## Paper 2

## Solution to Question 9


(a) (i) Let the coordinates of $P$ be $(x, y)$.

$$
\begin{aligned}
P C & =A C \\
\sqrt{(x+1)^{2}+(y-4)^{2}} & =\sqrt{(-1+4)^{2}+(4-0)^{2}} \\
(x+1)^{2}+(y-4)^{2} & =(-1+4)^{2}+(4-0)^{2} \\
x^{2}+2 x+1+y^{2}-8 y+16 & =25 \\
x^{2}+y^{2}+2 x-8 y-8 & =0
\end{aligned}
$$

Thus, the equation of the locus of point $P$ is $x^{2}+y^{2}+2 x-8 y-8=0$.
(ii) The locus of point $P$ passes through $B(-5, k)$.

Substitute $x=-5$ and $y=k$ into the equation.

$$
\begin{aligned}
(-5)^{2}+k^{2}+2(-5)-8 k-8 & =0 \\
25+k^{2}-10-8 k-8 & =0 \\
k^{2}-8 k+7 & =0 \\
(k-1)(k-7) & =0 \\
k=1 \quad \text { or } \quad k & =7
\end{aligned}
$$

(b) Since $A D$ is a tangent to the circle, then $A D$ is perpendicular to radius $A C$.

Gradient of $A C=\frac{4-0}{-1+4}=\frac{4}{3}$
So, gradient of $A D=-\frac{3}{4}$
Equation of $A D: \quad y-0=-\frac{3}{4}(x+4)$

$$
4 y+3 x+12=0
$$

At point $D, x=0$.

$$
\begin{aligned}
4 y+3(0)+12 & =0 \\
y & =-3
\end{aligned}
$$

So, the coordinates of point $D$ are $(0,-3)$.
$\triangle O A D$ is a right-angled triangle.

$$
\text { Area of } \begin{aligned}
\triangle O A D & =\frac{1}{2}(4)(3) \\
& =6 \mathrm{unit}^{2}
\end{aligned}
$$

