.
- Use the formula $\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$ to find the coordinates of $R$.
- Use the distance formula to find the locus of $S$.
- To determine whether the locus of $S$ intersects the $y$-axis, substitute $x = 0$ into the equation and use the discriminant $b^2 - 4ac$ to determine if there is any root.